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RESONANT FREQUENCY OF EQUILATERAL TRIANGULAR PATCH RESONATOR WITH SUPERSTRATE USING MODIFIED WOLFF MODEL
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#### Abstract

: A new model called modified Wolff model [MWM] recently developed has been used to calculate the resonant frequency of various modes of the equilateral triangular patch within $0.5 \%$ of reported experimental results.MWM also accounts for the effect of dielectric superstrate on the resonant frequency. MWM has already been used to calculate the resonant frequency of covered rectangular and circular patches within $0.5 \%$ accuracy. Thus MWM is suitable for


 microwave CAD.
## INTRODUCTION

The triangular patch resonator is an attractive microstrip element useful for the design of compact and broader bandwidth circulator, filter and oscillator. As a radiating element it has wide half-power beamwidth in both E\&H planes and has no nullin the endfire direction. The cavity model has been used by sevral authors [1-3] to calculate resonant frequency of various modes. Dahele and Lee [1] have presented experimental results on the resonant frequency of five modes and improved the cavity model by replacing side length of equilateral triangle by increased effective sidelength to account for the fringe field. However, they have not replaced $\epsilon r$ of the substrate by $\epsilon e f f$ to take care of inhomogenity of the medium. Garg and Long [2] have treated side of equilateral triangle as radius of a circle and obtained more accurate results on the resonant frequency. They have also maintained $\epsilon r$ in the expression for the resonant frequency. Gang [3] has shown that maintaining $\in r$, is not always right. Such assumption is valid for small h/L ratio. Moreover, the cavity model in its standard form is not suitable for calculation of resonant frequency with superstrate. Fullwave analysis results for equilaternal tringular patch with and without cover are not available. Recently authors have developed a modified Wolff model [MWM] to calculate the resonant frequency of the covered rectangular [4] and circular patches [5] with accuracy of $0.5 \%$ compared to experimental and SDA results. In this paper we have applied MWM to calculate the resonant frequency of the covered equilateral triangular patch shown in fig.(1a).

MODEL
Resonant frequency of the covered patch could be obtained from [3]

$$
\mathrm{fm}, \mathrm{n}, \mathrm{l}=\frac{2 V_{0}}{3 \text { Leff }(\epsilon d y n) 1 / 2}\left(\mathrm{~m}^{2}+\mathrm{mn}+\mathrm{n}^{2}\right) 1 / 2
$$

where $V o$ is velocity of light and the modal integers $m, n, l$ satisfy the condition $m+n+l=0$. Leff is the extended side of the triangular patch due to the fringe field. $\epsilon d y n$ ( $\epsilon r 1, \epsilon_{r 2}$, W,h1,h2) is the dynamic dielectric constant of the patch with superstrate obtained from

$$
\epsilon_{\mathrm{dyn}}\left(\epsilon_{\mathrm{r} 1}, \epsilon_{\mathrm{r} 2}, \mathrm{w}, \mathrm{~h} 1, \mathrm{~h} 2\right)=\frac{\text { Єydn } \left.\epsilon_{r} 1 \epsilon_{r} 2, \mathrm{w}, \mathrm{~h} 1, \mathrm{~h} 2\right)}{\operatorname{Cdyn}\left(\epsilon_{\mathrm{r} 1}=\epsilon_{\mathrm{r} 2}=1, \mathrm{~W}, \mathrm{~h} 1, \mathrm{~h} 2\right)}
$$

To calculate Leff and $\epsilon d y n$ the equilateral tringular patch has been reduced to an equivalent rectangular patch without changing the length of triangle, fig.(1b). Width $W$ of the equivalent rectangular patch is $0.433 L$.

$$
\begin{align*}
& \text { Cdyn }=\frac{\text { Co,stat }}{\gamma_{\mathrm{n}} \bar{\gamma}_{\mathrm{m}}}+\frac{\text { C , stat }}{\gamma_{\mathrm{n}}}  \tag{3}\\
& \gamma_{\mathrm{i}}=\left\{\begin{array}{ll}
1 & i=0 \\
2 & i \neq 0
\end{array} \quad i=\mathrm{n}, \mathrm{~m}\right. \tag{4}
\end{align*}
$$

where, the first term of equation (3) is dynamic central capacitance of the patch which is not influenced by the superstrate. Therefore,

$$
\text { Co,stat }=\begin{array}{|c}
\epsilon_{r o} \epsilon_{r 1 A}  \tag{5}\\
-
\end{array}
$$

where, $A$ is the area of the equilateral triangle. The fringe capacitance along the length $L$ is obtained from

$$
\begin{equation*}
\text { Ce,stat } \left.=\frac{1}{2} \frac{\mathrm{Z}\left(\mathrm{~W}, \mathrm{~h} 1, \mathrm{~h} 2, \epsilon_{\mathrm{r} 1}, \epsilon_{\mathrm{r} 2}=1\right) \mathrm{XL}}{\operatorname{vo} \mathrm{Z} 2\left(W, \mathrm{~h} 1, \mathrm{~h} 2, \epsilon_{\mathrm{r} 1}, \epsilon_{\mathrm{r} 2}\right)}-\text { Co, stat }\right] \tag{6}
\end{equation*}
$$

where $Z$ is the characteristic impedance of the line. The effective length Leff of the covered patch can be obtained from Long and Garg formula which has been adopted for covered patch also [4].

RESULTS
The calculated resonant frequency of the $L=10 \mathrm{~cm}$ equilateral patch on substrate $\epsilon \mathrm{r} 1=2.32, \mathrm{~h} 1=0.159 \mathrm{~cm}$ is shown in the table-1 along with measured resonant frequency and resonant frequency calculated by Garg-Long [2] DaheleLee[1], and Gang[3]. Gang attempted to rationalize the calculation of resonant frequency of triangular patch.

However, his method gives maximum average error i.e. $1.9 \%$ compared to the experimental values. MWM provides average error $0.26 \%$ and error remains always within $0.5 \%$. The method adopted by Garg and Long also gives good result but is applicable to the single layer structure only. Effect of the cover on various modes of the triangular Patoh is shown in fig. (2)

## CONCLUSION

The MWM can be effectively used to calculate resonant frequency of the multilayered rectangular, circular and equilateral triangular patches with the accuracy of fullwave analysis while retaining advantages of cavity model. The method is suitable for microwave CAD.

## REFERENCE

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2. G.Garg and S.A.Long, "An improved formula for the resonant frequencies of the triangular microstrip Patch antenna " IEEE Trans. Antenna and Propagat Vol. 6 AP-16, No. 4, PP. 570-571, April 88.
3. Xu Gang, "On the resonant frequencies of microstrip antennas", IEEE Trans. Antenna \& Propagat. Vol.AP-37, No. $2 \mathrm{AP}-16$, No. 4 PP. 245-247, Feb. 89
4. A.K.Verma ad Z.Rostamy, "Modified Walff model for resonance frequency of covered microstrip patch", Electron. Letters, Vol.27, No.20, PP.1850-1852 26 Sept. 91.
5. A.K.Verma and Z.Rostamy, "Modified walff model for resonance frequency of covered circular patch", Electron. Letters,Vol.27, No. 24, PP. 2234-2236, Nov. 91.

Table 1
Resonant Frequency of Equilateral Triangular Patch by various methods.

| TM | fr exp GHz | $\begin{aligned} & \text { Gang } \\ & \mathrm{fr}[3] \\ & \mathrm{GHz} \end{aligned}$ | $\underset{\%}{\text { Error }}$ | Dahele fr [1] GHz | $\begin{aligned} & \text { Error } \\ & \% \end{aligned}$ | $\begin{aligned} & \text { Garg } \\ & \mathrm{fr}[2] \\ & \mathrm{GHz} \end{aligned}$ | $\underset{\%}{\text { Error }}$ | MWM <br> fr <br> GHz | $\underset{\%}{\text { Error }}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 10 | 1.280 | 1.306 | 2.03 | 1.299 | 1.8 | 1.273 | 0.5 | 1.280 | 0.0 |
| 11 | 2.242 | 2.262 | 0.89 | 2.252 | 1.45 | 2.239 | 0.1 | 2.239 | 0.1 |
| 20 | 2.550 | 2.612 | 2.40 | 2.599 | 1.92 | 2.546 | 0.3 | 2.560 | 0.3 |
| 21 | 3.400 | 3.456 | 1.65 | 3.439 | 1.15 | 3.419 | 0.5 | 3.420 | 0.5 |
| 30 | 3.824 | 3.919 | 2.48 | 3.899 | 1.96 | 3.81 | 0.1 | 3.840 | 0.4 |



NORMALIZED SUPERSTRATE THICKNESS $\left(h_{2} / h_{1}\right)$ EFFECT OF SUPERSTRATE ON Fr

FIG. 2

(b)
(a): EQUILATERAL TRIANGULAR PATCH RESONATOR WITH SUPERSTRATE
(b) : EQUIVALENT RECTANGULAR PATCH

FIG . 1

