SCATTERING BY A FINITE PERIODIC ARRAY OF CYLINDERS

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INTRODUCTION

The periodic structures are used in many devices such as filters, polarizers, and diffraction elements, so that it is important to investigate the scattering characteristics of the periodic structures. Most papers have dealt with infinite periodic structures to which the Floquet theorem is applicable. Recently, Matsushima and Itakura [1] have reported the effects of edges of an array with a finite number of strips. To large periodic structures with a finite number of unit cells, few effective numerical techniques have been applied because of necessity of handling a large number of unknowns. Facq [2], however, showed that the method of moments is applicable to large finite periodic structures by noting that the coefficient matrix of a system of linear equations is block Toeplitz.

In this paper, we investigate the scattering of electromagnetic wave incident upon a large, but finite, number of equally spaced parallel conducting cylinders with arbitrary shape. Boundary element method [3] is used to transform an integral equation for the induced current into a matrix equation. To obtain the solution of the matrix equation, we can use the conjugate gradient method combined with the fast Fourier transform [4], which reduces the execution time in computing and saves the storage on a computer, by extending the coefficient block Toeplitz matrix to a circular matrix.

FORMULATION BY BOUNDARY ELEMENT METHOD

Consider the finite number of infinitely long perfectly conducting cylinders. Each cylinder is parallel to the z direction and is uniformly spaced from its neighbors by a distance d in the x direction as shown in Fig.1.

Let TE wave $\mathbf{E}^i(\boldsymbol{\rho})(= E^i(\boldsymbol{\rho})\hat{z})$ and TM wave $\mathbf{H}^i(\boldsymbol{\rho})(= H^i(\boldsymbol{\rho})\hat{z})$ incident upon the array where $\boldsymbol{\rho}$ is the position vector of the field point and $\hat{z}$ is the unit vector in the z direction. It is well known that the electric and magnetic integral equation for each induced surface current K are expressed as

$$E^i(\rho) = \frac{1}{4}\eta k \sum_{n=1}^{N} \int_{\Gamma_n} K(\rho_n')\, G(\rho_n'|\rho)\, dc_n' \qquad \text{for TE wave} \quad (1a)$$

$$H^i(\rho) = -\frac{1}{2}K(\rho) - \frac{j}{4}\sum_{n=1}^{N} \int_{\Gamma_n} K(\rho_n') \frac{\partial G(\rho_n'|\rho)}{\partial n'}\, dc_n' \qquad \text{for TM wave} \quad (1b)$$

$$G(\rho_n'\ \rho) = H_0^{(2)}(k|\rho - \rho_n'|) \quad (2)$$

where $k(=2\pi/\lambda)$ and $\eta(=\sqrt{\mu/\varepsilon})$ are the wave number and the intrinsic impedance in free space, respectively. Γ_n indicates the surface of the nth cylinder, $H_0^{(2)}$ is the zero-order Hankel function of the second kind, and ρ_n' is the position vector of the source point on the nth cylinder. $\partial/\partial n'$ implies the derivative in the direction of an unit outward nomal $\hat{n}(\rho_n')$ at ρ_n'.

According to the boundary element method, the surface of each cylinder is divided into Q elements. The αth element of the nth cylinder is indicated by $\Gamma_{n\alpha}$ and two nodal points of $\Gamma_{n\alpha}$ are represented by $P_{n\alpha}$ and $P_{n\alpha+1}$. The value of K along element $\Gamma_{n\beta}$ is assumed to be interpolated from the values $K_{n\beta}$ and $K_{n\beta+1}$ at the nodal points $P_{n\beta}$ and $P_{n\beta+1}$ as

$$K(\rho_n) = I^1_{n\beta}(\theta_n)K_{n\beta} + I^2_{n\beta}(\theta_n)K_{n\beta+1} \tag{3}$$

where

$$\left.\begin{array}{l} I^1_{n\beta}(\theta_n) = (\theta_{n\beta+1} - \theta_n)/(\theta_{n\beta+1} - \theta_{n\beta}) \\ I^2_{n\beta}(\theta_n) = (\theta_n - \theta_{n\beta})/(\theta_{n\beta+1} - \theta_{n\beta}) \end{array} \quad (\theta_{n\beta} \leqq \theta_n \leqq \theta_{n\beta+1}) \right\} \tag{4}$$

In Eq.(4), $\theta_{n\beta}$ is the azimuth angle of the vector from the origin O_n to the nodal point $P_{n\beta}$. We assume $\theta_{n1} = 0$ and $\theta_{nQ+1} = 2\pi$. Substituting Eq.(3) into Eqs.(1a) and (1b), we obtain

$$E^i(\rho) = \frac{1}{4}\eta k \sum_{n=1}^{N}\sum_{\beta=1}^{Q}\left[{}^1A^\beta_n(\rho)K_{n\beta} + {}^2A^\beta_n(\rho)K_{n\beta+1}\right] \tag{5a}$$

$$H^i(\rho) = -\frac{1}{2}K(\rho) - \frac{j}{4}\sum_{n=1}^{N}\sum_{\beta=1}^{Q}\left[{}^1B^\beta_n(\rho)K_{n\beta} + {}^2B^\beta_n(\rho)K_{n\beta+1}\right] \tag{5b}$$

where

$${}^sA^\beta_n(\rho) = \int_{\theta_{n\beta}}^{\theta_{n\beta+1}} I^s_{n\beta}(\theta'_n)\, G(\rho'_n|\rho)\, g(\theta'_n)\, d\theta'_n \tag{6a}$$

$$(s = 1, 2)$$

$${}^sB^\beta_n(\rho) = k\int_{\theta_{n\beta}}^{\theta_{n\beta+1}} I^s_{n\beta}(\theta'_n)\{\hat{n}(\rho'_n)\cdot\hat{R}(\rho'_n,\rho)\}H^{(2)}_1(k|\rho-\rho'_n|)\,g(\theta'_n)\,d\theta'_n \tag{6b}$$

$$g(\theta'_n) = \left[\{f(\theta'_n)\}^2 + \{df(\theta'_n)/d\theta'_n\}^2\right]^{1/2} \tag{7}$$

In Eq.(6b), $\hat{R}(\rho'_n,\rho)$ denotes an unit vector in the direction $\rho - \rho'_n$ and $H^{(2)}_1$ is the first-order Hankel function of the second kind. The boundary of the cross section of the nth cylinder is expressed by the curve $r_n = f(\theta_n)$ in the polar coordinate (r_n, θ_n) associated with the cylinder. Let the field point be coincided with the nodal point $P_{m\alpha}$ whose position vector is defined by $\rho_{m\alpha}$. Then Eqs.(5a) and (5b) lead to

$$E^i_{m\alpha} = \sum_{n=1}^{N}\sum_{\beta=1}^{Q} A^{\alpha\beta}_{mn} K_{n\beta} \tag{8a}$$

$$(1 \leqq m \leqq N,\ 1 \leqq \alpha \leqq Q)$$

$$H^i_{m\alpha} = \sum_{n=1}^{N}\sum_{\beta=1}^{Q} B^{\alpha\beta}_{mn} K_{n\beta} \tag{8b}$$

$$A^{\alpha\beta}_{mn} = \frac{1}{4}\eta k\left({}^1A^{\alpha\beta}_{mn} + {}^2A^{\alpha\beta-1}_{mn}\right) \tag{9a}$$

$$B^{\alpha\beta}_{mn} = -\frac{1}{2}\delta_{mn}\delta_{\alpha\beta} - \frac{j}{4}\left({}^1B^{\alpha\beta}_{mn} + {}^2B^{\alpha\beta-1}_{mn}\right) \tag{9b}$$

where δ_{pq} is the Kronecker delta function and ${}^sA^{\alpha\beta}_{mn}$ (${}^sB^{\alpha\beta}_{mn}$) and $E^i_{m\alpha}$ ($H^i_{m\alpha}$) are values of ${}^sA^\beta_n$ (${}^sB^\beta_n$) and E^i (H^i) at the nodal point $P_{m\alpha}$, respectively, with s = 1,2. Here we have assumed ${}^2A^{\alpha 0}_{mn} = {}^2A^{\alpha Q}_{mn}$ (${}^2B^{\alpha 0}_{mn} = {}^2B^{\alpha Q}_{mn}$). The equations

(8a) and (8b) are rewritten in the matrix form as

$$\mathbb{v} = \mathbb{Z}\,\mathbb{u} \tag{10}$$

In Eq.(10), Z is NQ X NQ matrix and **u** and **v** are NQ dimensional column vectors:

$$\mathbb{Z} = \begin{bmatrix} \mathbb{Z}_{11} & \mathbb{Z}_{12} & \cdots & \mathbb{Z}_{1N} \\ \mathbb{Z}_{21} & \mathbb{Z}_{22} & \cdots & \mathbb{Z}_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbb{Z}_{N1} & \mathbb{Z}_{N2} & \cdots & \mathbb{Z}_{NN} \end{bmatrix}, \quad \mathbb{u} = \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_N \end{bmatrix}, \quad \mathbb{v} = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_N \end{bmatrix} \tag{11}$$

where block Z_{mn} is Q X Q submatrix whose (α,β)th element is $A^{\alpha\beta}_{mn}$ ($B^{\alpha\beta}_{mn}$) and $\mathbf{u}_m$ and $\mathbf{v}_m$ are Q dimensional column vectors whose αth elements are $K_{m\alpha}$ and $E^i_{m\alpha}$ ($H^i_{m\alpha}$), respectively.

We can easily show that $A^{\alpha\ \beta}_{m+\nu\, n+\nu} = A^{\alpha\beta}_{mn}$ ($B^{\alpha\ \beta}_{m+\nu\, n+\nu} = B^{\alpha\beta}_{mn}$) for any integer ν. Therefore, it follows that Z is a block Toeplitz matrix. So, we can efficiently solve Eq.(10) by using the conjugate gradient method combined with the fast Fourier transform presented in reference [4].

NUMERICAL RESULTS

As examples, we consider the current distributions on an array of 32 identical circular cylinders. The radius a of each cylinder is 0.1λ and the space d between adjacent cylinders is $2/3\lambda$. Each cylinder is equally divided into 16 elements. For the TM plane wave incident upon an array at the angle 90° (normal incidence) and 60° (at which the grating anomaly occurs), Figs.2(a), (b), and (c) show the current distributions on the left, middle, and right of an array, respectively. Figure 3 show the current distributions for the TE plane wave with the same incident angle as TM plane wave. In Figs.2 and 3, the interval $(0,\pi)$ is an illuminated region and the interval $(-\pi,0)$ is a shadow region. The great change of the current distribution at the edges of the array of cylinders is seen in case of incidence at the angle causing the grating anomaly.

CONCLUSIONS

The current distribution of a finite periodic array of cylinders has been analyzed by the boundary element method. Solving the matrix equation, we have used the conjugate gradient method combined with the fast Fourier transform, which can greatly reduce the execution time in computing and drastically save the storage on a computer. From numerical examples, we showed that the edge effect on the current distribution is remarkable at the incident angle causing the grating anomaly.

REFERENCES

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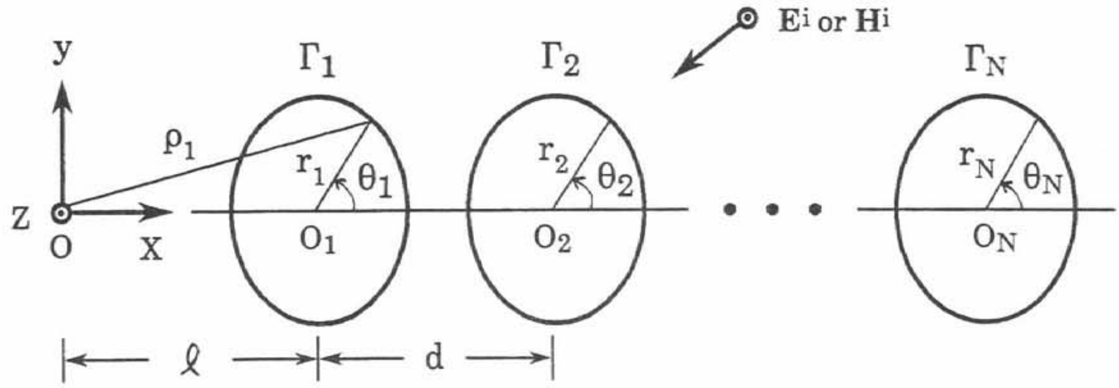
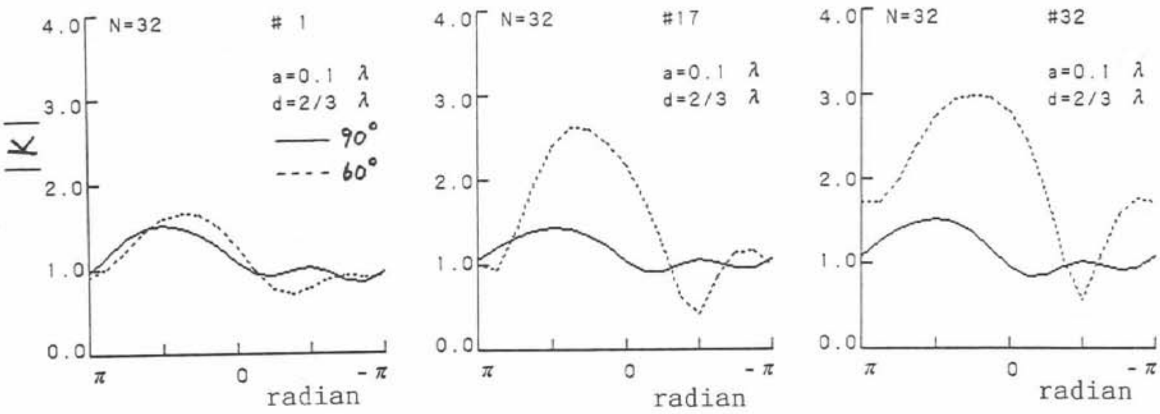
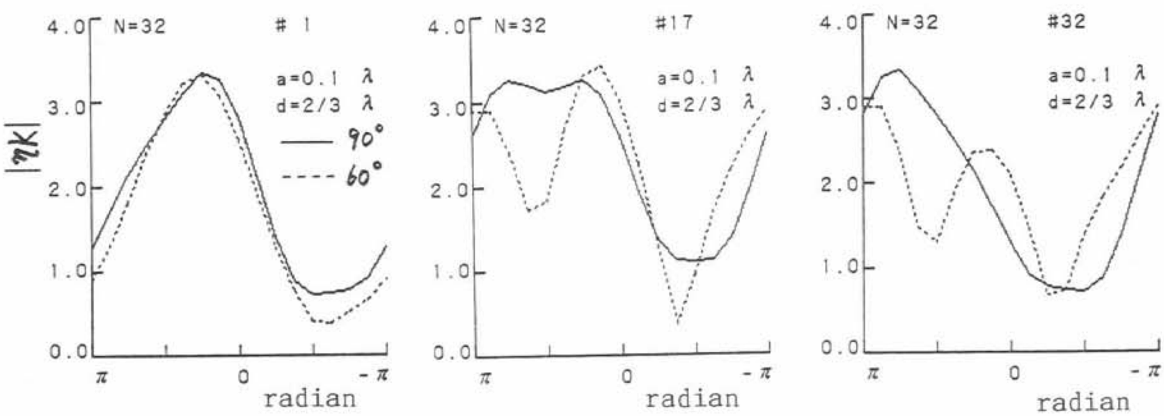


Fig.1. Finite periodic array of cylinders



(a) left (b) middle (c) right

Fig.2. Current distribution of the array of cylinders
illuminated by a TM plane wave



(a) left (b) middle (c) right

Fig.3. Current distribution of the array of cylinders
illuminated by a TE plane wave