AN MMSE DOWNLINK BEAMFORMING WITH FEEDBACK

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Abstract – For the space division multiple access (SDMA) downlink channel, multiple transmit antennas at the base station and multiple receive antennas at mobile stations can be used. In this letter, optimal weight vectors for the multiple transmit antennas are derived under the minimum mean square error (MMSE) criterion. It is assumed that the base station knows the channel parameters. Interestingly, we observe that the number of receive antennas at mobile stations does not affect the MMSE performance. On the other hand, we show that, in order to achieve a reasonable performance, the number of transmit antennas should be close to or greater than the number of downlink signals.

I. Introduction

The use of multiple antennas creates a number of possibilities for wireless communications. For example, one can consider Space Division Multiple Access (SDMA) by using multiple antennas. Multiple users can share the same frequency band and are separated by exploiting the space dimension [2]. In SDMA systems, assuming that the base station is equipped with multiple transmit and receive antennas, there are two possible ways to perform beamforming: one is the uplink beamforming and the other is the downlink beamforming. The uplink beamforming has been relatively well addressed in the literature, while the downlink beamforming is not yet. The results of some studies in this area can be found in [3] [5] [1]: using the downlink channel information at the Base Station, the downlink beamforming weight vectors have been obtained in [3] [5] for a single-user. The multiuser downlink beamforming has been formulated and solved using a semidefinite optimization technique in [1]. Unfortunately, there is no closed form solution to this problem.

In this letter, we consider the multiuser downlink beamforming with known channel parameters. We assume that mobile stations are equipped with multiple receive antennas. Thus, the channel parameters form a matrix. From this, it is assumed that the base station knows the downlink channel matrices for all mobile stations. Later in this letter, we will relax this assumption. By formulating the Minimization Mean Square Error (MMSE) setup, we will derive a *closed* form solution of the downlink beamforming weight vectors and discuss the results.

II. Signal model and problem formulation

We assume flat fading channels. Let $\mathbf{H}_q$ denote the matrix channel for the qth downlink channel (the channel from the base station to the qth mobile station). We assume that the base station transmit the signals with L_T transmit antennas and mobile stations are equipped with L_R receive antennas. Then, the size of the matrix channel $\mathbf{H}_q$ is $L_R \times L_T$. Let $\mathbf{w}_k$ denote the weight vector for the kth multiple transmit antennas. Then, the transmitted signal can be written as

$$\mathbf{y}(t) = \sum_{k=1}^{K} \mathbf{w}_k s_k(t), \tag{1}$$

where $s_k(t)$ is the data signal to be transmitted to the kth mobile station and K is the number of the downlink signals. Then, the qth mobile station receives the following vector signal

$$\begin{aligned} \mathbf{r}_q(t) &= \mathbf{H}_q \mathbf{y}(t) + \mathbf{n}(t) \\ &= \sum_{k=1}^{K} \mathbf{H}_q \mathbf{w}_k s_k(t) + \mathbf{n}(t), \end{aligned} \tag{2}$$

where $\mathbf{n}(t)$ is the background noise vector. Let $\mathbf{c}_q$ be the weight vector for combining the signals on the L_R receive antennas at the qth mobile station. Then, the received signal is written as

$$\begin{aligned} x_q(t) &= \mathbf{c}_q^H \mathbf{r}_q(t) \\ &= \sum_{k=1}^{K} \mathbf{c}_q^H \mathbf{H}_q \mathbf{w}_k s_k(t) + \mathbf{c}_q^H \mathbf{n}(t). \end{aligned} \tag{3}$$

The Signal-Interference-plus-Noise Ratio (SINR) can be written as

$$\gamma_q = \frac{|\mathbf{c}_q^H \mathbf{H}_q \mathbf{w}_q|^2}{\sum_{k \neq q} |\mathbf{c}_q^H \mathbf{H}_q \mathbf{w}_k|^2 + \sigma^2 ||\mathbf{c}_q||^2}, \tag{4}$$

where σ^2 is the noise variance ($E[\mathbf{n}(t)\mathbf{n}^H(t)] = \sigma^2 \mathbf{I}$). Obviously, it is desirable to find the weight vectors $\{\mathbf{w}_q\}$ and $\{\mathbf{c}_q\}$ that maximize the SINR's. That is, under the Maximization SINR (MSINR) criterion, the weight vectors $\{\mathbf{w}_q\}$ and $\{\mathbf{c}_q\}$ can be computed. This is a joint optimization problem with multiple objective functions. However, there is no general analytical method for computing those vectors. In [1], some algorithms are proposed to find the weight vectors $\{\mathbf{w}_q\}$ with $L_R = 1$ (i.e., a single receive antenna). Unfortunately, closed form solutions are not available in [1]. In the next section, we will consider an alternative approach, allowing to compute exactely the weight vectors that correspond to a closed form solution under the MMSE criterion.

III. Computation of the weight vectors for transmitting

Assuming that the weight vectors at the reception, namely $\{\mathbf{c}_q\}$ are given, we show in this section how the weight vectors at the transmission $\mathbf{w}_q$ can be computed. We also assume that the base station knows all channel matrices $\{\mathbf{H}_q\}$. As mentioned in the previous section, there is no closed form solution for computing the weight vectors under the MSINR criterion. Thus, instead of using the MSINR criterion, we will consider the MMSE criterion that allows to find a closed form solution for $\{\mathbf{w}_q\}$.

Let $\mathbf{g}_q = \mathbf{H}_q^H \mathbf{c}_q$. Then, the received signals are written as

$$x_q(t) = \sum_{k=1}^{K} \mathbf{g}_q^H \mathbf{w}_k s_k(t) + \mathbf{c}_q^H \mathbf{n}(t), \; q = 1, 2, \cdots, K. \tag{5}$$

The MSE is defined as

$$\text{MSE} = E\left[\sum_{q=1}^{K} |s_q(t) - x_q(t)|^2\right]. \tag{6}$$

Let $\mathbf{x}(t) = [x_1(t)\ x_2(t)\ \cdots\ x_K(t)]^T$ and $\mathbf{s}(t) = [s_1(t)\ s_2(t)\ \cdots\ s_K(t)]^T$. From (5), we can show that

$$\mathbf{x}(t) = \begin{bmatrix} \mathbf{g}_1^H \\ \mathbf{g}_2^H \\ \vdots \\ \mathbf{g}_K^H \end{bmatrix} [\mathbf{w}_1\ \mathbf{w}_2\ \cdots\ \mathbf{w}_K]\, \mathbf{s}(t) + \begin{bmatrix} \mathbf{c}_1^H \mathbf{n}(t) \\ \mathbf{c}_2^H \mathbf{n}(t) \\ \vdots \\ \mathbf{c}_K^H \mathbf{n}(t) \end{bmatrix}. \tag{7}$$

Hence, the MSE is written as

$$\begin{aligned} \text{MSE} &= E[(\mathbf{x}(t) - \mathbf{s}(t))^H(\mathbf{x}(t) - \mathbf{s}(t))] \\ &= \sigma_s^2 trace((\mathbf{I} - \mathbf{G}^H\mathbf{W})^H(\mathbf{I} - \mathbf{G}^H\mathbf{W})) + \sigma^2 \sum_{k=1}^{K} ||\mathbf{c}_k||^2 \\ &= \sigma_s^2 ||\mathbf{I} - \mathbf{G}^H\mathbf{W}||_F^2 + \sigma^2 \sum_{k=1}^{K} ||\mathbf{c}_k||^2, \end{aligned} \tag{8}$$

where σ_s^2 is the variance of the signal, $\mathbf{G} = [\mathbf{g}_1\ \mathbf{g}_2\ \cdots\ \mathbf{g}_K]$, and $\mathbf{W} = [\mathbf{w}_1\ \mathbf{w}_2\ \cdots\ \mathbf{w}_K]$. Therefore, the (conditional) MMSE solution of $\{\mathbf{w}_q\}$ is the weight vectors which minimizes $||\mathbf{I} - \mathbf{G}^H\mathbf{W}||_F^2$. If $L_T > K$ and $rank(\mathbf{G}) = K$, there are infinite solutions for $\mathbf{W}$ which satisfy $\mathbf{I} = \mathbf{G}^H\mathbf{W}$. For the case that $L_T \leq K$, we have the following result.

Theorem 1: Suppose that that $L_T \leq K$ and $rank(\mathbf{G}) = r$. Let $\{\mathbf{u}_\ell\}$ and $\{\mathbf{v}_\ell\}$ be the left and right singular vectors corresponding to the singular values, $\sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_r$ of $\mathbf{V}$. The MMSE solution of $\mathbf{W}$ is given as

$$\mathbf{W}_{MMSE} = \mathbf{U}\mathbf{\Sigma}^\dagger\mathbf{V}^H, \tag{9}$$

where $\mathbf{U} = [\mathbf{u}_1\ \cdots\ \mathbf{u}_{L_T}]$, $\mathbf{V} = [\mathbf{v}_1\ \cdots\ \mathbf{v}_K]$, and $\mathbf{\Sigma}^\dagger$ is a diagonal matrix of size $L_T \times K$ and given as

$$\mathbf{\Sigma}^\dagger = diag(\sigma_1^{-1}, \sigma_2^{-1}, \cdots, \sigma_r^{-1}, 0, \cdots, 0).$$

If $r = K$, we get

$$\mathbf{W}_{MMSE} = \left(\mathbf{G}\mathbf{G}^H\right)^{-1}\mathbf{G}. \tag{10}$$

Clearly, $\mathbf{W}_{MMSE}$ is the pseudo-inverse of $\mathbf{G}$.
Proof: See [4] (pp. 139).
From (10), we can have a closed form solution for the weight vectors $\{\mathbf{w}_q\}$. It shows that we do not need any optimization techniques under the MMSE criterion, while some optimization techniques have to be used under some different setup (e.g., the MSINR criterion with constraints) in [1].

In general, if $rank(\mathbf{G}) = r$, the MSE can be written as

$$\text{MSE} = \sigma_s^2 \max(K - r, 0) + \sigma^2 ||\mathbf{C}||_F^2, \tag{11}$$

where $\mathbf{C} = [\mathbf{c}_1\ \cdots \mathbf{c}_K]$. In addition, if $rank(\mathbf{G}) = L_T$, we have

$$E[\mathbf{e}(t)\mathbf{e}^H(t)] = \sigma_s^2(\mathbf{I} - \mathbf{G}^H(\mathbf{G}\mathbf{G}^H)^{-1}\mathbf{G}) + \sigma^2\mathbf{G}^H\mathbf{G},$$

where $\mathbf{e}(t) = \mathbf{x}(t) - \mathbf{s}(t)$. As shown in (11), it is observed that the number of the receive antennas does not strongly affect the MSE performance. It indicates that the mobile stations in SDMA systems do not need to have multiple receive antennas.

Note that the MSE in (11) is the sum of the individual MSE's. Let $r = L_T$. The Average individual MSE (AMSE) can be computed as

$$\text{AMSE} = \frac{\text{MSE}}{K} = \frac{\sigma_s^2 \max(K - L_T, 0)}{K} + \frac{\sigma^2 ||\mathbf{C}||_F^2}{K}.$$

Thus, the AMSE decreases as K gets larger while $K - L_T$ is fixed. It shows that the AMSE of the SDMA system with $K = 20$ and $L_T = 18$ is much smaller than that with $K = 5$ and $L_T = 3$. Consequently, larger numbers of users and transmit antennas are desirable.

IV. Discussions

- *Determination of the combining weight vectors* $\{\mathbf{c}_q\}$:
  This is an important point for the following reason: we have seen that the performance of the proposed method directly relies on r, the rank of matrix $\mathbf{G}$. Indeed, r should be as large as possible in order to achieve a good performance. However, if the rank of the channel matrix $\mathbf{H} = [\mathbf{H}_1\ \mathbf{H}_2\ \cdots\ \mathbf{H}_k]$ is denoted by $r_{\mathbf{H}}$, then obviously, $r \le r_{\mathbf{H}}$. It is then desirable to choose the coefficients $\{\mathbf{c}_q\}$ such as we have $r = r_{\mathbf{H}}$. To achieve this with certainty, the coefficients $\{\mathbf{c}_q\}$ should then be computed from the base-station. It has to be pointed out that the transmit diversity in general is such as the rank of matrix $\mathbf{H}$ will be L_T. Therefore, in any cases, the performance of MSE criterion will be clearly limited by the number of transmit antenna which should be as close as possible to the number of users.
- *Feedback information*: In order to decide the weight vectors $\{\mathbf{w}_q\}$, we assume that the base station knows the downlink channel matrices and the weight vectors $\{\mathbf{c}_q\}$. Thus, the mobile stations have to transmit the downlink channel matrices and the combining weight vectors. The transmission of this information may result in a sacrifice of the uplink channel capacity. Fortunately, as shown in Theorem 1, in order to construct $\{\mathbf{w}_q\}$, the mobile stations only need to send the vectors $\{\mathbf{g}_q\}$. With given $\mathbf{c}_q$ and estimated $\mathbf{H}_q$, the mobile stations can compute $\mathbf{g}_q$'s and transmit these vectors. If there are more than one receive antennas at mobile stations, this approach is quite desirable to minimize the sacrifice of the uplink channel capacity, because it is not required to send the K channel matrices of size $L_R \times L_T$.

V. Conclusion

We have presented in this letter a new concept of using space diversity at the transmission in a multi-user case environment, so that the Mean Square Error between the transmitted signal and received signal is minimized. The interesting result of this study is that the performance of the proposed method does not depend on the number of receive antenna but clearly depends on the number of transmit antenna which should approach the number of users in the system, for optimal performance. The basic results presented here raise a number of interesting questions. Indeed, this method relies on a simple and general unconstrained optimization problem, where a number of parameters such as the weight vectors at the receiver have not been specified. However, it is clear that in order to ensure the best achievable performance, these parameters should be also chosen accordingly, which will probably lead to a modified constrained optimization problem.

References

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