A Partially Adaptive Eigenbased Method for DOA Estimation of Fractional -Bandwidth Wideband Signals

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1. Introduction

The use of eigenbased techniques for direction-of-arrival (DOA) estimation has been studied extensively in recent years. Many methods and algorithms have been reported in the literatures. [1] - [4] are some of the eigenbased methods. In their treatments, signals are assumed to be purely wideband or narrowband. However, there are situation in which we have to deal with mixed signal environment. For example in a wideband application one may encounter a fractional bandwidth or narrowband interference. This paper describes an algorithm used to detect interferences in a mixed signal environment.

2. Eigenbased DOA Estimation

Consider a uniform circular array consisting with L sensor elements and J tapped delays in each channel. Let $\underline{W}_l = (\underline{w}_1^T , \underline{w}_2^T , ... , \underline{w}_J^T)^T$, be the LJ-dimensional weight vector which is designed to achieve a closest approximation to the desired response, over a frequency band of interest $[f_l , f_u]$ in a particular look direction, where $[\underline{w}_k]_i = w_{(k-1)L+i}$, k=1,2,....,J and i= 1, 2, ..., L. The frequency response of the weight vector to a plane wave front impinging from (in general) a three-dimensional location, θ is given by

$$H_u (f, \theta) = \underline{W}_l^H \underline{S}(f, \theta) \tag{1}$$

where $\underline{S}(f, \theta)$ is a LJ-dimensional time delayed steering vector given by

$$\underline{S} (f, \theta) = [1, \exp(-j2\pi fT_d), ..., \exp(-j2\pi f(J-1)T_d)]^T \otimes \underline{\alpha}(f, \theta) \tag{2}$$

where $\underline{\alpha}(f, \theta)= [\exp(j2\pi f\tau_1), \exp(j2\pi f\tau_2), ..., \exp(j2\pi f\tau_L) ^T$ is the array manifold and $\{\tau_k, k=1, 2, ..., L\}$ is the propagation delay from the array origin to the k^{th} sensor for the source location θ; T_d is the temporal sample delay and $\otimes$ denotes Kronecker product. The coefficient vector $\underline{W}_l$ is chosen such that integral of the squared magnitude of the error between the desired look direction response, $A(f, \theta_0)$ and the response of the weight vector, $H_u(f, \theta_o)$ over a frequency band of interest is minimized, namely

$$\underset{\underline{W}_l}{\text{minimize}}\ e_l^2$$

where
$$e_l^2 = \int_{f_l}^{f_u} |A(f, \theta_o) - H_u(f, \theta_o)|^2 \, df \tag{3}$$

The solution to this optimisation problem is

$$\mathbf{Q}_l \underline{W}_l = \underline{P}_l \tag{4}$$

where $\mathbf{Q}_l$ is the LJxLJ dimensional matrix and $\underline{P}_l$ is the LJ-dimensional vector given by

$$[Q_l]_{k,l} = \Psi \{ (\tau_j - \tau_i) + (m-n)T_d \} \quad (5)$$
$$k = i + (m-1)L, \qquad l = j + (n-1)L$$
$$i, j = 1, 2, ..., L, \qquad m, n = 1, 2, ..., J.$$

$$\underline{P}_l = [p_1^T, p_2^T, ..., p_J^T]^T \quad (6)$$

where

$$\Psi (\tilde{\tau}) = f_u \operatorname{sinc}(2\pi f_u \tilde{\tau}) - f_l \operatorname{sinc}(2\pi f_l \tilde{\tau} \quad (7)$$

$$[\underline{p}_k]_i = \int_{f_l}^{f_u} \frac{1}{2} \{ A^*(f,\theta_0) \exp[j2\pi f(\tau_i-(k-1)T_d)] + A(f,\theta_0) \exp[-j2\pi f(\tau_i-(k-1)T_d)]\}df$$
$$i = 1, 2, ..., L \qquad k = 1, 2, ..., J \quad (8)$$

and * denote the complex conjugate. The weight vector $\underline{W}_l$ can then be determined using equation (4) with the assumption that the required response at the look direction be a flat power spectrum over the frequency band of interest, i.e. $\{A(f, \theta_0)=\exp(j2\pi f\tau_o), f\in(f_l , f_u) \}$. A search routine may be incorporated to find a optimum τ_o.

Since $\mathbf{Q}_l$ matrix is obtained by integrating the power response $\rho(f, \theta)$ of a multichannel tapped delay line filter over the frequency band of interest $[f_l, f_u]$, it is a positive semidefinite symmetric matrix and hence can be factorised as

$$\mathbf{Q}_l = \Gamma\Lambda\Gamma^T \quad (9)$$

where $\Gamma = [\mathbf{U}_1 , \mathbf{U}_2]$ is the LJxLJ dimensional eigenvector matrix , and $\mathbf{U}_1$ is consisting of a set of eigenvectors spanned in the range space corresponding to the desired look direction, and $\mathbf{U}_2$ is another set of eigenvectors spanned in the null space. Λ is a diagonal matrix whose elements are equal to the eigenvalues of the $\mathbf{Q}_l$ matrix. Let the diagonal matrix be given by $\Lambda = \operatorname{diag} [\lambda_1, \lambda_2, ..., \lambda_{LJ}]$ with $\lambda_1 \geq \lambda_2 \geq .. \geq \lambda_p \geq \lambda_{p+1} \geq \lambda_{p+2} ...... \geq \lambda_{LJ} \geq 0$. The first group of the eigenvalues, $\{ \lambda_i, i = 1, 2, ..., p \}$ have magnitudes much greater than zero and are spanned in the range space of the desired look direction. In a system consisting of 10 sensor elements and 5 delay taps, the dimension of $\mathbf{Q}_l$ is 50 but p is only closed to 10. Hence most of the eigenvectors in the $\mathbf{Q}_l$ matrix are spanned in the null space. This is clearly shown in the frequency response plot in Fig 1. The desired look direction is assumed at 180^0. For a wideband signal impinging from the desired look direction, the frequency responses of each eigenvector in the range space show that the signal is preserved in the range of frequency interest. On the other hand, each eigenvector in the null space behalf as a spatial filter to the signal arrived from the desired look direction.

Let $\mathbf{U}_2$ be a LJxM transformation matrix consisting of some or all of the null space eigenvectors. The transformed signal covariance matrix, $\mathbf{R}_U$ is given by

$$\mathbf{R}_U = E \{ [\mathbf{U}_2^H \underline{X}(t)]. [\mathbf{U}_2^H . \underline{X}(t)]^H \} = U_2^H . \mathbf{R} . U_2 \quad (10)$$

where $\mathbf{R}$ is the signal covariance matrix given by

$$\mathbf{R} = E \{ \underline{X}(t) . \underline{X}^H(t) \} = \sum_{i=1}^{m} \mathbf{R}_i^D + \mathbf{R}^W \quad (11)$$

In equation (11) we assume that there are m directional wideband sources and also that the noise is white and uncorrelated. $\mathbf{R}_i^D$ is the matrix formed by all the signals and $\mathbf{R}^W$ is a matrix represents all the white noise. For simplicity we neglect the effects due to manifold errors.

The dimension of $\mathbf{R}_U$ is only MxM and hence it is easier to decompose $\mathbf{R}_U$ as compared to the signal covariance matrix **R** which has a dimension of LJxLJ. Decomposing $\mathbf{R}_U$ and let $\underline{e}_m$ be an eigenvector of $\mathbf{R}_U$ in the noise sub-space. Since an eigenvector in the noise sub-space is orthogonal to a steering vector corresponding to the signal sub-space, it follows that $\| \underline{e}_m^H \cdot \mathbf{U}_2^H \cdot \underline{S}(f, \alpha)\| \approx 0$ if there is a signal impinging from the direction α. Hence the DOA of the signals can be determined by identifying the optimum peaks in the spatial plot, i.e.

$$\rho(\alpha) = 10 \log_{10} \{ 1/ \| [\underline{e}_m^H \cdot \mathbf{U}_2^H \cdot \underline{S}(f, \alpha)] \|^2 \} \qquad (12)$$

To cater for the DOA estimation in a mixed signal environment comprising wideband signals as well as fractional bandwidth signals, the spatial plot may be obtained by modifyint equation (12) as follows :-

$$\rho(\alpha) = 10 \log_{10} \{ \sum_{i=1}^{N_f} 1/ \| [\underline{e}_m^H \cdot \mathbf{U}_2^H \cdot \underline{S}(f_i, \alpha)] \|^2 \} \qquad (13)$$

where N_f is the number of frequency segments in the frequency range of interest $[f_l , f_u]$. In our simulation, with N_f set to 10, fractional bandwidth signal 1% bandwidth can be detected by the algorithm.

3. Simulation Results

In all our simulations we assume that the system was a double-ring 10 elements uniform circular array with outer diameter d_o = 10 r and inner diameter d_i = 8 r , where r is the inter-ring spacing given by $r = \mu_u \cdot \lambda_u$ and λ_u is the wavelength corresponding to the upper cutoff frequency in the frequency band of interest, and μ_u is a dimensionless spatial sampling factor which is set at 0.25 in the simulation. The sampling frequency was set as reference at 1 p.u. and the frequency band of interest is [0.125, 0.25]. Five taps in each element output were used with the a tapped-delay time T_d equal to the sampling time.

In the first experiment we deal with only wideband sources. The source scenario consists of three uncorrelated wideband signals with frequency band of [0.1215, 0.25] impinging from 180^0 (the desired look direction), 80^0 and 220^0. The power of each source are respectively -6, 6 and 0 dB. The noise is assumed to be white and its power at -20 dB. To estimate the DOA of the sources under this source scenario, it is evident that one may not have to use all the eigenvectors in the null space. The number of null eigenvectors required to form $\mathbf{U}_2$ is determined by the frequency band as well as the number of sources. Figure 2 shows the results using 15, 20 and 25 null eigenvectors. Although fewer than 15 null eigenvectors may be used to form $\mathbf{U}_2$, the result obtained may not be convincing especially in an weak signal environment.

In the second experiment we dealt with mixed signals environment. The source scenario consists of a desired signal with bandwidth [0.125, 0.25], and two closely spaced signals, one with bandwidth equal to 10% at a centre frequency of 0.1875 and the other with a bandwidth of 1 % with centre frequency varied from 0.14 to 0.22. The incident angles of the two closely spaced sources are 226° and 229° respectively and the desired look direction is 180°. The power of each source is assumed at 0 dB and the white noise is -20 dB. In the simulation, we use only 20 null eigenvectors. The spatial plot depicted in figure 3 shows clearly the DOA of the two closely spaced, fractional bandwidth sources.

4. Conclusion

This paper has presented a method in estimating the DOA of sources under pure wideband environment and mixed environment with fractional bandwidth signals. The method presented is fast and efficient because the decomposition process involves only matrix with size less than halve of normal matrix size.

5. References

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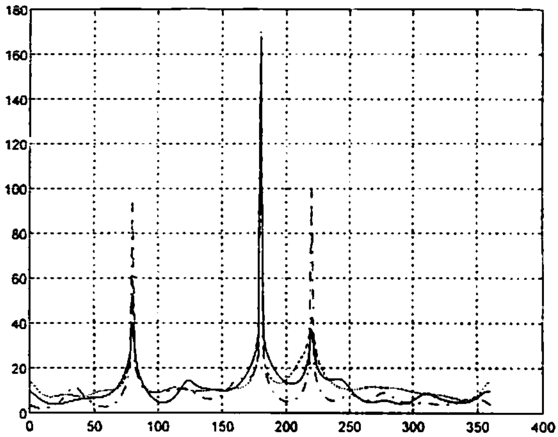

Fig 2 : Spatial spectrum plot of three wideband sources impinging from 80°, 180° and 220°. Number of null eigenvectors used are 15 for solid line, 20 for dashed line and 25 for dashdot line.

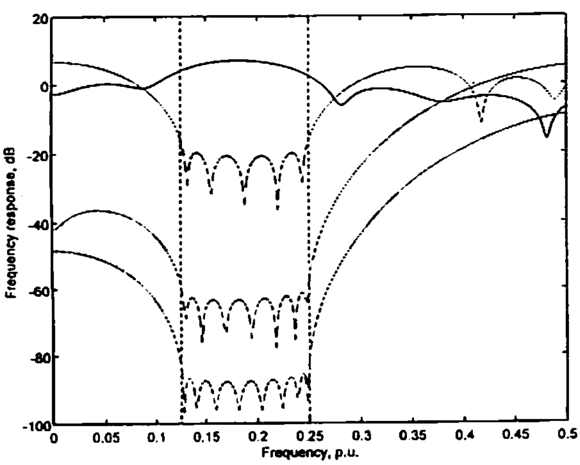

Fig 1 : Frequency responses of the eigenvectors corresponding to, (from the top) λ_1, λ_{11}, λ_{16} and λ_{50} respectively. Frequency range of interest is [0.125, 0.25]; sampling frequency =1.

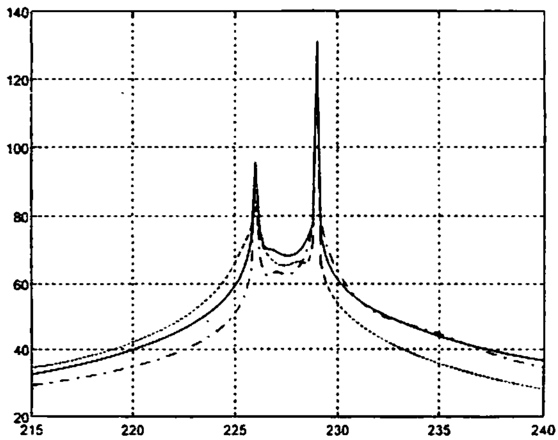

Fig 3 : Spatial spectrum plot for closely spaced mixed signal envoronment. A 10% bandwidth source with centre frequency 0.1875 impinging from 226°, and a 1% bandwidth source with centre frequency 0.1875 (solid), 0.14 (dashed) and 0.22 (dashdot) respectively impinging from 229°. The desired source is at 180°.