SOME FEATURES OF STATISTICAL CHARACTERISTICS OF SCATTERED ELECTROMAGNETIC WAVES IN TURBULENT MAGNETOPLASMA

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1. Introduction

Analyses of statistical characteristics of small amplitude electromagnetic waves passing through a plane layer of turbulent plasma in natural and laboratory conditions are actual problem in many applications from the practical point of view [1,2]. The peculiarities of the angular (spatial) power spectrum (APS) of scattered radiation in turbulent collisional magnetoplasma with diagonal permittivity tensor have been investigated in [3–5]. Statistical characteristics (broadening of the spatial spectrum and displacement of its maximum) in collisional magnetoplasma is analysed in this paper using geometrical optics approximation.

2. Formulation of the problem

The features of magnetoplasma, when the imposed magnetic field is directed towards the Z–axis are described by permittivity tensor [1]

$$\varepsilon_{xx} = \varepsilon_{yy} = \eta' - i\eta'' = 1 - \frac{v(1-is)}{(1-is)^2 - u}\ ;\ \ \varepsilon_{xy} = -\varepsilon_{yx} = \mu' - i\mu'' = i\frac{v\sqrt{u}}{(1-is)^2 - u}\ ;\ \ \varepsilon_{zz} = \varepsilon' - i\varepsilon'' = -\frac{i\,v\sqrt{u}}{1-u}\ ;$$

$$\varepsilon_{xz} = \varepsilon_{zx} = \varepsilon_{yz} = \varepsilon_{zy} = 0\,, \qquad (1)$$

where $v = \omega_p^2/\omega^2$, $u = \omega_H^2/\omega^2$ and $s = \nu_{eff}/\omega$ are non–dimensional plasma parameters, ν_{eff} is the effective collisional frequency, $\omega_p^2 = 4\pi e^2 N/m$ is the plasma frequency, $\omega_H = |e|H/mc$ is the angular gyrofrequency for the magnetic field, *N* is the electron concentration, ω is the angular frequency of electromagnetic field, *e* and *m* are the charge and mass of an electron, *c* is the speed of light in the vacuum. We shall consider high–frequency and non–resonance frequency band, i.e. $(1-\sqrt{u})^2 >> s^2$.

In geometrical optics approximation, phase fluctuation $S(\mathbf{r})$ satisfies following stochastic differential equation:

$$\mathrm{Det}\left[(\nabla S)^2\delta_{ik} - \frac{\partial S}{\partial x_i}\frac{\partial S}{\partial x_k} - k_0^2\varepsilon_{ik}\right] = 0\ , \qquad (2)$$

where: ε_{ik} are permittivity tensor components determined by equation (1), $k_0 = \omega/c$ is the absolute value of the wavevector. We suppose that inhomogeneous layer having thickness z_0 satisfies the conditions: $z_0 < L_0$, $z_0/k_0\ell^2 << 1$ (L_0 is the length of path passing by the wave, ℓ is the characteristic scale of large–scale inhomogeneities). Permittivity fluctuations are caused by fluctuations of both electron concentration *N* and magnetic field *H*. Fluctuation components of

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permittivity are small with respect to their mean values: $\varepsilon_{ik} = \bar{\varepsilon}_{ik} + \delta\varepsilon_{ik}$, where $\bar{\varepsilon}_{ik}$ is the mean value of ε_{ik}; fluctuating component is determined as:

$$\delta\varepsilon_{ik} = \left(\frac{\partial\varepsilon_{ik}}{\partial N}\right)_H N_1 + \left(\frac{\partial\varepsilon_{ik}}{\partial H}\right)_N H_1 \ , \tag{3}$$

N_1 and H_1 are fluctuations of both electron concentration and magnetic field strength, respectively, which are random functions of the spatial coordinates. We suppose that the mean values do not depend on spatial coordinates. In this case, the solution could be sought as $S = k_0 n(\boldsymbol{\tau}\mathbf{r}) + \varphi_1(\mathbf{r})$, where *n* is the refractive index, $\boldsymbol{\tau}$ is the unit vector towards the direction of wave propagation, φ_1 is the phase fluctuation. We suppose that the vector $\boldsymbol{\tau}$ is lying in the YZ plane. Substituting *S* into equation (2), assuming that phase fluctuations are small, from the zero-th approximation *n* could be determined; in the first approximation with respect to $\delta\varepsilon_{ik}$, the phase fluctuation satisfies the following stochastic differential equation:

$$a_y \frac{\partial\varphi_1}{\partial y} + a_z \frac{\partial\varphi_1}{\partial z} = A_\nu \delta\nu + A_h \delta h \, , \tag{4}$$

where:

$$a_y = a'_y + i a''_y, \ \ a_z = a'_z + i a''_z, \ \ a'_y = n\sin\Theta\left[(\eta' - n_1^2)(\eta' + \varepsilon') + \sin^2\Theta \cdot n_1^2(\varepsilon' - \eta') - \mu''^2\right] ,$$

$$a''_y = n\sin\Theta\left\{(\eta' + \varepsilon')(n_2^2 - \eta'') - (\eta' - n_1^2)(\eta'' + \varepsilon'') + \sin^2\Theta\left[n_1^2(\eta'' - \varepsilon'') - n_2^2(\varepsilon' - \eta')\right] - 2\mu'\mu''\right\} ,$$

$$a'_z = n\cos\Theta\left[2\varepsilon'(\eta' - n_1^2) + \sin^2\Theta \cdot n_1^2(\varepsilon' - \eta')\right] ,$$

$$a''_z = n\cos\Theta\left\{2\left[\varepsilon'(n_2^2 - \eta'') - \varepsilon''(\eta' - n_1^2)\right] + \sin^2\Theta\left[n_1^2(\eta'' - \varepsilon'') - n_2^2(\varepsilon' - \eta')\right]\right\} , \tag{5}$$

Θ is the angle between the direction of wave propagation and imposed magnetic field, $\delta\nu = N_1 / N_0$ and $\delta h = H_1 / H_0$ are relative fluctuations of concentration and magnetic field, respectively. Explicit expressions for the other coefficients have been omitted due to the limited space. Refractive index for collisional plasma is determined by the following expression [1]

$$n^2 = 1 + \frac{2v(1 - v - is)}{2(1 - is)(1 - v - is) - u\sin^2\Theta \pm \sqrt{u^2\sin^4\Theta + 4u(1 - v - is)^2\cos^2\Theta}} \ . \tag{6}$$

The sign "+"refers to the ordinary wave and the sign "–" refers to the extraordinary wave. In the medium with large–scale inhomogeneities, i.e. at $\lambda / 2\pi\ell << 1$ (λ is the wavelength) amplitude fluctuations are small enough and the main influence of inhomogeneities are revealed due to the phase distortion. Applying the boundary condition at $\varphi_1 |_{z=0} = 0$, the solution of equation (4) and, hence, the transversal correlation function of the phase, at arbitrary correlation function of concentration $W_N(\boldsymbol{\rho}) = < N_1(\mathbf{r}) N_1(\mathbf{r}') >$ and magnetic field $W_H(\boldsymbol{\rho}) = < H_1(\mathbf{r}) H_1(\mathbf{r}') >$ fluctuations, could be found using Fourier transformation:

$$W_\varphi(\rho_x, \rho_y, z) = < \varphi_1(x, y, z)\varphi_1^*(x + \rho_x, y + \rho_y, z) > = 2\pi z \int_{-\infty}^{\infty} d\text{æ}_x d\text{æ}_y \exp(i\text{æ}_x\rho_x + i\text{æ}_y\rho_y) \times$$

$$\times \frac{1}{2a\,\text{æ}_y}\left[1 - \exp(-2a\,\text{æ}_y z)\right]\left[\frac{D^2 + E^2}{N_0^2} W_N\left(\text{æ}_x, \text{æ}_y, -\frac{a'_y}{a'_z}\text{æ}_y\right) + \frac{F^2 + G^2}{H_0^2} W_H\left(\text{æ}_x, \text{æ}_y, -\frac{a'_y}{a'_z}\text{æ}_y\right)\right] \ , \tag{7}$$

where D, E, F and G coefficients are complicated functions of permittivity components. Statistical independence of $\delta\nu$ and δh are taken into account.

Two-dimensional spectral density of correlation function of fluctuation of both concentration and magnetic field has the form:

$$W(æ_x, æ_y, \xi) = \frac{\ell_\perp \overline{\ell}}{4\pi} \exp\left(-\frac{æ_x^2 \ell_\perp^2 + æ_y^2 \overline{\ell}^2}{4} \right) \exp\left(-\frac{\xi^2 \overline{\ell}^2}{\ell_\perp^2 \ell_\parallel^2} \right) \exp\left[-i \frac{\overline{\ell}^2 (\ell_\perp^2 - \ell_\parallel^2) \sin\alpha \cos\alpha}{\ell_\perp^2 \ell_\parallel^2} æ_y \xi \right], \quad (8)$$

where $\overline{\ell} = \ell_\perp \ell_\parallel / \sqrt{\ell_\perp^2 \sin^2\alpha + \ell_\parallel^2 \cos^2\alpha}$. If the wave is propagating along the external magnetic field, $\Theta = 0$, at propagation along the direction of nonisotropic inhomogeneities, $\Theta = \alpha$. In medium with high elongated inhomogeneities toward Z–axis, $\alpha = 0$.

If the regular phase difference between the two observation points located on the line normal to the layer is neglected, correlation function of the field may be written as [3–5]: $W_E(\rho_x, \rho_y, z) = E_0^2 < \exp\{i\varphi_1(x, y, z) - i\varphi_1^*(x + \rho_x, y + \rho_y, z)\} >$. In the most interesting case, at strong phase fluctuations $< \varphi_1 \varphi_1^* > >> 1$, we may assume that they are normally distributed [6]. The APS of scattered radiation, which is under the great practical importance, is determined as the Fourier transformation from the correlation function of scattered field [6]. Knowledge of correlation function of the phase allows us to calculate the broadening of the APS and displacement of its maximum using the expressions [3–5]:

$$\Delta k_x = \frac{1}{i} \frac{\partial W_\varphi}{\partial \rho_x}\bigg|_{\rho_x = \rho_y = 0}, \quad < k_x^2 > = -\frac{\partial^2 W_\varphi}{\partial \rho_x^2}\bigg|_{\rho_x = \rho_y = 0}, \quad < k_y^2 > = -\frac{\partial^2 W_\varphi}{\partial \rho_y^2}\bigg|_{\rho_x = \rho_y = 0}. \quad (9)$$

Dependences of broadening of the APS and displacement of its maximum versus angle Θ in the YZ and XZ planes at $\alpha = 15^0$ and at parameter of anisotropy $\mu = \ell_\perp / \ell_\parallel = 1:2.7$ are presented in Fig.1–3. From these figures it follows that collision between the particles leads to asymmetry of the APS for both scattered waves; for the extraordinary wave crevasse has strongly pronounced form in the YZ plane. In all figures curve 1 corresponds to the extraordinary wave, curve 2 corresponds to the ordinary wave and curve 3 to the non–collisional plasma $s = 0$.

Expression for transversal correlation function of the phase, when tensor components ε_{ik} are functions of Z, has been obtained. The results could be valid for the route of the arbitrary length. Using the well–known representation of regular variation of electron concentration in F_2–layer of the ionosphere [1,7], we consider the lower part of this layer, where the variation of the mean concentration has the parabolic form: $N(z) = N_0[1 - (z/L)^2]$ (L is the effective thickness of the lower part of the *F*–layer). Two limiting cases, when relative dispersion of electron concentration $\delta N = \sqrt{< N_1^2 > / N_0^2} = \text{const}$ and $< N_1^2(z) > = \text{const}$, have been studied at $\delta N \simeq 2{,}5 \cdot 10^{-3}$ (morning and daytime) and at $\delta N \simeq 3{,}8 \cdot 10^{-3}$ (at night) [1]. Inhomogeneities lead to the variation of group velocity of a signal, which does not coincide with the light speed *c* in vacuum. Corrections to the group and phase paths, caused by the inhomogeneities, are coincided within the sign at wave propagation in ionosphere and troposphere. Therefore, their dispersions are the same $\sigma_g^2 = \sigma_\varphi^2 = < \varphi_1^2 > / k_0^2$ [7]. Dispersion of the phase at $h = 250$ km, $\omega = 40$ MHz, $z_0 \sim 1{,}5 \cdot 10^2$ km, $\ell \sim 1$ km is equal to $< \varphi_1^2(z_0) > \sim (1 \div 2.3)$. Hence, $\sigma_\varphi \approx (7{,}5 \div 11.5)$ m, which is in a good agreement with experimental data obtained on the basis of other methods [7]. Figure 4 illustrates the dependence of relative intensity of fluctuations $< \varphi_1^2(z_0) >$ versus the thickness of the scattered layer. Solid line corresponds to the case $(\delta N)^2 = \text{const}$ and dashed line to the case $< N_1^2 > = \text{const}$. It is known [8] that the intensity of phase fluctuations is gradually increasing in the range of $(z_0 / L) \sim 0 \div 1$ at waves reflection from the ionospheric layer in proportion to the distance passing by the wave in turbulent plasma. Figure 4 vividly shows, that the substantial difference between these curves could be observed at $(z_0 / L) \sim (0{,}35 \div 0{,}6)$. Intensity of phase fluctuations at $(\delta N)^2 = \text{const}$ always exceeds intensity of phase fluctuations at $< N_1^2 > = \text{const}$.

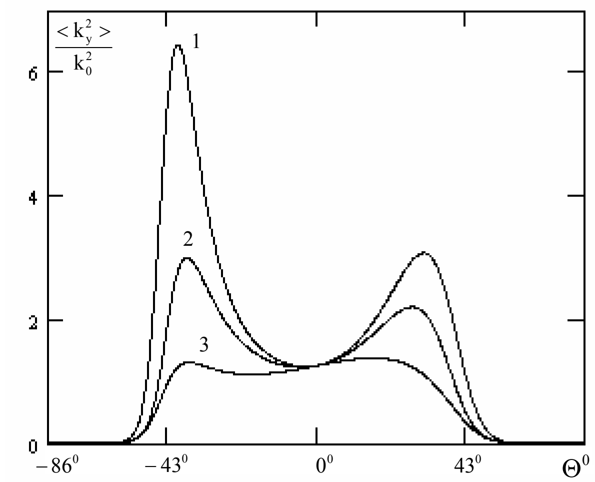

Fig.1. The dependence of broadening of the APS versus angle Θ^0 in the YZ plane at $\alpha = 15^0$; $\mu = 1:2.7$.

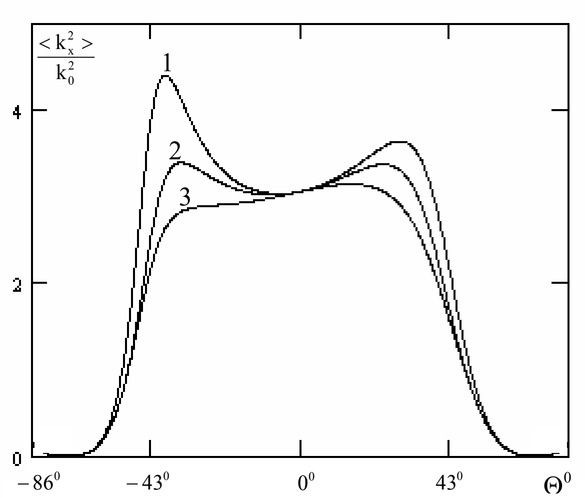

Fig.2. The dependence of broadening of the APS versus angle Θ^0 in the XZ plane at $\alpha = 15^0$; $\mu = 1:2.7$.

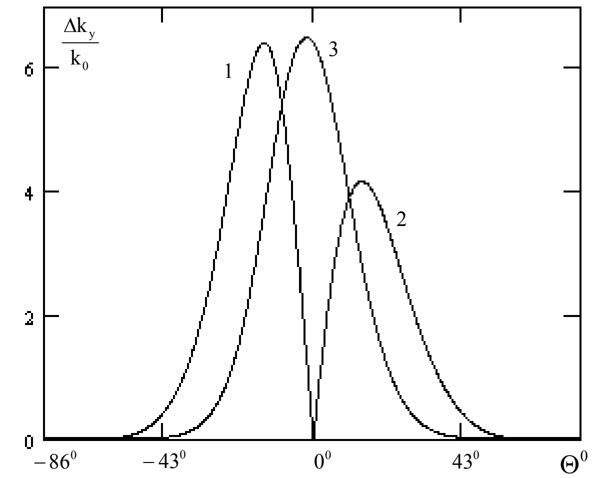

Fig.3. The dependence of displacement of maximum of the APS versus angle Θ^0 at $\alpha = 15^0$; $\mu = 1:1.3$.

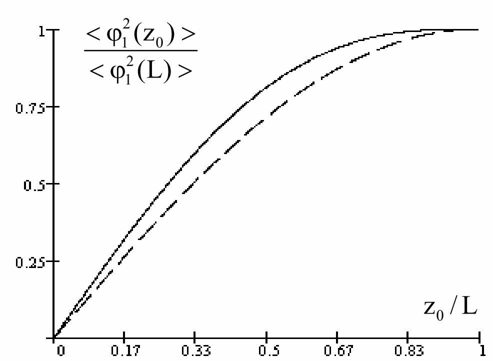

Fig.4. The dependence of intensity of phase fluctuation $<\varphi_1^2(z_0)>$ versus non–dimensional spatial parameter z_0 / L.

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