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Determination of Propagation Loss in the Troposphere with Piece-wise Linear Boundary by Parabolic Equation Method

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Abstract— **Slope and surface roughness change the direction of scattered field with respect to incident field and spread the scattered wave. Thus beam width of scattered field may become greater than the maximum allowable angle of propagation for the parabolic equation method. This scattered field causes interference effect on the wireless systems. Interference analysis requires the computation of aforementioned scattered field. It is proposed that the parabolic equation be formulated in such a manner that it can consider the widening of beam width of scattered field, so that it may analyze the radio wave propagation over rough surfaces with arbitrary roughness factor. In this paper, based on the parabolic equation method, a new formulation for determination of radio wave propagation over sloped surfaces will be presented. Spectral domain of scattered field was separated from spectral domain of incident field. This method can consider the radio wave propagation with larger beam widths. Despite the available methods, by this method, only PE algorithm determines the maximum angle for radio wave propagation. Our method has some similarities with ray tracing methods, whereby acceleration techniques can be used for accelerating the proposed algorithm.**

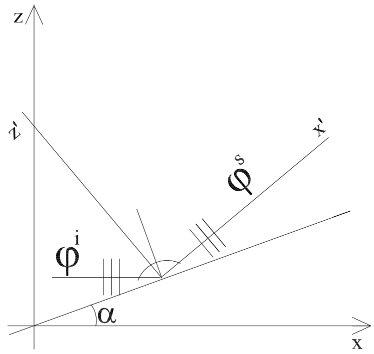

Fig. 1. Geometry of Problem.

I. Introduction

Recent advances in airborne and satellites mappings provide creation of high resolution databases of terrains. In the majority of databases terrain height is saved in the raster format, where terrain is sampled and then heights of sample points are saved in the database. The extracted path between transmitter and receiver from these databases, is a piece-wise linear function of range. Therefore, presenting suitable algorithms for evaluation of radio wave propagation over piece-wise terrains is of great importance.

The parabolic equation method as an effective approximation has been applied to propagation of radio waves in various media. In this method, by neglecting the backward scattered field, a fast marching algorithm is derived for determining radio wave propagation.

Algorithms for determination of radio wave propagation over rough surface with the parabolic equation method, fall into terrain masking method and conformal mapping method.

In the terrain masking method, it is assumed that the wave impinging on terrain is absorbed by it. So, in the split step method, when the terrain height at the new range step is higher than previous step, the field is truncated at terrain height. Also for propagation modeling over terrains,where the new terrain height is lower than terrain height at previous step, a block of zeroes is padded into field[1].

The aim of conformal mapping method is to map the media with lower rough terrain into a media with flat boundary. It is obvious that in this process the parabolic partial differential equation will remain as a parabolic equation. Only the refractive index of media can be changed. Therefore, the approximations, which were assumed in the derivation of split step algorithm, must remain valid. Donhue and Kuttler [2-6] divided the mapping algorithms into Global Conformal Mapping, Piece-wise Conformal Map, Continuous Shift Map and Piece-wise Linear Shift Map methods.In the Global Conformal Mapping, a map is used for the entire domain of interest, while in the Piece-wise Conformal Map method, the domain is divided into some sub-domains and conformal mapping is applied in each domain individually. Therefore, the derivations of maps are different in sub-domains. Local variations of terrain is smaller and it are possible to apply this method for terrains with larger slopes.

In the Continuous Shift Map method, for a boundary with terrain profile of $z = T(x)$, the following transform is applied

$$x' = x, z' = z - T(x) \quad (1)$$

whereby it is seen that the new partial differential equation can be solved with split-step method.In the Piece-wise Linear Shift Map method profile of terrain between two successive range steps is approximated by a linear function and a simple propagator is derived. In this paper a new algorithm for analyzing radio wave propagation over sloped terrain will be presented.After presenting our algorithm, we implement this algorithm for mobile coverage planning and compare our results with other methods.

II. Problem Formulation

Now we consider a sloped surface with slope of α . Dielectric constant and conductivity of surface are ε and σ(Fig.1). The refractive index of medium , n , is close to unity. A wave propagated at x direction is incident on this surface. A $e^{-j\omega t}$ time dependence for field is considered.

Incident and scattered fields must satisfy following wave equation

$$\nabla^2\varphi + k^2n^2\varphi = 0 \quad (2)$$

From geometrical optic it is well known that if a horizontal field impinges onto a surface with slope of α, scattered field angle is 2α with respect to surface. Spectral domain analysis of problem confirms this fact. Parabolic equation solution of radio wave propagation requires limitation of beam width of incident and scattered fields by a finite maximum value. In the usual formulation of radio wave propagation by parabolic equation method, the same spectral domain is considered for incident and scattered fields. Therefore, changing direction of scattered field due to slope causes the spreading of total(scattered+incidenc)field. Consequently, slope causes further limitation on the incident field beam width(see Fig. 2). To eliminate this problem, in this paper we consider separate spectral domains

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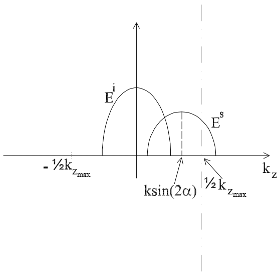

Fig. 2. Spectral Domain Analysis of Scattered Field.

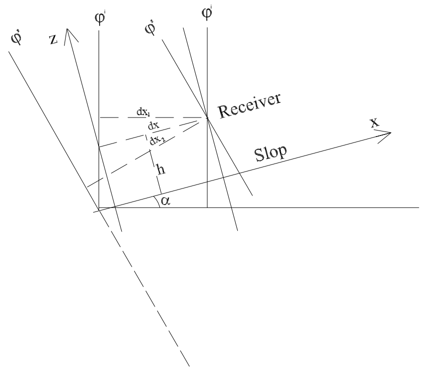

Fig. 3. Geometry of Problem of mobile receiver over surface with constant slop.

for incident and scattered fields. An algorithm similar to analyzing backward scattered field due to knife edge algorithm is used[7].

$$\begin{aligned}\varphi(x,z) &= \varphi^i(x,z) + \varphi^s(x,z)\\ \varphi(x,z) &= u^i(x,z)e^{jkx} + u^s(x',z')e^{jkx'}\end{aligned} \tag{3}$$

where i and s stand for incident and reflected fields, x' is direction of propagation of scattered field, z' is right hand normal to x'. x' and z' in Fig.1 can be derived as follow

$$\begin{aligned}x' &= x\cos(\alpha) + z\sin(\alpha)\\ z' &= -x\sin(\alpha) + z\cos(\alpha)\end{aligned} \tag{4}$$

by substituting (3) in (2), neglecting backward scattered field and considering the satisfaction of wave equation with φ^i and φ^s, the following partial differential equation can be derived for u^i and u^s[8].

$$\frac{\partial^2 u^i}{\partial z^2} + 2jk\frac{\partial u^i}{\partial x} + k^2(n^2-1)u^i = 0 \tag{5}$$

$$\frac{\partial^2 u^s}{\partial z'^2} + 2jk\frac{\partial u^s}{\partial x'} + k^2(n^2-1)u^s = 0 \tag{6}$$

By means of split step algorithm we can solve these parabolic equations[8].

$$u^i(x+\Delta x,z) = e^{-jk\frac{n^2-1}{2}\Delta x}F^{-1}\left(e^{-j\frac{2\pi^2\Delta x}{k}}F(u^i(x,z))\right) \tag{7}$$

$$u^s(x'+\Delta x',z') = e^{-jk\frac{n^2-1}{2}\Delta x'}F^{-1}\left(e^{-j\frac{2\pi^2\Delta x'}{k}}F(u^s(x',z'))\right) \tag{8}$$

F and F^{-1} are Fourier transform pairs which are defined as follows:

$$U(x,p) = F\left(u(x,z)\right) = \int_{-\infty}^{+\infty} u(x,z)e^{-j2\pi pz}dz \tag{9}$$

$$u(x,z) = F^{-1}\left(U(x,p)\right) = \int_{-\infty}^{+\infty} U(x,p)e^{j2\pi pz}dz \tag{10}$$

Combination of (7) and (8) via (3) should be performed with some care. When one of the expressions loses its validity at observation point, it must be replaced with zero. When the scattered wave term loses its validity, this algorithm reduces to terrain masking algorithm[1],which can evaluate the scattered field in scattering field dominant regions. With similar argument, this algorithm can be derived for wide angle split step algorithm, where the propagatore term, ($e^{-j\frac{2\pi^2p^2}{k}\Delta x}$), in (7) and (8) changes into $e^{(-j\sqrt{k^2-(2\pi p)^2}-k)\Delta x}$ By computation of initial field for scattered field in the whole domain, this field can be computed. This initial field must satisfy the boundary condition in the vicinity of slope. For extracting of initial field, we write initial incident field as weighting summation of plane waves.

$$u^i(0,z) = \int_{-\infty}^{+\infty} f^i(p)e^{j2\pi pz}dz$$

Each incident plane wave ($f^i(p)e^{j2\pi pz}$) impinges on the slope at the angle of $\psi = \alpha + \arcsin\left(\frac{2\pi p}{k}\right)$. Scattered field due to this plane wave can be derived as follows

$$f^s(p) = \Gamma_{V,H}(\psi)f^i(p)$$

where $\Gamma_{V,H}(\psi)$ are reflection coefficients from slope and can be derived as follows

$$\begin{aligned}\Gamma_V(\psi) &= \frac{\sin\psi - \sqrt{\varepsilon_{rc}-\cos^2\psi}}{\sin\psi - \sqrt{\varepsilon_{rc}+\cos^2\psi}}\\ \Gamma_H(\psi) &= \frac{\varepsilon_{rc}\sin\psi - \sqrt{\varepsilon_{rc}-\cos^2\psi}}{\varepsilon_{rc}\sin\psi - \sqrt{\varepsilon_{rc}+\cos^2\psi}}\end{aligned} \tag{11}$$

V stands for vertical polarization and H stands for horizontal polarization. ψ and ε_{rc} are incident angle and complex dielectric constant and can be calculated as follows

$$\begin{aligned}\psi &= \alpha + \sin^{-1}\left(\frac{2\pi p}{k}\right)\\ \varepsilon_{rc} &= \varepsilon_r - j60\lambda\sigma\end{aligned} \tag{12}$$

Therefore, initial field for scattered field is derived as follows

$$u^s(0,z') = F^{-1}\left[\Gamma_{V,H}(\psi)F(u^i(0,z))\right]|_{z'=-z} \tag{13}$$

For flat surfaces, these reflection coefficients agree with those mentioned in text books. Also for propagation over stratified surfaces, proper reflection coefficient can be derived[9].

III. IMPLEMENTATION OF MARCHING ALGORITHM FOR MOBILE COVERAGE PREDICTION

For radio coverage prediction in mobile communication system, propagation loss at a predefined height from surface must be determined. For example in Fig.3, the height of receiver from earth is h. If the range step of receiver is denoted by dx, then the range step of incident field(dx_1) and scattered field (dx_2) are computed as follows:

$$dx_1 = dx\cos(\alpha) \tag{14}$$

$$dx_2 = dx\cos(\alpha) + 2h\sin(\alpha) \tag{15}$$

The algorithm, at first computes the initial field for the scattered field by (11). Then by substituting the step sizes calculated by (14) and (15) into (7) and (8), u^i and u^s at observation point are evaluated. By means of (3), the field at observation point can be computed. u^i and u^s are the initial fields for next step. For determination of propagation loss in the presence of slope discontinuity, the profile following coordinates and scattering field coordinates become similar which are shown in Fig.4

The origin of the profile following coordinate is

$$|^{x_0(1-\cos(\alpha))}_{-x_0 sin(\alpha)}$$

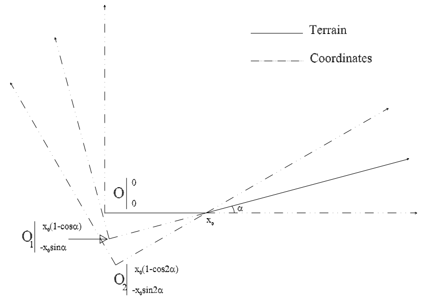

Fig. 4. Geometry of Problem of mobile receiver over slop discontinunity.

A similar algorithm can be used for propagation modeling. However, in this case the origin of incident and scattered field coordinates differ. Therefore,(4) changes into

$$x\prime = (x - x_0(1 - \cos(2\alpha)))\cos(2\alpha) + (z + x_0\sin(2\alpha))\sin(2\alpha) \quad (16)$$
$$z\prime = -(x - x_0(1 - \cos(2\alpha)))\sin(2\alpha) + (z + x_0\sin(2\alpha))\cos(2\alpha)$$

IV. NUMERICAL RESULTS

In order to investigate the efficiency of above mentioned algorithm for radio wave propagation over sloped surfaces, the problem of radio wave propagation over surfaces with gentle slope, surface with large slope and in the presence of slope discontinuity will be solved with this algorithm and will be compared with results of other methods.
Fig.5 compares the results of our algorithm with results of piecewise linear algorithm[8] for perfect conducivity ($\Gamma(\psi) = -1$) up going($\alpha = 5°$ at fig.1) and down going ($\alpha = -5°$ at fig.1) slope at $f = 1GHz$. For clarity, the results of piece-wise linear method have 20 dB offset. It is seen that results of the two methods agree for both slopes. Fig. 6 compares the results of our algorithm with Janswammy's algorithm [6] for lossy ($\sigma = 4$ and $\varepsilon_r = 4$) surface at $f = 1MHz$. It is seen that the results of two algorithm coincide. In the Fig.7 we computed the basic propagation loss over a constant slope($\alpha = 30°$). Compare these results with the results of 2 ray and piece-wise linear method. Fig.8 shows the propagation loss over bi-linear terrain.Two section of terrain have $\Delta\alpha = 5°$ difference in slope. It is seen that our results coincide with the results of piece-wise linear method. Fig.9 and Fig.10 show similar results for $\Delta\alpha = 10°$ and $\Delta\alpha = 15°$.

V. CONCLUSION

In this paper ,based on the parabolic equation method, a new algorithm for modeling of radio wave propagation over sloped surface was developed. Spectral domain of scattered field was separated from spectral domain of incident field. Despite other available methods, our proposed method can evaluate the propagation of low beam width wave over arbitrary slope. Also it can incorporate the effect of troposphere and vegetation[10].
When the angle of propagation of scattered field becomes very different from the incident field, algorithm reduces to terrain masking method. Despite the terrain masking method, our proposed algorithm can predict the scattered field propagating far from incident field accurately.
It is seen that the results of our algorithm agree well with the results of former methods for radio wave propagation over terrains. Also, it can predict propagation loss over surfaces with larger slopes compared with other methods.
This algorithm has some similarities with ray tracing algorithm. Therefore at professional software based on proposed algorithm, accelerating methods which proposed for ray tracing method can be used[11].

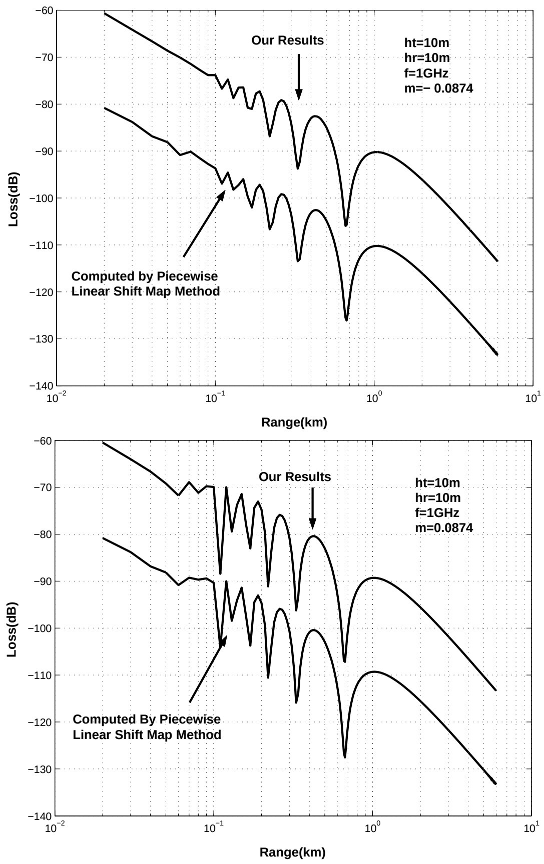

Fig. 5. Analyzing of radio wave propagation over constant slop($\alpha = 5°$) with algorithm presented in this paper and piece-wise linear algorithm. for clarity, the results of piece-wise linear method have 20 dB offset.

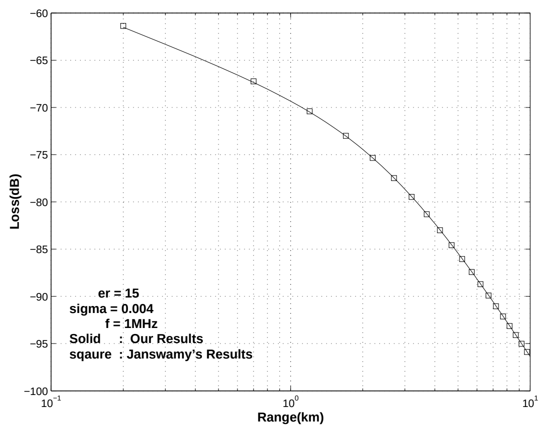

Fig. 6. Analyzing of radio wave propagation over lossy ground .

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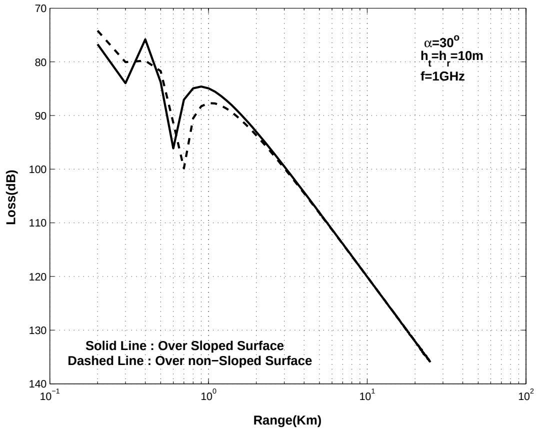

Fig. 7. Computation of Basic Propagation Loss over Large Slop($\alpha = 30°$).

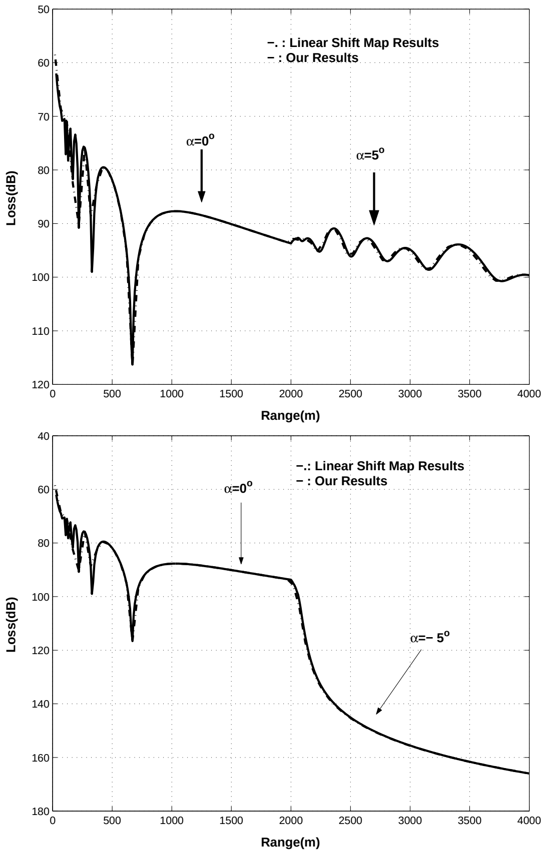

Fig. 8. Comparison of results of algorithm with Linear Shift map for Bilinear Slop($|\Delta\alpha| = 5°$).

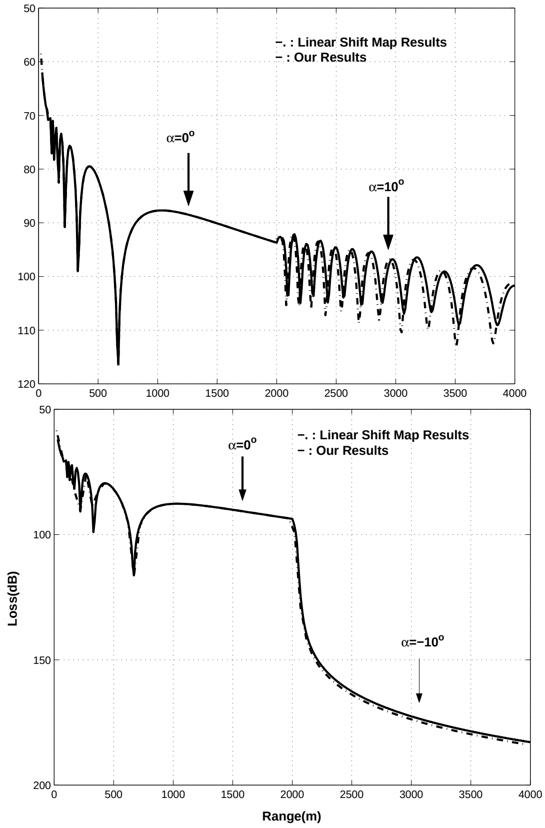

Fig. 9. Comparison of results of algorithm with Linear Shift map for Bilinear Slop($|\Delta\alpha| = 10°$).

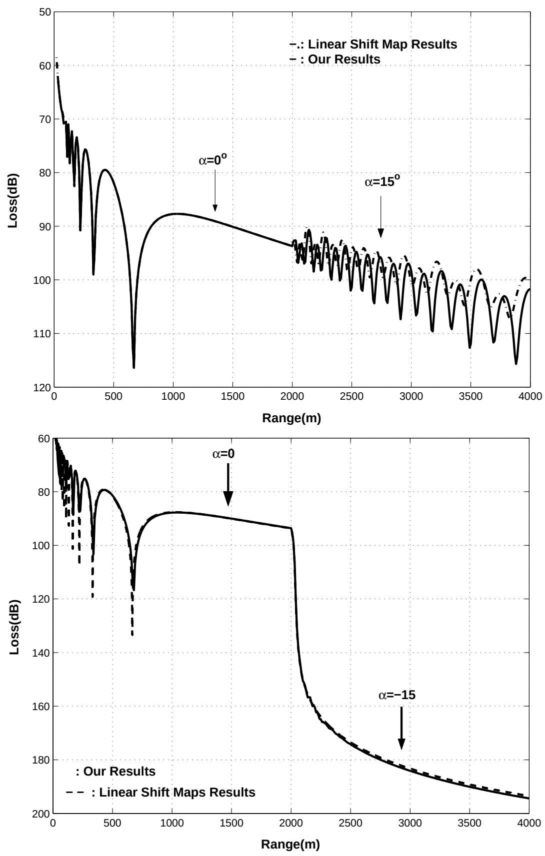

Fig. 10. Comparison of results of algorithm with Linear Shift map for Bilinear Slop($|\Delta\alpha| = 15°$).

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