# Electromagnetic Transmission Through a Narrow Slit Backed by a Nearby Conducting Strip 

Jong-Ig Lee* and Young-Ki Cho**<br>* Division of Information System Eng., Dongseo Univ.<br>** School of Electronics and Electrical Eng., Kyungpook National Univ.<br>E-mail : ykcho@ee.knu.ac.kr

## 1. Introduction

The problems of electromagnetic transmission through apertures in a thick conducting screen have been considered by Harrington et al. [1,2]. In the studies [1,2], it has been found that, for a certain screen thickness, the slit becomes resonated and extraordinary large transmission may occur.

In this study, the problem of electromagnetic coupling through a slit, in an infinite conducting screen, backed by a nearby conducting strip is considered (see Fig. 1). A simple equivalent circuit for the problem specialized to the case of a narrow slit is developed. It is found that when a narrow slit is resonated due to an appropriately placed nearby conducting strip, the effective slit width becomes $1 / \pi$ wavelengths, independent of the width of the narrow slit. The differences between two contrastive types of maximum coupling (or slit resonance), i.e., cavity-type and parasitic-type, through the narrow slit are described. In addition, the essential similarities between the resonance phenomena observed in the present structure of a narrow slit in an infinite conducting screen and the previously studied narrow slit in a thick conducting screen [1] are discussed.

## 2. Theory

The cross-sectional view of the proposed geometry is shown in Fig. 1 where $d$ is distance between the conducting strip at $z=d$ and infinite conducting screen at $z=0$, and $X_{0}$ is the lateral offset of the strip from the slit $\operatorname{center}(x=0)$. The $y$-component of the TM polarized plane wave (i.e., $E_{y}=0$ ) which is incident on the slit with an incident angle $\theta_{0}$ can be given by $H_{y}^{i}=H_{0} \exp \left(-j \underline{k_{1}} \cdot \underline{\rho}\right) \quad$ where $\quad \underline{k_{1}}=k_{1}\left(\sin \theta_{0} \hat{x}+\cos \theta_{0} \hat{z}\right) \quad, \quad \underline{\rho}=x \hat{x}+z \hat{z} \quad, \quad k_{n}=k_{0} \sqrt{\varepsilon_{r n}}$, $k_{0}=2 \pi / \lambda_{0}$, and $\lambda_{0}$ is the free space wavelength.

Employing the equivalence principle, the original problem is divided into two equivalent situations, as shown in Fig. 2 in which the slit is closed with a perfect conductor and the electric field originally present in the slit is supplied by attaching current sheets $\pm \underline{M}\left[= \pm \hat{y} E_{x}^{a}(x)= \pm \hat{y} E_{x}(x, 0)\right.$ over the slit] on both sides of the slit. Expressing the fields in the regions 1 and 2 in terms of the fields due to the incident TM wave, the magnetic current sheets $\pm \underline{M}$ over the slit, and the induced current $\underline{J}\left[=J_{x}(x)\right]$ over the conducting strip, one can find that the expressions for the $y$-component magnetic field in each region might be in the form

$$
\begin{equation*}
H_{y 1}=H_{y 1}^{i}+H_{y 1}^{M} \tag{1}
\end{equation*}
$$

$$
\begin{equation*}
H_{y 2}=-H_{y 2}^{M}+H_{y 2}^{J} . \tag{2}
\end{equation*}
$$

In (1) and (2), superscripts (i, M, and J) denote the fields due to the incident TM wave, the equivalent magnetic currents $\pm \underline{M}$, and the induced electric current $\underline{J}$, respectively.

Enforcing boundary conditions on the electromagnetic fields over the slit [i.e., $\left.H_{y 1}\left(z=0_{-}\right)=H_{y 2}\left(z=0_{+}\right)\right]$and the electric field over the strip (i.e., $E_{x 2}=-E_{x 2}^{M}+E_{x 2}^{J}=0$ ), we can obtain the coupled integro-differential equations [3] for the electric field $E_{x}^{a}(x)$ over the slit and
the induced current $J_{x}(x)$ over the strip as

$$
\begin{array}{ll}
{\left.\left[H_{y 1}^{i}+H_{y 1}^{M}\right]\right|_{z=0}=\left.\left[-H_{y 2}^{M}+H_{y 2}^{J}\right]\right|_{z=0_{+}},} & |x|<a / 2, \\
{\left[-E_{x 2}^{M}(\underline{\rho})+E_{x 2}^{J}(\underline{\rho})\right]=0, \underline{\rho} \in S,} & \text { oveross the slit (3) } \\
{\left[\begin{array}{l}
\text { on }
\end{array}\right.} \\
\hline
\end{array}
$$

In order to solve the coupled equations (3) and (4) by use of the method of moments, the unknown distributions $E_{x}^{a}(x)$ and $J_{x}(x)$ are expanded in terms of pulse function and piecewise sinusoidal function, respectively, and Galerkin's scheme is employed to reduce (3) and (4) to a linear equation system.

From knowledge of the distributions $E_{x}^{a}(x)$ and $J_{x}(x)$, the coupled power $P_{T}$ through the slit into the region 2 can be computed and then the effective slit width $a_{e f f}$ which is the coupled power $P_{T}$ normalized w.r.t the incident power $P_{I}$, per unit length in the $y$-direction, is obtained as

$$
\begin{equation*}
a_{e f f}=\frac{P_{T}}{P_{I}}, \quad P_{I}=\frac{1}{2} \eta_{1}\left|H_{0}\right|^{2} \cos \theta_{0} . \tag{5}
\end{equation*}
$$

When the problem is specialized to the case of a narrow slit (i.e., $k_{1} a \ll 1$ ) with no nearby conducting strip present, the problem can be solved approximately by assuming the electric field distribution $E_{x}^{a}(x)$ over the slit to be uniform as $E_{x}^{a}(x) \approx V_{a} / a$. Then the equation (3) is reduced to the scalar equation for the potential difference $V_{a}$ across the slit as

$$
\begin{equation*}
\left(Y_{1}+Y_{2}\right) V_{a}=I_{s}=2 H_{0} \tag{6}
\end{equation*}
$$

in which

$$
\begin{equation*}
Y_{i}=G_{i}+j B_{i}=\frac{k_{i}}{2 \eta_{i}}\left[1-j \frac{2}{\pi} \log \left(k_{i} a C\right)\right], \mathrm{C}=0.1987 . \tag{7}
\end{equation*}
$$

Note that the conductance $G_{i}\left(=k_{i} / 2 \eta_{i}\right)$ is the typical value of conductance appears in the various types of narrow slit problems such as narrow transverse slit in a parallel-plate waveguide (PPW) [4], wide transverse slit in a PPW with small guide height [4], and narrow slit in a flanged PPW [3]. Furthermore the susceptance $B_{i}$ becomes approximately equal to that of a narrow slit in a flanged PPW with small guideheight because, in this case of small guideheight, most of the reactive power near the slit is confined in the exterior region rather than in the interior waveguide region [3].

Fig. 3a shows the equivalent circuit for the approximation to a narrow slit. In this case of narrow slit without nearby scatterer, the conductance component is much smaller than the susceptance component, i.e., $G_{i} \ll B_{i}$, and so the coupled power $P_{T}\left(=0.5 \operatorname{Re}\left\{V_{a} I_{2}^{*}\right\}\right)$ transferred to the load $Y_{2}$ in Fig. 3a is very small because of poor impedance matching. However when a nearby scatterer such as a conducting strip is placed near the narrow slit in region 2, as shown in Fig. 1, the equivalent admittance $Y_{2}$ would be changed to $Y_{2}{ }^{\prime}$ while the admittance $Y_{1}$ is remained constant. Owing to an appropriately placed nearby scatterer, the resonance of a narrow slit or the maximization of the coupled power $P_{T}$ through the slit might be achieved when $Y_{2}{ }^{\prime}=Y_{1}^{*}=G_{1}-j B_{1}$ and then the total admittance $Y_{S}{ }^{\prime}\left(=Y_{1}+Y_{2}{ }^{\prime}\right)$ of the slit at resonance becomes $Y_{S}{ }^{\prime}=2 G_{1}$ as shown in Fig. 3b. Accordingly the available coupled power and the maximum effective slit width are

$$
\begin{gather*}
P_{T \max }=\eta_{1}\left|H_{0}\right|^{2} / k_{1} \text { when } Y_{2}=Y_{1}^{*}=G_{1}-j B_{1}  \tag{8}\\
\max \text { of } \mathrm{a}_{e f f}=P_{T \max } / P_{I}=\lambda_{1} / \pi \cong 0.3183 \lambda_{1} \tag{9}
\end{gather*}
$$

independent of the slit width $a$, as expected.

It is interesting to note that, as given in (8) and (9), the maximum effective slit width of a narrow slit in the present geometry is the same as the transmission width of a narrow slit at resonance in a thick conducting screen [1]. It is also worth while to mention that, from the viewpoint of equivalent circuit representation, the maximum coupling condition $Y_{S}{ }^{\prime}=2 G_{1}$ in the present problem as well as maximum transmission condition through a narrow slit in a thick conducting screen [1] is the same as the maximum radiation condition $Y_{i n}=2 G$ (input admittance at resonance) in the transmission line model [5] of the rectangular microstrip patch antenna. Here $G$ is the conductance of the radiating edge of the rectangular microstrip patch antenna.

## 3. Results and discussion

The effective slit widths $a_{\text {eff }}$ against the lateral strip offset $X_{0}$ for the cases of $a=0.01 \lambda_{0}$ and $a=0.001 \lambda_{0}$ are shown in Fig. 4 where two contrastive types of resonance phenomena (maximization of $a_{\text {eff }}$ ) have been observed. In Fig. 4, the variations of $a_{\text {eff }}$ against $X_{0}$ for the cases of solid line and dashed are quite different from each other, which is very similar to the variations observed in the cavity-type and parasitic-type couplings in a simplified aperture coupled and proximity coupled microstrip patch antenna structures [6,7]. Hence we use the same terminology cavity-type and parasitic-type couplings (and resonances) as in the previous studies [6,7] since most of their representative characteristics also appear in the present coupling problem.

Both types of variations have their own maximum values of $a_{\text {eff }}$ which are almost equal to the expected one $\lambda_{0} / \pi \cong 0.3183 \lambda_{0}$ in (9), though the offset points where the resonances (or maximum couplings) occur are different from each other. For the case of cavity-type coupling(solid line), the structure has a nearby conducting strip placed very close to the slit, the variation of $a_{\text {eff }}$ is very sensitive to the offset $X_{0}$, and the strip length $L$ is approximately half wavelength as in the microstrip patch resonator [5]. On the contrary, for the case of parasitic-type coupling, the resonance occurs at offsets near to the non offset point $X_{0}=0$, the variation is insensitive to the offset at the points near to the maximum coupling point ( $X_{0}=0$ ), the strip is longer than half wavelength, and the strip is more distant from the slit than in the case of cavity-type coupling. As the slit width $a$ is decreased, the strip lengths $L$ should be increased, in order to obtain the resonances of both types, while the strip-to-slit distance $d$ is decreased, which gives more sensitive variation in $a_{\text {eff }}$ against the offset $X_{0}$.

In Fig. 5 the variations of $a_{e f f}$ for various values of slit width $a$ are plotted, which shows again the available effective slit width $a_{e f f}$ is approximately $1 / \pi$ wavelengths irrespective of the actual slit width $a$, as expected in (9).

## 4. Conclusion

The problem of electromagnetic coupling through a slit in an infinite conducting screen backed by a conducting strip is considered for the case that the TM polarized plane wave is incident on the slit. From a simple equivalent circuit for the case of narrow slit, the maximum effective slit width is found to be $1 / \pi$ wavelengths independent of the actual slit width. Two contrastive types of maximum coupling phenomena (cavity-type and parasitic-type) through the narrow slit are observed as in the simplified aperture coupled microstrip patch antenna geometry previously studied.

## References

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Fig. 1. Cross sectional view of the proposed geometry.

a. case with no conducting strip present. b. case with conducting strip present (at resonance).

Fig. 3. Equivalent circuit for narrow slit case.


Fig. 4. Effective slit width $\left(\varepsilon_{r 1}=\varepsilon_{r 1}=1\right)$.
(solid line : cavity type, dashed line : parasitic type).

