NUMERICAL SOLUTION OF SCATTERING FROM CONDUCTING OBJECTS OF RECTANGULAR CROSS-SECTION

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INTRODUCTION - In electromagnetic scattering problems involving conducting objects surface integral equations are often used to study the far and near fields parameters. For three dimensional objects of complex shape surface patches or wire-grids are used to model the scattering surface. Techniques such as moment methods are then applied to reduce the integral equations to a matrix equation, in order to determine the surface current and subsequently the far scattered fields. In the present study the surface integral equation in conjunction with moment methods are also used to investigate the scattering from the complex structures of rectangular shape cross-sections. However, a mapping function is used to analytically describes the object's cross-sectional contour, so that entire domain basis functions of Fourier type can be used to represent the surface current.

The details of the formulation of the E- and H-field integral equations for the electromagnetic scattering from bodies of arbitrary cross-sections were presented in[1-2]. This formulation is based on conformal transformation of object's cross-section to a region outside a circle. Theoretically, the basis functions in the Galerkin method are not the eigenfunctions of the integral operator, enforcing the current modes to be coupled on the surface. As a result, an infinite number of modes contribute to the scattered field. These current modes are related to the wavelength and in turns to the electrical size of the scatterer. In the Raleigh and the resonance regions only a few modes are needed to characterize the electromagnetic behavior of the conducting body. Therefore, one can obtain a numerical solution to the scattered field by including a finite number of modes. The required transformation for the scattering from conducting square surfaces was formulated in[2]. The purpose of this study is to focus on the formulation of the scattering from conducting objects of rectangular cross-section, in order to investigate the variation of the radar cross section due to the polarization of the incident wave and the effect of the mode coupling as a result of perturbation of the cross section from cylinder.

ANALYSIS - For objects of rectangular cross-section, the required transformation is given by[3]

$$\frac{dz}{d\zeta} = C \frac{[(\zeta^2 - a^2)(\zeta^2 - a^{-2})]^{\frac{1}{2}}}{(1 + \zeta^2)^2} \qquad \begin{array}{l} z = x + iy \\ \zeta = u + iv \end{array}$$

which transform the cross-section of the scatterer in the z-plane into the upper half of the ζ-plane, with the boundary transforming into the real axis. A transformation of the form $\zeta = -\tan\frac{\tau}{2}$, with $\tau = \chi + i\xi$ transforms the half-plane region into a (χ,ξ) region outside a circle with the boundary itself being the circle. In the above, C is a constant which depends upon the scale and orientation of the rectangle. The required metric coefficient of the transformation for the rectangular cross-section has the form of

$$\left| \frac{dz}{d\tau} \right| = h(\xi) = \frac{b}{L} \left| (\sin^2\alpha - \sin^2\xi)^{\frac{1}{2}} \right|$$

where, α is a constant which depends on the width to length ratio of the rectangle, and $2b$ is the width of the rectangle. The cross-sectional contour of the rectangle is defined by $\chi = 0$ for $0 \le \xi \le 2\pi$ and may be obtained from

$$x = \frac{b}{2L}[\cos\xi + \sum_{m=1}^{\infty} D_m \cos(2m-1)\xi]$$

$$y = \frac{b}{2L}[\sin\xi - \sum_{m=1}^{\infty} D_m \sin(2m-1)\xi]$$

$$\rho(\xi) = [x(\xi)^2 + y(\xi)^2]^{\frac{1}{2}} \quad , \quad \phi(\xi) = \tan^{-1}\left[\frac{y(\xi)}{x(\xi)}\right]$$

Here, L is a constant and is defined as,

$$L = E - k'^2 K \quad , \quad k'^2 = 1 - k^2 \quad , \quad k = \sin\alpha$$

k is the root of the transcendental equation defined as:

$$(E - k'^2 K)a = (E' - k^2 K')b$$

where, $2a$ is the length of the rectangle and other terms are defined as

$$E = \int_0^{\frac{\pi}{2}} \sqrt{1 - k^2\sin^2\xi}\, d\xi \quad , \quad E' = \int_0^{\frac{\pi}{2}} \sqrt{1 - k'^2\sin^2\xi}\, d\xi$$

$$K = \int_0^{\frac{\pi}{2}} \frac{d\xi}{\sqrt{1 - k^2 \sin^2\xi}} \quad , \quad K' = \int_0^{\frac{\pi}{2}} \frac{d\xi}{\sqrt{1 - k'^2 \sin^2\xi}}$$

The constants D_m are defined as

$$D_m = - \frac{a_m}{2m - 1}$$

and a_m are given in terms of Legendre polynomials as

$$a_0 = 1 \qquad a_1 = -\mu \qquad , \mu = \cos 2\alpha$$

$$a_m = P_m(\mu) - 2\mu P_{m-1}(\mu) + P_{m-2}(\mu)$$

or they can be obtained from the recursive relation given by

$$(m + 1)a_{m+1} - (2m - 1)\mu a_m + (m - 2)a_{m-1} = 0.$$

NUMERICAL RESULTS - Fig. 1 shows the coordinate geometry of the conducting rectangular cross-section. Fig. 2 illustrates the radar cross section of a rectangular plate of $\frac{b}{a} = .5$ and $t = .0317\lambda$ for the plane wave incident along the width of the rectangle. To obtain the numerical result 3 expansion functions are used to model the current along the generating curve of the body. 60 sampling points are used for the sampling of the surface current along the azimuth. Six impulse approximations are used to model the triangular base function along the generating curve. A nonvanishing half-triangle is chosen at an edge. The computed result is compared with the numerical result obtained from the surface patch technique available in the literature[4]. The first mode approximation reasonably replicates the back scattering cross section. Representative results using the dense matrix and selected mode approximations for the convergence of the solution is under investigation.

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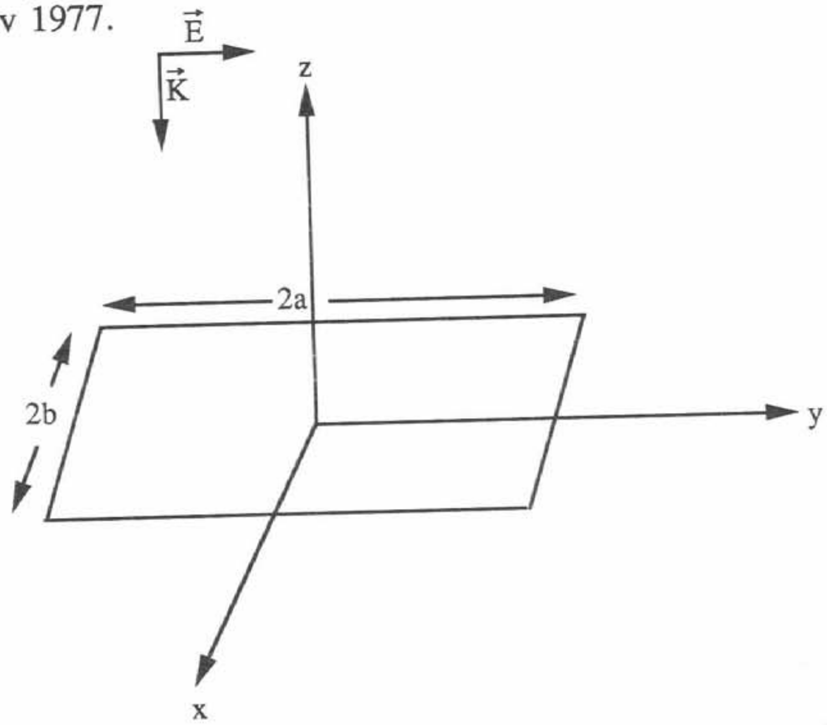

Fig. 1. Coordinate geometry for a conducting rectangular plate.

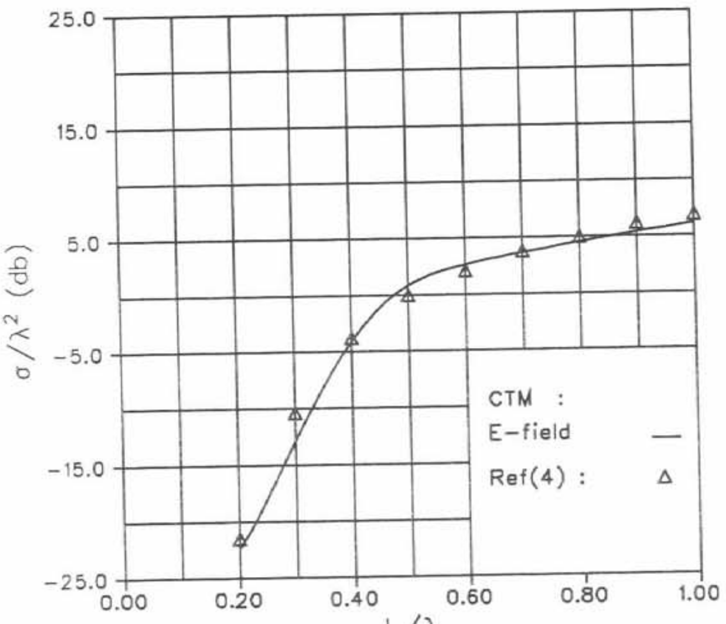

Fig. 2. : Back scattering cross-section vs L/λ for a conducting rectangular plate of $t=.0317\lambda$, $L=2a$, NP=13, MT=3, Nm=1