UTD Analysis of an Inclined Rectangular Aperture Antenna Mounted on an Open-Ended Cavity Excited by a Probe

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Abstract

This paper presents the Uniform Theory of Diffraction (UTD) analysis of an inclined rectangular aperture antenna mounted on an open-ended cavity excited by a probe. This work is taken into account the uniform theory of diffraction to improve the results from the previous works by using the dyadic Green function approach. The geometry of the problem is made up of the rectangular aperture mounted on inclined open-ended cavity. This structure is fed by a probe inside the cavity. The study starts with the consideration of the geometry of the diffraction such as the incident field, the shadow boundaries and the relative parameters. Subsequently, the diffracted fields and the total fields that radiated from the antenna are evaluated to precise the radiation pattern from the aperture antenna. The results in this work are significant to further design the aperture antenna from the open-ended cavity fed by probe in the microwave and the wireless communication applications.

1. INTRODUCTION

At presently, there are many researches for the antenna development in order to improve the performance of the antenna to meet the demand of the microwave and wireless communication systems. In this paper, the antenna is the rectangular aperture mounted on an inclined open-ended cavity with the probe fed inside the cavity. This work is continued from the works in [1-4] to improve the results from the previous ones. In the previous work, the antenna is simulated by using the Numerical Electromagnetic Code (NEC) [1] whose the results are not flexible for the parametric study. Therefore, the analysis of this antenna with the normal aperture [2-3] and the inclined aperture [4] is performed using the dyadic Green function approach, respectively. To precise the result in those works, the diffraction on a rectangular aperture antenna mounted on an open-ended cavity excited by a probe is investigated in this paper, in case of the inclined aperture. The diffraction theory used in paper is the two-dimensional Uniform Theory of Diffraction (UTD) and the geometry of the problem is illustrated in Fig.1. It is made up of the rectangular aperture mounted on one side of the cavity and inclined it. The other walls of the cavity are perfect electric conductor that is the same as the normal aperture in [2-3]. The differences between them are the dimension of the cross section of the aperture and the origin of *xyz* coordinates at the middle of aperture.

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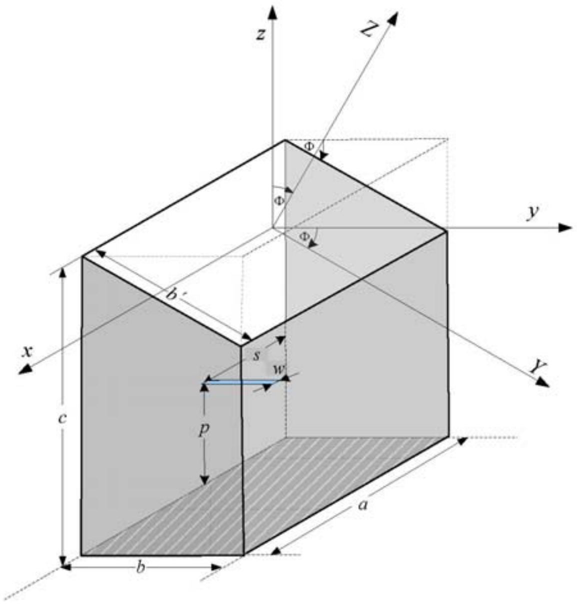
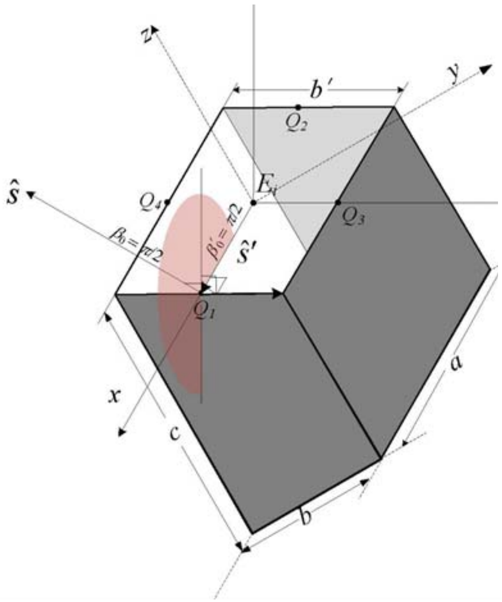
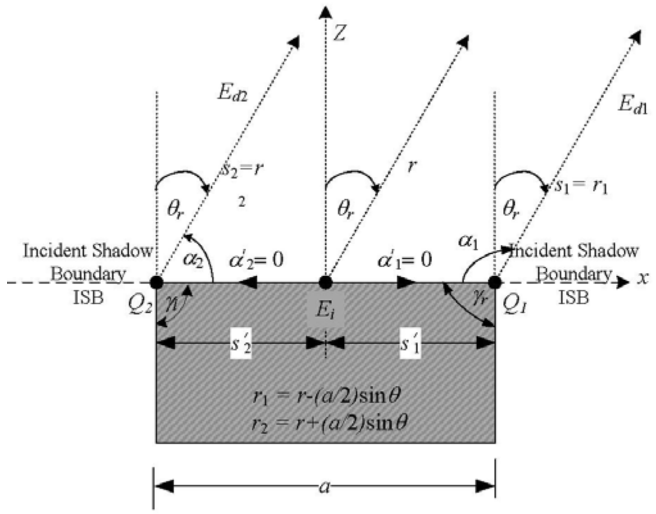


Fig. 1: An inclined rectangular aperture antenna fed by probe

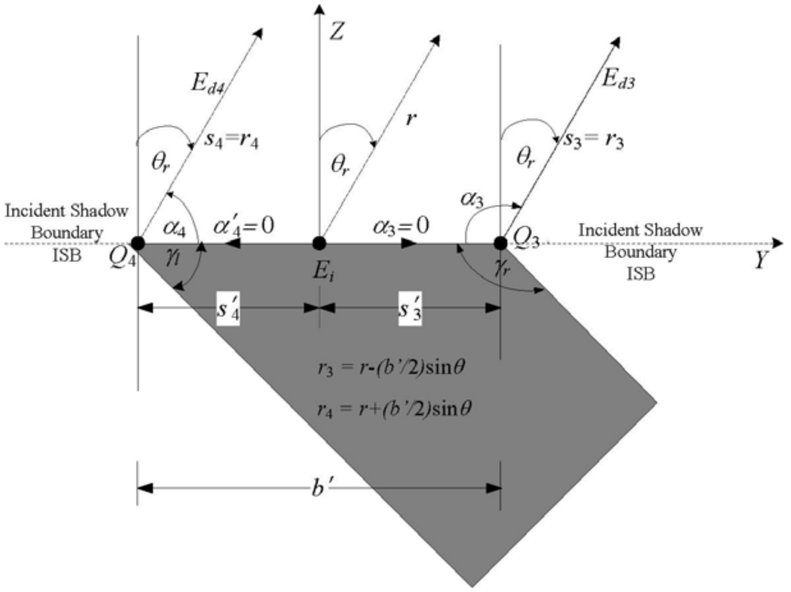
Fig. 1 illustrates the configuration of the inclined open-ended cavity used for this work, where a and b parameters are the dimension of the aperture cross section with the length of the cavity c, the length of the probe l located at (s, 0, p) with the width of the probe w.
In the analysis of the inclined rectangular aperture from open-ended cavity, the principle of the rotation of axis is used to solve the problem. The procedure of the analysis for the inclined rectangular aperture is discussed in more detail in [4].



(a) Three-dimensional view



(b) Two-dimensional view for *xZ*-plane



(c) Two-dimensional view for *YZ*-plane

Fig. 2: Diffraction on the antenna structure

2. Geometry Of The Diffraction Analysis

In order to analyze the problem using the two-dimensional UTD diffraction, the shadow boundaries and relative parameters are considered first. From the geometry of the problem in the fig.1 and consider in the *xYZ* coordinates. It can be defined the shadow boundaries and incident field that radiated from the inclined rectangular aperture antenna excited by a probe inside the open-ended cavity as shown in fig. 2. From fig. 2(a), it illustrates the diffracted ray on the structure of an antenna at Q_1. The angle of the incident ray at the edge (β_0') and the angle of the diffracted ray at the edge (β') are identical that equal to $\pi/2$. At the other point, the β'_0 and β_0 are similar at Q_1.

For fig. 2 (b) and (c), the two-dimensional view of the antenna in *xZ*-plane and *YZ* –plane are shown with diffracted ray, shadow boundaries and relative parameters. In each of the both plane, there are two diffracted points with the angle of the incident ray (α': measured in a plane perpendicular to the edge) is zero.

In the *xZ*-plane, the both of interior angle (γ_r and γ_l) equal to $\pi/2$, thus *n* equals to 1.5 and the angles of the diffracted ray (α) are in range $0 \le \alpha \le 3\pi/2$. But in the *YZ*-plane, the interior angle (γ_r and γ_l) are not equal which $\gamma_r = \Phi + (\pi/2)$ and $\gamma_l = \Phi - (\pi/2)$, therefore, the n-parameters equal to 1.5 - (Φ/π) and 1.5 + (Φ/π), respectively. For the angles of the diffracted ray, α_3 are in range $0 \le \alpha_3 \le (\pi/2) + \Phi$ and α_4 are in range $0 \le \alpha_4 \le 2\pi - \Phi$. The summary of the various angles and value of *n* in the geometry of the diffraction on the structure of the antenna are listed in Table 1

TABLE 1: SUMMARY OF THE VARIOUS ANGLES AND VALUE OF N

Parameters	Diffracted ray point			
	Q_1	Q_2	Q_3	Q_4
β'_0	$\pi/2$	$\pi/2$	$\pi/2$	$\pi/2$
β_0	$\pi/2$	$\pi/2$	$\pi/2$	$\pi/2$
α'	0	0	0	0
α	0 to $3\pi/2$	0 to $3\pi/2$	0 to $(\pi/2)+\Phi$	0 to $2\pi-\Phi$
γ	$\pi/2$	$\pi/2$	$\Phi+(\pi/2)$	$\Phi-(\pi/2)$
n	1.5	1.5	1.5 - (Φ/π)	1.5 + (Φ/π)

Then, the two-dimensional UTD diffraction coefficients will be considered, for the structure in fig. 2, they can be written as

$$D_s(L,\alpha,n) = 0 \ , \tag{1}$$

$$D_h(L,\alpha,n) = \frac{e^{-j\pi/4} F\left[kLa(\alpha)\right]}{n\sqrt{2\pi k}} \left(\frac{2\sin(\pi/n)}{\cos(\pi/n) - \cos(\alpha/n)} \right), \tag{2}$$

where D_s and D_h are the soft and hard two-dimensional UTD diffraction coefficients, respectively, and L is the distance parameter, can be determined for several types of illumination as

$$L = \begin{cases} s\sin^2\beta_0 & \text{for plane wave incidence} \\ \dfrac{s's}{s+s'} & \text{for cylindrical wave incidence} \\ \dfrac{s's\sin^2\beta_0}{s+s'} & \text{for conical and spherical wave incidence.} \end{cases}$$

The definition of function $a(\alpha)$ can be written as

$$a(\alpha) = 2\cos^2\left(\frac{2n\pi N - \alpha}{2}\right), \tag{3}$$

in which N is the integer, in this case, it is defined as zero ($N = 0$).

For the function $F[x]$ is called as the transition function, it is can be defined as

$$F(x) = 2j\sqrt{x}e^{jx} \int_{\sqrt{x}}^{\infty} e^{-ju^2} du \, . \tag{4}$$

Note that the integral part of the transition function resembles a Fresnel integral. The transition function, function $a(\alpha)$ and their relative parameters are discussed in more detail in [5].

Therefore, the diffracted field can be written as

$$\overline{E}_d(s) = \overline{E}_i(Q_D) \cdot \overline{\overline{D}} A(s) e^{-jks}, \tag{5}$$

where $\overline{E}_i(Q_D)$ is the incident field at the diffracted point Q_D on the edge, $\overline{\overline{D}}$ is the diffraction coefficient, for this case, can be given in eq.1-2 and $A(s)$ is the spreading factor, can be classified as

$$A(s) = \begin{cases} \dfrac{1}{\sqrt{s}} & \text{for plane and conical wave incidence} \\ \dfrac{1}{\sqrt{s\sin\beta_0}} & \text{for cylindrical wave incidence} \\ \sqrt{\dfrac{s'}{s(s+s')}} & \text{for spherical wave incidence.} \end{cases}$$

3. RADIATED FIELD FROM THE APERTURE

Once the shadow boundaries are known, the related parameters and the two-dimensional UTD diffraction coefficients are determined. Therefore, the diffracted fields and the total field can be evaluated in this section.

A. *xZ plane* $P(r, \theta_r, \phi_r = 0)$

In the view of symmetrical structure in this plane, the fields can be considered only in the region $0 \le \theta_r \le \pi$. From fig.2 (b), there are two diffracted points that are Q_1 and Q_2. The diffracted angles at both of two diffracted points (α_1 and α_2) can be expressed in θ term as

$$\alpha_1 = \frac{\pi}{2} + \theta_r, \tag{6}$$

$$\alpha_2 = \frac{\pi}{2} - \theta_r \cdot \tag{7}$$

Therefore, the diffracted fields can be considered as follows

-The diffracted field at Q_1

The diffracted field at Q_1 can be expressed as

$$E_{d1}(\theta_r) = E_i(Q_1)\frac{D_h(L,\alpha_1)}{2}A(s)e^{-jkr_1}. \tag{8}$$

It is found that the diffraction coefficient is divided by two because $\alpha' = 0$ in this case, so-called grazing incidence, the diffraction coefficients are divided by two.

- The diffracted field at Q_2

The diffracted field at Q_2 can be expressed as

$$E_{d2}(\theta_r) = E_i(Q_2)\frac{D_h(L,\alpha_2)}{2}A(s)e^{-jkr_2}. \tag{9}$$

-The second-order diffracted field at Q_1

In fact, there is discontinuity in the pattern at $\theta_r = \pi/2$, the transition at the shadow boundary. To solve this problem, the high-order diffraction terms are included. Therefore, in this case, the second-order diffracted term is necessary to solve this problem. The second-order diffracted field that emanates from Q_1 due to illumination from E_{d2} can be expressed as

$$E_{12}(\theta_r) = E_{d2}(Q_1)\frac{D_h(L,\alpha_{12})}{2}\frac{e^{-jkr_{12}}}{\sqrt{r_{12}}}. \tag{10}$$

-The incident field E_{d2} at Q_1

The incident field E_{d2} at Q_1can be expressed as

$$E_{d2}(Q_1) = E_i(Q_2)\frac{D_h(L,\alpha_2 = 0)}{2}\frac{e^{-jka}}{\sqrt{a}}. \tag{11}$$

-The total field at an observation point

The total field at an observation point then becomes

$$E_t = \begin{cases} E_i + E_{d1} + E_{d2} + E_{d12} & \text{for } 0 < \theta_r \le \pi/2 \\ E_{d1} + E_{d12} & \text{for } \pi/2 < \theta_r \le \pi, \end{cases} \tag{12}$$

where E_i is the field that radiated from the inclined aperture antenna at an observation point $P(r,\theta_r,\phi_r = 0)$, can be written as

$$E_i(\theta_r) = \begin{cases} E_{ap}(\theta_r)\dfrac{e^{-jkr}}{\sqrt{r}} & \text{for } \theta_r \le \pi/2 \\ 0 & \text{for } \theta_r > \pi/2, \end{cases} \tag{13}$$

and $E_{ap}(\theta)$ is the radiated field from the inclined rectangular aperture mounted on one side of the cavity [4].

The value of the parameters from eq.8-12 can be listed in table 2.

TABLE 2 : SUMMARY OF PARAMETERS IN EQUATION 8-12 FOR *XZ*-PLANE

	Parameters		
Diffracted field	$A(s)$	L	$a(\alpha)$
A. The diffracted field at Q_1	$1/\sqrt{r}$	$a/2$	$1-\sin\theta$
B. The diffracted field at Q_2	$1/\sqrt{r}$	$a/2$	$1+\sin\theta$
C. The second-order diffracted field at Q_1		a	$1-\sin\theta$
D. The incident field E_{d2} at Q_1		$a/3$	2

Note: assume that the incident wave as the cylindrical wave and the incident fields are given by

$$E_i(Q_1) = E_i(Q_2) = E_{ap}(\theta_r)\frac{e^{-jk(a/2)}}{\sqrt{a/2}}. \tag{14}$$

B. YZ- plane $P(r,\theta_r,\phi_r = \pi/2)$

From fig.2 (c), there are two diffracted points that are Q_3 and Q_4. The structure of the antenna in this plane is not symmetry and the interior angles of the wedge at Q3 and Q_4 are not identical. Therefore, the fields are considered in all of the ranges ($0 \le \theta_r \le 2\pi$).

The diffracted angles at both of two diffracted points (α_3 and α_4) can be expressed in θ_r term as

-At Q_3

The diffracted angle α_3 is defined in two ranges as

Range 1: $0 \le \theta_r \le \pi - \Phi$

$$\alpha_3 = \frac{\pi}{2} + \theta_r. \tag{15}$$

Range 2: $3\pi/2 \le \theta_r < 2\pi$

$$\alpha_3 = \theta_r - \frac{3\pi}{2}. \tag{16}$$

-At Q_4

The diffracted angle α_4 is defined in two ranges as

Range 1: $0 \le \theta_r \le \pi/2$

$$\alpha_4 = \frac{\pi}{2} - \theta_r. \tag{17}$$

Range 2: $\pi - \Phi < \theta_r < 2\pi$

$$\alpha_4 = \frac{5\pi}{2} - \theta_r. \tag{18}$$

Therefore, the diffracted fields can be considered as follows

-The diffracted field at Q_3

The diffracted field at Q_3 can be expressed as

$$E_{d3^{\pm}}(\theta_r) = E_i(Q_3)\frac{D_h(L,\alpha_{3^{\pm}})}{2}A(s)e^{-jkr_3}, \qquad (19)$$

where $\pm$ are represented for the range 1 and 2 at Q_3, respectively.

-The diffracted field at Q_4

The diffracted field at Q_4 can be expressed as

$$E_{d4^{\pm}}(\theta_r) = E_i(Q_4)\frac{D_h(L,\alpha_{4^{\pm}})}{2}A(s)e^{-jkr_4}, \qquad (20)$$

where $\pm$ are represented for the range 1 and 2 at Q_4, respectively.

-The second-order diffracted field at Q_3

The second-order diffracted field at Q_3 can be expressed as

$$E_{d34^{\pm}}(\theta_r) = E_{d4}(Q_3)\frac{D_h(L,\alpha_{34^{\pm}})}{2}\frac{e^{-jkr_{34}}}{\sqrt{r_{34}}}, \qquad (21)$$

where $\pm$ are represented for the range 1 and 2 at Q_3, respectively and $\alpha_{34^{\pm}} = \alpha_{3^{\pm}}$.

-The incident field E_{d4} at Q_3

The incident field E_{d4} *at* Q_3 can be expressed as

$$E_{d4}(Q_3) = E_i(Q_4)\frac{D_h(L,\alpha_4=0)}{2}\frac{e^{-jkb'}}{\sqrt{b'}}. \qquad (22)$$

- The second-order diffracted field at Q_4

The second-order diffracted field at Q_4 can be expressed as

$$E_{d43^{\pm}}(\theta_r) = E_{d3}(Q_4)\frac{D_h(L,\alpha_{43^{\pm}})}{2}\frac{e^{-jkr_{43}}}{\sqrt{r_{43}}}, \qquad (23)$$

where $\pm$ are represented for the range 1 and 2 at Q_4, respectively and $\alpha_{43^{\pm}} = \alpha_{4^{\pm}}$.

-The incident field E_{d3} at Q_4

The incident field E_{d3} *at* Q_4 can be expressed as

$$E_{d3}(Q_4) = E_i(Q_3)\frac{D_h(L,\alpha_3=0)}{2}\frac{e^{-jkb'}}{\sqrt{b'}}. \qquad (24)$$

-The total field at an observation point

The total field at an observation point then becomes

$$E_t = \begin{cases} E_i + E_{d3^+} + E_{d4^+} + E_{d34^+} + E_{d43^+} & \text{for } 0 \le \theta_r \le \pi/2 \\ E_{d3^+} + E_{d34^+} & \text{for } \pi/2 < \theta_r \le \pi - \Phi \\ E_{d3^-} + E_{d43^-} & \text{for } \pi - \Phi \le \theta_r \le 3\pi/2 \\ E_i + E_{d3^-} + E_{d4^-} + E_{d34^-} + E_{d43^-} & \text{for } 3\pi/2 \le \theta_r \le 2\pi. \end{cases} \qquad (25)$$

The value of the parameters from eq.19-25 can be listed in table 3.

TABLE 3: SUMMARY OF PARAMETERS IN EQUATION 19-25 FOR *YZ*-PLANE

Diffracted field	Parameters		
	$A(s)$	L	$a(\alpha)$
- The diffracted field at Q_3 (in both of range 1 and 2)	$1/\sqrt{r}$	$b'/2$	$1-\sin\theta_r$
- The diffracted field at Q_4 (in both of range 1 and 2)	$1/\sqrt{r}$	$b'/2$	$1+\sin\theta_r$
- The second-order diffracted field at Q_3 (in both of range 1 and 2)		b'	$1-\sin\theta_r$
- The incident field E_{d4} at Q_3 (in both of range 1 and 2)		$b'/3$	2
- The second-order diffracted field at Q_4 (in both of range 1 and 2)		b'	$1+\sin\theta_r$
- The incident field E_{d3} at Q_4 (in both of range 1 and 2)		$b'/3$	2

Note : assume that the incident wave as the cylindrical wave and the incident fields are given by

$$E_i(Q_3) = E_i(Q_4) = E_{ap}(\theta_r)\frac{e^{-jk(b/2)}}{\sqrt{b/2}}. \qquad (26)$$

Finally, the relations for the various angles that used to transform the far field $E(\theta_r, \phi_r) \rightarrow E(\theta, \phi)$ can be written as

For yz plane

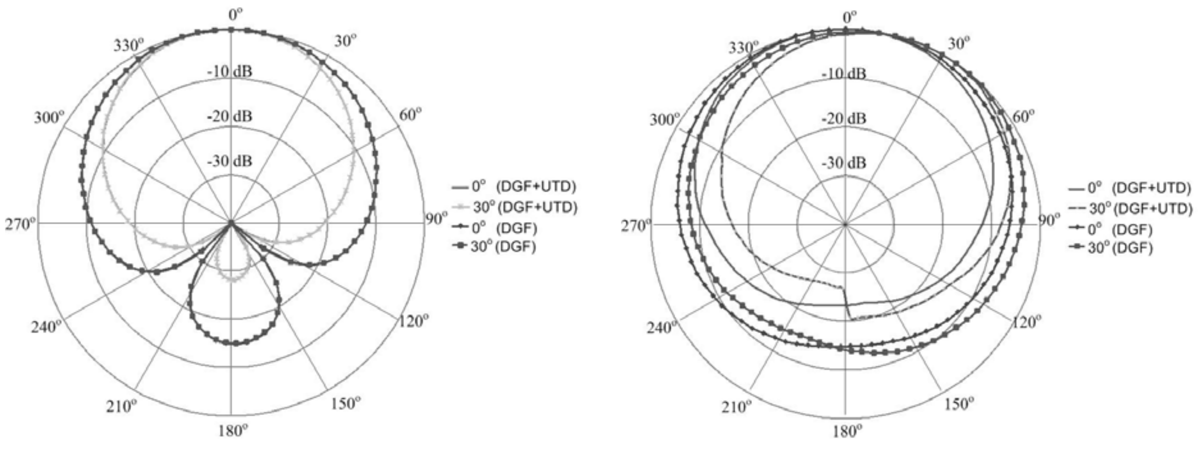
$$\theta_r = \theta - \Phi \quad \text{and} \quad \phi_r = \phi = \frac{\pi}{2}. \qquad (27)$$

For xz plane

$$\theta_r = \theta \quad \text{and} \quad \phi_r = \phi = 0. \qquad (28)$$

4. RADIATION PATTERN

In this section, the comparison of the radiation pattern calculated using only dyadic Green function approach and using the dyadic Green function approach together with the diffraction theory is performed. To improve the results from the previous works [1-4], therefore, the radiated fields from the rectangular aperture mounted on the cavity in those works are taken as the incident fields in this research. By following the procedure in the previous work and using the diffraction theory in this work, the radiation pattern is obtained in fig.3.



(a) xz-plane (b) yz-plane

Fig. 3: Radiation patterns from the normal (0°) and inclined (30°) rectangular apertures by using the dyadic Green function approach and diffraction theory with $a= 0.7\lambda$, $b=0.35\lambda$, $c= 0.75\lambda$, $l= 0.25\lambda$, $p=0.25\lambda$, $s= 0.35\lambda$ and $w=0.015\lambda$

From fig.3, it is found that the radiation pattern which is calculated using only dyadic Green function approach and using the dyadic Green function approach together with the diffraction theory are similar, both *xz* and yz plane. However, the amplitudes of the fields are different, especially in the back lobe radiation because the total field in this boundary is only the diffracted field, thus the level of the field in the back lobe is lower. Moreover, it is found that the maximum field is at the angle of Φ for the radiation pattern in *yz*-plane, because the configuration in this plane is inclined. On the other hand, the maximum field is at the angle of 0^o for the radiation pattern in *xz*-plane. It is noted that the radiation pattern in *yz*-plane for case of inclined (30°) aperture is discontinuous at $\theta = \pi$ because the two diffracted point in Fig.2(c) is on the wedge whose the interior angle (γ) is not identical. To solve this problem, the third-order diffraction terms or higher are included. So, it can use the theory of the three-dimensional diffraction to analyze this problem.

5. Conclusions

This paper presents the UTD analysis of an inclined rectangular aperture antenna mounted on an open-ended cavity excited by a probe. This work is continued from the parametric studies of the normal and the inclined rectangular apertures from the open-ended cavity fed by a probe using the dyadic Green function approach to improve the results by using the uniform theory of diffraction from those previous works. The geometry of the problem is made up of the rectangular aperture mounted on of the open-ended cavity and inclined it with the probe fed inside the cavity. The study starts with the consideration of the geometry of the diffraction such as the incident field, the shadow boundaries and the relative parameters. Then, the diffracted fields and the total field that radiated from the antenna are evaluated to precise the radiation pattern from the aperture antenna. The results in this work are significant to further design the aperture antenna from the open-ended cavity fed by probe in the microwave and the wireless communication applications.

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