# A CMA Adaptive Array on a Rectangular Conducting Plate

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#### I. Introduction

Since the constant modulus algorithm (CMA) was first proposed by J.R.Treichler et al. in 1983[1] and applied to the array by R.Gooch et al. in 1986[2], the performance of the CMA adaptive array has been analyzed in various aspects. Although the CMA adaptive array was even studied with an experiment by M.Fujimoto et al. in 1995[3], the most of the analyses have been based on the assumption that the array elements are omnidirectional and in free space. In 1995, the performance of a CMA adaptive array with dipole elements was analyzed, when the array was placed on an infinite ground plane and the mutual coupling (MC) effects among the array elements were taken into account[4].

In this paper, the performance of a CMA adaptive array on a finite sized rectangular conducting plate has been investigated. The mutual coupling effects among the array elements and the diffraction effects caused by the edges of the conducting plate have been taken into account in the calculation by the hybrid method of MM (Moment Method) and GTD (Geometrical Theory of Diffraction). The performance with the consideration of diffracted field is different from that with the array on an infinitely large ground plane where MC between antenna elements is consider.

### II. CMA Adaptive Array

A CMA adaptive array is an adaptive system suitable for mobile communication, where it can successfully suppress interference under multipath environment. The system output y(k) at the kth sampling instant can be written as

$$y(k) = W^{T}(k)X(k) \tag{1}$$

where X(k) is the vector of the system input, and W(k) is the vector of adjustable weights. The superscript T denotes transpose.

The CMA adaptive array eliminates the amplitude fluctuation of the array output due to the interference. The cost function to be minimized can be expressed as

$$J(W) = E[||y(k)|^2 - \sigma^2|^2]$$
 (2)

where  $E[\cdot]$  denotes the expectation and  $\sigma$  is the amplitude of the array output in the absence of interference. The steepest descent method is employed to minimize the cost function. The weight vector which adjusts the output of the array is updated according to the following equation.

$$W(k+1) = W(k) - 4\mu E\{\{|y(k)|^2 - \sigma^2\} X^*(k) y(k)\}$$
(3)

where  $\mu$  is a positive step size and \* denotes complex conjugate.

#### III. Computational Method

When we take into account mutual coupling effects between the array elements and the diffracted field caused by the edges of the conducting plate, the current vector *I* on the array elements can be obtained by solving the following matrix equation[5].

$$ZI = V \tag{4}$$

where Z is the generalized impedance matrix and V is the induced voltage vector due to the incident field. The mnth element of the matrix Z is  $Z_{mn} = Z_{mn} + Z_{$ 

$$Z_{mn} = -f \quad W_m \cdot E_{n^s} \quad dl \qquad \qquad Z_{mn} = -f \quad W_m \cdot E_{n^s} \quad dl \qquad (5)$$

and

$$V_{m} = \int_{lm} W_{m} \cdot E^{j} dl \qquad V_{m} = \int_{lm} W_{m} \cdot E^{j} e dl \qquad (6)$$

where  $l_m$  is the segment on which the *m*th expansion function is located.  $W_m$  is the *m*th testing function. Here the piecewise sinusoidal function is used for both the expansion function and the testing function.  $E_{n^s}$  is the scattered field from the *n*th current expansion function and E is the incident field.  $Z_{mn^l}(n=m)$  is the lumped load impedance corresponding to the output terminal of each element of the array where  $Z_{mn^l}$  is connected ( $Z_{mn^l}=0$  when  $n\neq m$  or m is not equal to the number of the expansion function at the output terminal of the antenna elements).  $E_{n^{s_s}}$  and  $E^s$  are respectively the diffracted fields of  $E_{n^s}$  and  $E^s$  due to the edges of the rectangular conducting plate and are calculated by GTD[6]. Here the corner diffraction is ignored because it is very weak when the distance between the antenna and the corner is larger than  $0.3\lambda[7]$ .

The element output voltage X(t) (i=1,2, ..., N) can be obtained by the multiplication of the current at the output terminal by the corresponding load impedance.

### IV. Simulation

Fig.1 shows the configuration of an equally spaced linear array with four quarter wavelength monopoles.  $(r, \theta, \Phi)$  is a spherical coordinate system and d is the interelement spacing. The radius of each monopole is  $0.002\lambda$ , where  $\lambda$  is the wavelength.

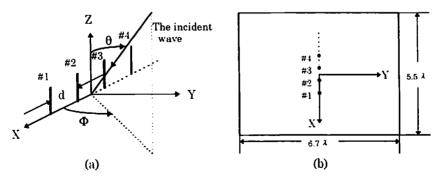
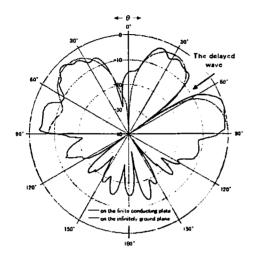


Fig.1 The configuration of the linear array with four equally spaced quarter wavelength monopoles (a) the coordinate system (b) a finite rectangular conducting plate.

The incident wave is assumed to be a plane wave modulated by a  $\pi/4$  shift QPSK signal with a symbol duration T. For simplicity, the signal bandwidth is not considered in the computation. Each antenna is terminated by a  $50\Omega$  load. In the calculation, 15 snapshots are used to update the weight vector. Each snapshot is taken at the center of a symbol of the first arriving signal. For the cost function in eq.(2), it is assumed that  $\sigma=1$  and the initial weight vector is  $W\sigma=[1.0.0.0]^T$ . Since all the output powers of the antenna elements are not the same any more when mutual coupling effects are taken into account, the SNR is defined as the ratio of the power at the receiving terminal of the first element of the array to that of thermal noise and set to be 40 dB. The electric field magnitude ratio of the first to the second incident wave is 3 dB and the time delay of the second wave is T compared with the first wave.



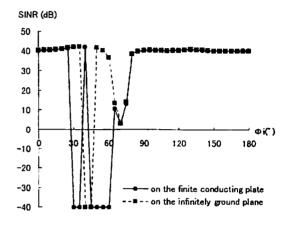


Fig.2 The receiving patterns after convergence when the array is placed on the ground plane and on the finite conducting plate.  $\theta_d$ =80°,  $\Phi_d$ =40°,  $\theta_i$ =55° and  $\Phi_i$ =40°.

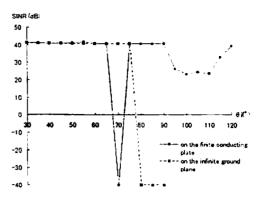
Fig.3 The SINR patterns after convergence when the array is placed on the ground plane and on the finite conducting plate.  $\theta_d=\theta_i=80^\circ$ ,  $\Phi_d=70^\circ$ , with  $\Phi_1$  varying from  $0^\circ$  to  $180^\circ$ .

Fig.2 shows the receiving patterns of the CMA adaptive array after 200 iterations when the array is placed on the ground plane and on the finite conducting plate. The direct wave comes from  $\theta_d$ =80°,  $\Phi_d$ =40° and the delayed wave from  $\theta_i$ =55°,  $\Phi_i$ =40°. It can be seen that both patterns have the deep nulls in the direction of the delayed wave. The fluctuation of the receiving pattern when the array on the finite conducting plate is due to the diffracted field from the four edges of the plate.

Fig.3 shows the SINR patterns of the array on the infinite ground plane and on the finite conducting plate. The incident directions of the desired and delayed waves are  $\theta_4$ =80°,  $\Phi_4$ =70° and  $\theta_i$ =80°,  $\Phi_i$ =0°~180°. respectively. When the array is on the finite conducting plate, the array catches the first wave except at  $\Phi_i$ =30°, 35° and 45°~60°. While the array is on the infinite ground plane, it catches the delayed wave at  $\Phi_i$ =40°, 45°. From this figure it can be seen that the two arrays have different capture property. It depends on the initial receiving pattern whether the array catches the direct wave or the delayed wave.

Fig.4 shows the SINR patterns after 200 iterations with the finite plate and the infinite ground plane. The first wave is incident from  $\Phi_d$ =90°,  $\theta_d$ =80°, while the incident direction of the delayed wave varies from  $\theta_i$ =30° to  $\theta_i$ =120° in the  $\Phi_i$ =50°~230° plane.

It can be seen that with the infinite ground plane the array catches the first wave when 30°≤θi≤75° and catches the second wave when 80°≤θi≤90°. But with the finite plate the array catches the first wave for all  $\theta_i$  except at  $\theta_i$ =70°. This is because the initial pattern has the maximum value at this angle. Since the initial received power at  $\Phi_i$ =50° is less than that at  $\Phi_4=90^\circ$  with the finite plate when  $80^\circ \le \theta_i \le 90^\circ$ , the array does not catch the second wave in these directions. When 95°≤0i≤115°, the SINR did not reach 40 dB after 200 iterations. Because the initial received power of the Fig.4 second wave is much less than that of the first infinite plane after 200 iteration, when first wave the convergent speed is slow. The SINR reached 40 dB at 0i=120°, because the initial second wave varies from 0i=30° to 120° in SIR is already 40 dB. This can be seen from the initial receiving pattern.



SINR patterns with finite and wave comes from  $\Phi_d=90^\circ$ ,  $\theta_d=80^\circ$  and the Φ<sub>i</sub>=50°~ 230° plane.

#### V. Conclusion

The performance of a CMA adaptive array with the consideration of mutual coupling effects among the array elements and the diffraction effects caused by the edges of a rectangular conducting plate was investigated. The simulation shows that the mutual coupling and diffracted fields have significant effects on the capture property of the CMA adaptive array.

## VI. Acknowledgment

We are grateful to Mr. Mitoshi Fujimoto at Toyota Central Research & Development Lab. for his useful advice to this work.

#### References VII.

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