

**VIRTUAL RAY OF DIFFRACTION: H-POLARIZED DIFFRACTION COEFFICIENTS OF COMPOSITE WEDGE**

Se-Yun Kim  
 Korea Institute of Science and Technology  
 P.O.BOX 131, Cheongryang, Seoul, Korea  
 E-mail: [ksy@imrc.kist.re.kr](mailto:ksy@imrc.kist.re.kr)

1. Introduction

In spite of significant advances in the application of numerical techniques in electromagnetic problems, there have not been comparable achievements in the physical understanding of the scattering and diffraction. Hence a new technique may be needed to obtain analytical expression for the solution of canonical problems. One of such canonical structures, of which exact solution is not available until now, is penetrable wedge. Some heuristic diffraction coefficients of penetrable wedges were constructed by modifying the exact diffraction coefficients of the corresponding perfectly conducting wedge[1]. Those solutions provided acceptably accurate results in some limited cases, but their diffraction coefficients could not satisfy the edge condition at the tip of the composite wedge. A new method, the virtual ray of diffraction(VRD)[2], is suggested to solve the diffraction by the composite wedge illuminated by H-polarized plane wave, as shown in Fig.1.

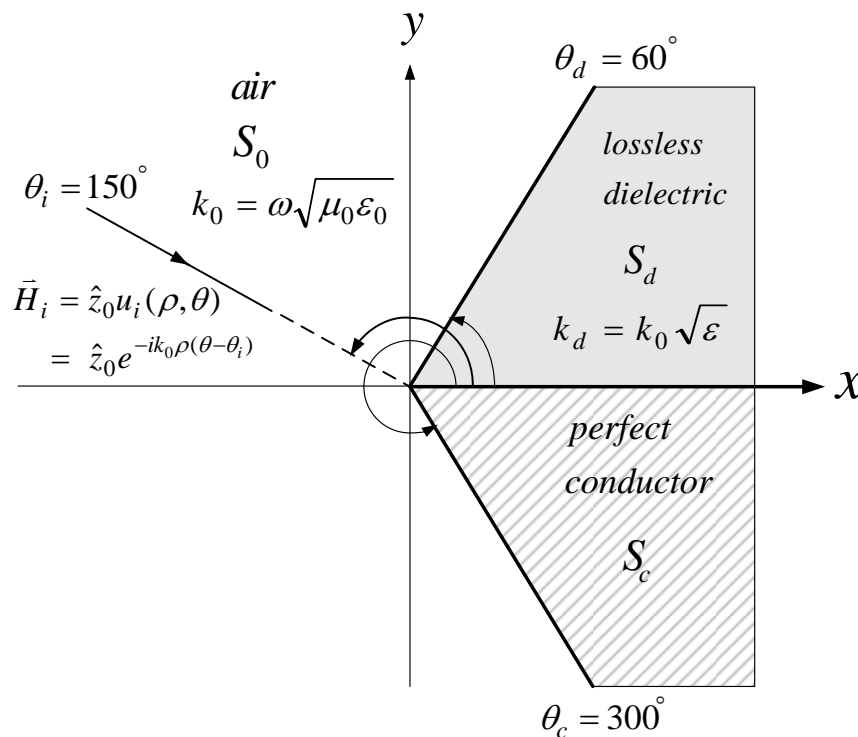


Fig. 1. Geometry of composite wedge consisting of perfect conductor and lossless dielectric illuminated by H-polarized plane wave

The basic concept of the method is inspired from the one-to-one relation between geometrical rays and the corresponding diffraction coefficients[3]. It is well recognized that the conventional PO(physical optics) solution to the composite wedge consists of the exact GO(geometrical optics) term and the edge-diffracted field. In particular, the PO diffraction coefficients are expressed by sum of cotangent functions, of which amplitudes and poles are exactly identical to the amplitudes and propagation

direction of the corresponding geometrical rays in the physical regions, respectively. However, the PO diffraction coefficients reveal erroneous due to those large deviation from zero in the complementary regions.

## 2. Virtual Ray of Diffraction

As the permittivity of its dielectric part increases to infinite, the composite wedge may become a perfectly conducting wedge. In this limiting case, the exact diffraction coefficients are expressed by sum of four cotangent functions with the angular period  $2\pi\nu_\infty$ , where  $\nu_\infty$  can be derived from the edge condition at the tip of the perfectly conducting wedge[4]. In contrast, the PO diffraction coefficients consist of two cotangent functions with the angular period  $2\pi$ . The PO diffraction coefficients may be changed into sum of two cotangent functions among the exact diffraction coefficients only if the angular period is adjusted from  $2\pi$  to  $2\pi\nu_\infty$ . Then the arguing point is whether two additional rays exist in conjunction with two remaining cotangent functions of the exact diffraction coefficients or not. We find that the amplitudes and propagation angles of two additional rays can be obtained only by extending the conventional ray-tracing technique under the assumption that reflections occur on both boundaries of the perfectly conducting wedge. According to the incident angle, only one of two wedge boundaries may be illuminated by an incident plane wave. Even in such incident case, the concept of the complementary region renders the other boundary to be illuminated by the incident ray with the same amplitude but propagation angle of  $2\pi$  angular shift. The amplitudes and propagation angles of two rays reflected by two wedge boundaries are also calculated by employing the conventional ray-tracing technique. This new ray-tracing law provides an extended GO solution consisting of two actual rays in the physical region and two virtual rays in its complementary region. Employing the one-to-one correspondence between geometrical rays and cotangent functions, one may construct the exact diffraction coefficients routinely.

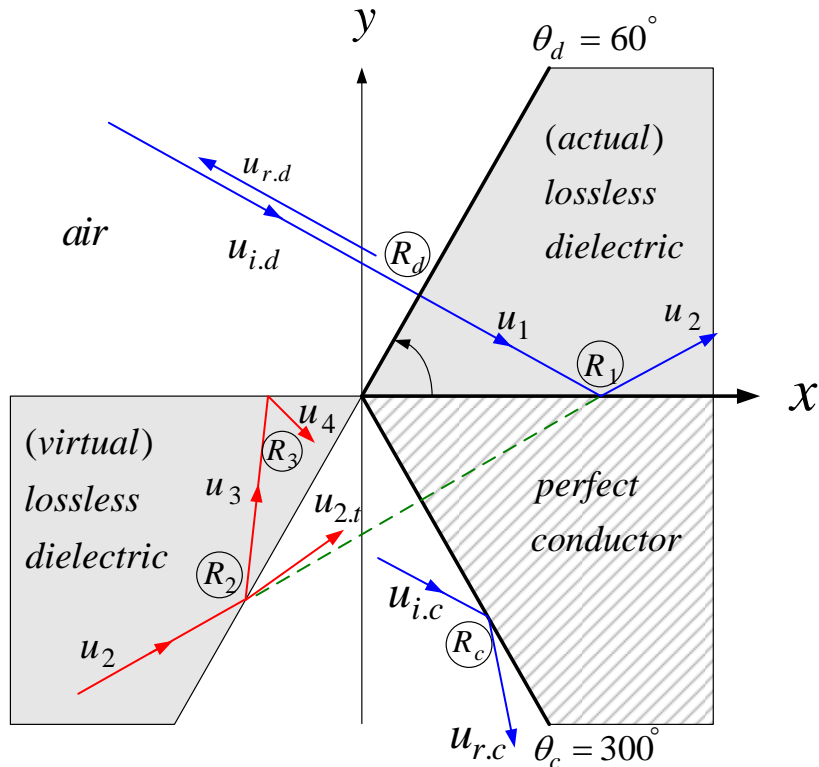


Fig. 2 Ray-tracing for actual rays in physical region and virtual rays in complementary region

The above extended ray-tracing procedure was applied to the diffraction by a composite wedge[5]. In this paper, we consider the incidence of H-polarized plane wave[6], as shown in Fig. 1. The

conventional ray-tracing provides the ordinary GO field as sum of actual rays in the physical region. All of the actual rays inside the dielectric part are also obtained exactly. After termination of internal reflections by actual rays, additional multiple reflections inside the dielectric part generate a number of virtual rays, as shown in Fig.2. But it is not clear how to terminate the virtually internal reflections. We impose two additional conditions to virtual rays. The first condition is so clear that all of the virtual rays should be located only in the complementary regions. The second condition is that the final reflection of virtual rays occurs on the conducting boundary. The last condition is required to satisfy the boundary condition at the conducting boundary. According to the one-to-one correspondence between rays and cotangent functions, the VRD diffraction coefficients are constructed directly by sum of cotangent functions, of which total number is equal to the total number of actual and virtual rays. The amplitude and pole of each cotangent function are taken by the amplitude and propagation angle of the corresponding ray, respectively. And the angular period of the cotangent functions is adjusted to  $2\pi\nu_\varepsilon$ , where  $\nu_\varepsilon$  can be derived from the edge condition at the tip of the composite wedge with relative permittivity  $\varepsilon$  in its dielectric part.

### 3. H-polarized Diffraction Coefficients

To show the validity of the suggested method, the VRD diffraction coefficients in Fig. 4 are compared with the PO diffraction coefficients in Fig. 3 for  $\theta_d = 60^\circ$ ,  $\theta_c = 300^\circ$ , and  $\theta_i = 150^\circ$  as  $\varepsilon$  increases from 1.1 to 1000. It should be noted that the diffraction coefficients for  $\varepsilon = 1$  and infinite are the exact solutions for the perfectly conducting wedges corresponding to  $\theta_d = 0^\circ$  (dotted black line) and  $60^\circ$  (bold black line), respectively. The PO diffraction coefficients in Fig. 3 cannot approach to the exact solution of  $\varepsilon = 1$  even if  $\varepsilon$  decreases to 1.1. And the PO diffraction coefficients of large  $\varepsilon$  in Fig. 3 intersect the exact solution of  $\varepsilon = \infty$ . In particular, all of the PO diffraction coefficients suffer from large deviation from two exact diffraction coefficients on the conducting boundary of  $w = \theta_c = 300^\circ$ . In Fig. 3, one may find that the PO diffraction coefficients cannot become zero in the complementary regions of  $0^\circ < w < 60^\circ$  and  $300^\circ < w < 360^\circ$ .

In contrast, the VRD diffraction coefficients in Fig. 4 approach the corresponding exact diffraction coefficients monotonically as  $\varepsilon$  decreases to 1.1 or increase to 1000. Fig. 4 illustrates the smooth transition from the VRD diffraction coefficients for high  $\varepsilon$  ( $>10$ ) to those for low  $\varepsilon$  ( $<1.3$ ). Such a smooth transition can be implemented by virtually total reflections inside the dielectric part. Hence one may conclude that the VRD technique provides highly accurate diffraction coefficients of a composite wedge consisting of arbitrary dielectric and perfect conductor. According to our formulation of dual integral equations[3], the exact diffraction coefficients have to become zero in the complementary regions. Compared with the PO diffraction coefficients, the VRD diffraction coefficients satisfy the null-field condition quite well in the complementary regions in  $0^\circ < w < 60^\circ$  and  $300^\circ < w < 360^\circ$ .

### REFERENCES

- [1] A. J. Booyesen and C. W. I. Pistorius, "Electromagnetic scattering by a two-dimensional wedge composed of conductor and lossless dielectric," *IEEE Trans. Antennas Propagat.*, vol. AP-40, no. 11, pp. 1702-1708, 1999.
- [2] S. Y. Kim, "Virtual ray of diffraction for composite wedge," *Proc. KJJC on AP/EMC/EMT*, pp. 57-60, Seoul, Korea, Nov. 22-23, 2004.
- [3] S. Y. Kim, J. W. Ra, and S. Y. Shin, "Diffraction by an arbitrary angled dielectric wedge: Parts I and II," *IEEE Trans. Antennas Propagat.*, vol. AP-39, no. 9, pp. 1272-1292, 1991.
- [4] J. Meixner, "The behaviour of electromagnetic fields at edges," *IEEE Trans. Antennas Propagat.*, vol. AP-20, pp. 442-446, 1972.
- [5] S. Y. Kim, "A convenient expression for the diffraction coefficients of a wedge composed of a conductor and a lossless dielectric," *Microwave Opt. Technol. Lett.*, vol. 13, pp. 216-219, 1996.
- [6] S. Y. Kim, "H-polarized diffraction by a wedge consisting of perfect conductor and lossless dielectric," *IEICE Trans. Electron.*, vol. E80-C, no. 11, pp. 1407-1413, Nov. 1997.

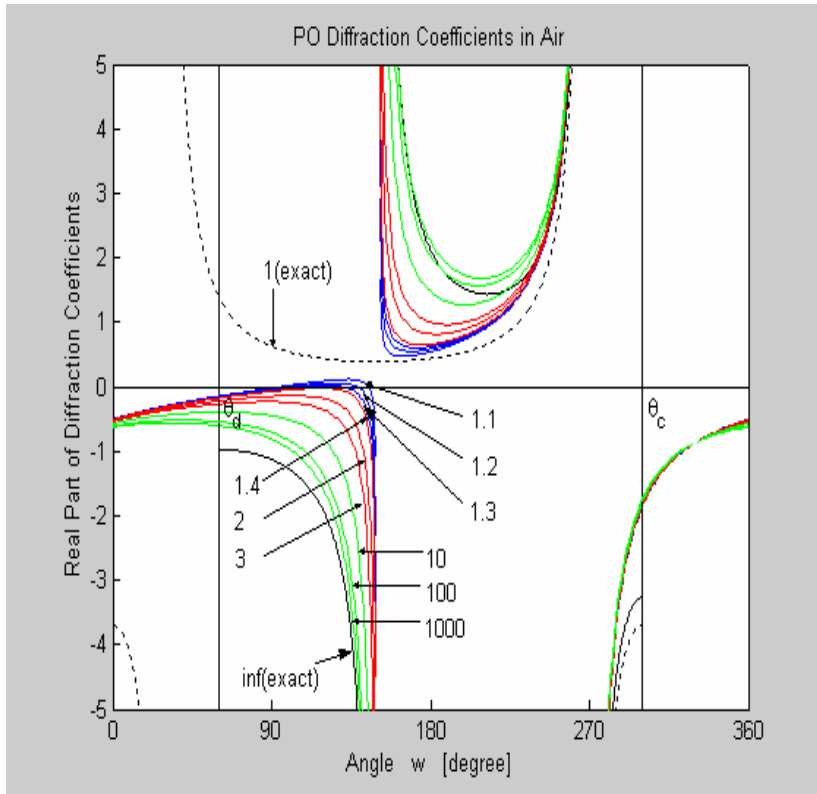


Fig. 3 PO diffraction coefficients for  $\theta_d = 60^\circ$ ,  $\theta_c = 300^\circ$ , and  $\theta_i = 150^\circ$

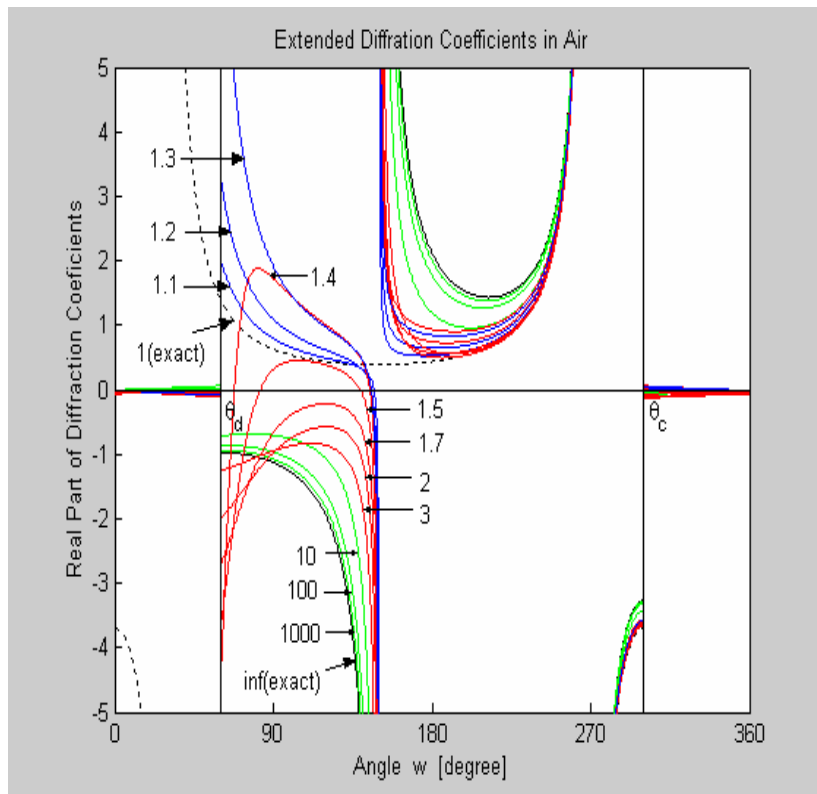


Fig. 4 VRD diffraction coefficients for  $\theta_d = 60^\circ$ ,  $\theta_c = 300^\circ$ , and  $\theta_i = 150^\circ$