PHYSICAL AND SURFACE THEORY OF DIFFRACTION

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Abstract. The aim of this investigation is to treat the surface wave traveling on a three dimensional curved surface having edges. The essentials of a method which was constructed to obtain the cross-section of tarfets by considering the surface current components induced on the bodies were established. The explicit expressions for scattering of high frequency fields by perfect conducting curved objects with edge were obtained via surface currents induced on the body. The contributions of various difraction phenomena to surface current were given explicitly. The results explain the contribution of surface currents to scattered field.

INTRODUCTION

Most of RCS calculation techniques combine the well-known results of GO,GTD, PO, PTD, and MOM by including equivalent currents and uniform asymptotic theories. None of the prediction techniques handles the surface traveling wave adequately because there has been no adequate treatment of the surface traveling wave phenomena that can be implemented in routine computations for complex objects, as far as we can glean from the open literature.

Our aim is to treat the surface wave traveling on a three dimensional curved surface having edges. Scattering by inflection, tip, and corner points were excluded in this treatment. We trace a way beginning with the canonical problems cited in |1| and |2| where first, the contributions on surface current of various diffraction phenomena on the perfect conducting spherical sheet were obtained, and then explicit expressions of scattered field including traveling waves were derived by the induced surface currents. Here we extended the physical and geometrical results in spherical case to surfaces having different principal radii by combining with the physical behaviour of phenomena. The generalized results has been collected as Physical and Surface Theory of Diffraction (PASTD) |3|.

ESSENTIALS OF THE THEORY

We consider surfaces consisted by regular convex and concave parts which are connected with continuous and closed regular edges. Finite number of such parts constitutes the whole surface S as $S=S_{convex}\cup S_{concave}$ where $S_{convex}=\cup_a S^a_{convex}$ and $S_{concave}=\cup_a S^a_{concave}$. Indices convex and concave intend the convex and concave portions of S respectively. Planar parts of S are included in S_{convex}. We will use symbols **n** and T_M for outward unit normal vector and tangential plane of S at $M\in S$ with appropriate indices (Fig.1). **n** and two principal directions **t** and **b** in plane T_M will address the ray coordinates. The components of field parallel to **t** and **b** will be denoted with indices $\parallel$ and $\perp$ at M, respectively. Here and in what follows bold face little letters denote unit vectors.

The diffraction phenomenon at M addresses the principal direction **t** (Fig.2a). Then **t** defines second principal direction as $\mathbf{b}=\mathbf{n}\wedge\mathbf{t}$. The triplet **n**, **t**, **b**, which we call observable directions of M, establishes the trihedron for ray trajectory around M. If a ray originated by the diffraction phenomena at M reaches to point P then we call M as an observation point of P. The lines parallel to the observable directions of an observation point of P constitute the adequate reference directions at P to obtain the scattered field from the observation point of P to P. We call the coordinate lines defined by abovesaid reference directions as observatory reference system at P observed by M.

The observatory reference system at P will change with adequate translations from one observation point to another. However, following two items, that address the scattered field and target position, illustrate the adequateness of observatory reference system:

i) $\vec{H}_\parallel$ and $\vec{H}_\perp$ at the diffraction point M contribute the components $\vec{J}_\perp$ and $\vec{J}_\parallel$

of surface current at M, respectively, which are transferred and, propagate as following law:

$$\begin{pmatrix} \vec{J}_{\perp}^{phen}(M) \\ \vec{J}_{\parallel}^{phen}(M) \end{pmatrix} = \begin{pmatrix} \vec{J}_{\perp}^{phen}(M^{0}) \\ \vec{J}_{\parallel}^{phen}(M^{0}) \end{pmatrix} \tilde{T}_{phen}^{(e\ ray)}(M^{0}) \Pi_{phen}^{(e\ ray)}(M^{0}, M; \rho^{phen}) \qquad (1)$$

Here phen and ray will be the symbols which intend the scattering phenomenon at observation point and the incident current to M^{0}, respectively. $M^{0} \in S$ is the observation point of M. The upper indice e in parantheses means that the quantity is defined for electric current and/or electric field. If $J_{\perp}$ and/or $J_{\parallel}$ is zero at M^{0} then the derivative of $J_{\perp}$ and/or $J_{\parallel}$ w.r. to path length effects the transfer in (1) instead of $J_{\perp}$ and/or $J_{\parallel}$, respectively. ρ denotes the principal radii of S which affect the diffraction process.

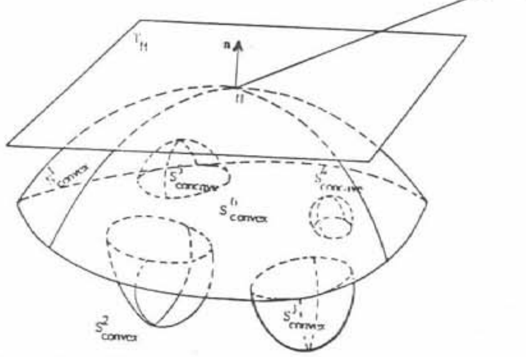


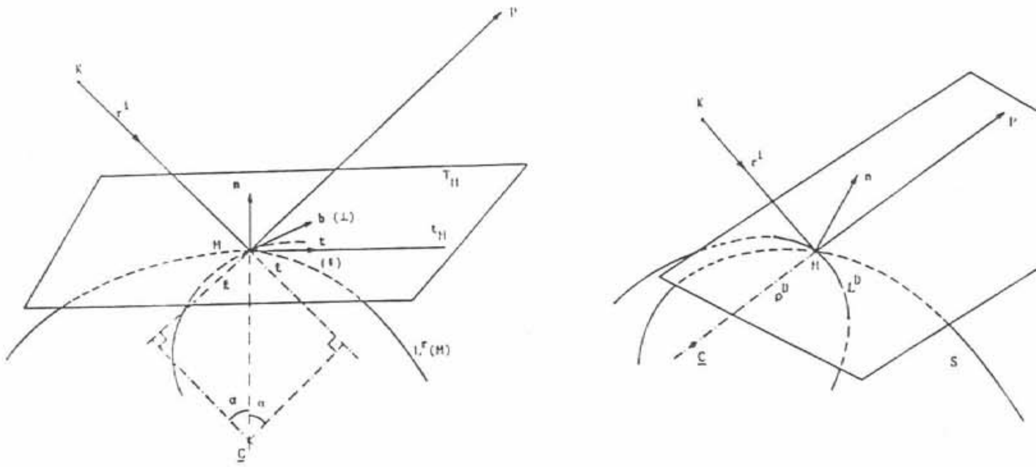
Fig.1. The target.

ii) $\vec{J}_{\parallel}$ and $\vec{J}_{\perp}$ at M which are observed from P contribute the components of $\vec{H}_{\perp}$ and $\vec{H}_{\parallel}$ of magnetic field at P, respectively as

$$\begin{pmatrix} \vec{H}_{\parallel}^{phen}(P) \\ \vec{H}_{\perp}^{phen}(P) \end{pmatrix} \sim n \wedge \begin{pmatrix} \vec{J}_{\perp}^{phen}(M) \\ \vec{J}_{\parallel}^{phen}(M) \end{pmatrix} C_{phen}^{(m)}(M) \Pi_{phen}^{(m)}(M, P; \rho^{phen}) \qquad (2)$$

Here the upper indice m in parantheses denotes the relation with magnetic field. The certain matrix coefficients C and $\tilde{T}$ include the convertion and transfer rule of surface current at an observation point M and a diffraction point M^{0} to the magnetic field and diffracted current at the observation point, respectively. The matrix Π determines the amplitude and phase change of the converted field or transferred current during propagation from observation point. In what follows bold face capital letters are used to denote matrices. We will use $\mathbf{F}(P) \equiv (\vec{H}_{\parallel}(P) \ \ \vec{H}_{\perp}(P))$ and $\mathbf{J}(M) \equiv (\vec{J}_{\perp}(M) \ \ \vec{J}_{\parallel}(M))$ for brevity. We call observatory components of magnetic field and surface current $\mathbf{F}(P)$ and $\mathbf{J}(M)$, respectively.

Tableau 1 demonstrates the contribution of diffraction phenomena chains. The column incident ray trajectory addresses the $\mathbf{J}(P^{0})$ and/or $\mathbf{J}(M^{0})$. s, c, and w denote space ray, creeping mode, and whispering gallery mode, respectively. The second letter in doubly lettered scripts keeps incident field and/or current in and, first letter means the phenomenon at observation point. r, e, and o are assigned to reflection, edge diffraction, and diffraction on surface phenomena



a b

Fig.2. (a) Observatory reference system, (b) reflection.

TABLEAU 1
The Guide to Iterative Diffractions

phenomenon / incident ray	r	oc	os	ec	ew	ow	es
s	$C^{r}(M)$ $\Pi^{(m)}_{MP}$ (1)	$\bar{T}^{(cw)}_{wc}$ Π_c (2)	-	$\bar{T}^{(es)}_{ec}$ Π_c (3)	$\bar{T}^{(es)}_{ew}$ $\Pi^{(m)}_w$ (4)	2	$\bar{T}^{(es)}_{es}$ (5)
c	-	1	$C^{(mc)}_{os}$ $\Pi^{(m)}_s$ (6)	$\bar{T}^{(ec)}_{ec}$ Π_c (7)	$\bar{T}^{(ec)}_{ew}$ $\bar{\Pi}^{(m)}_w$ (8)	-	$\bar{T}^{(ec)}_{es}$ (9)
w	-	1	-	$\bar{T}^{(ew)}_{ec}$ $\bar{\Pi}_{wc}$ (10)	$\bar{T}^{(qp)}_{ew}$ $\bar{\Pi}^{(mp)}_w$ (11)	$C^{(mq)}_o$ $\Pi^{(mq)}_w$ (12)	$\bar{T}^{(ew)}_{es}$ (13)
ec	-	1	$C^{(m)}_c$ $\Pi^{(m)}_s$ (6)	(7)	(8)	-	(9)
ew	-	-	-	(10)	(11)	$C^{(m)}$ $\Pi^{(m)}_w$	(13)
cw	-	-	-	(10)	(11)	(14)	(13)
cs	(1)	(2)	-	(3)	(4)	2	$C^{(mc)}_{es}$ $\Pi^{(m)}_s\Pi^{(m)}_c$
es	(1)	(2)	- (3)	(4)		2	$C^{(mc)}_{es}$ $\Pi^{(m)}_c\Pi^{(m)}_c$
ws	(1)	(2)	- (3)	(4)		2	$C^{(mw)}_{es}$ $\Pi^{(m)}_w\Pi^{(m)}_c$
wc	-	1		(7)	(8)	-	(9)
ww	-	-	-	(10)	(11)	$C^{(mp)}_{ew}$ $\bar{\Pi}^{(mp)}_w$	(13)

respectively. The row phenomenon addresses the $F(P)$ and/or $J(M)$. The Tableau 1 is versatile to calculate the contribution of secondary diffractions by iterations. The more complete descriptions and exact expressions of the certain transfer, convertion, and propagation matrices in (1) and (2) are given in |3| according to the diffraction processes and the ray path.

Contribution of Reflection

Let $\tilde{P}(r^i,\mathbf{n})$ denotes the plane containing incident ray r^i and $\mathbf{n}$. The line $t_M=\tilde{P}(r^i,\mathbf{n})\cap T_M$ defines a principal direction in the plane T_M. Unit vector $\mathbf{t}$ is in the direction which goes far away from r^i (Fig.2a). We conclude the following results for reflected field from M by the extention of results in |2|:

$$F^r(P)=\mathbf{n}\Lambda J^{go}(M)C_r^{(m)}(M)\Pi_{MP}^{(m)}e^{ikMP},\quad P\in\tilde{P}(r^i,\mathbf{N}) \tag{3a}$$

$$C_r^{(m)}(M)=\frac{1}{2}\begin{pmatrix}(-\mathbf{e}_{MK}\cdot\mathbf{n})^{-1} & 0\\ 0 & -1\end{pmatrix} \tag{3b}$$

$$\Pi_{MP}^{(m)}=D(-\rho^D(M),\overline{MP})\left(\frac{\ell\,\overline{KM}}{(\overline{KM}+\overline{MP})\ell+2\overline{KM}\;\overline{MP}}\right)^{1/2}\begin{pmatrix}\mathbf{e}_{PM}\cdot\mathbf{e}_{PC} & 0\\ 0 & 1\end{pmatrix} \tag{3c}$$

where k is the wave number of surrounding medium. Here $\rho^D(M)$ is the principal curvature radius of the curve $L^D(M)=\tilde{P}_\perp(r^i,\overline{MP})\cap S$ at M where $\tilde{P}_\perp(r^i,\overline{MP})$ is the plane perpendicular to $\tilde{P}(r^i,\mathbf{n})$ and $MP\in\tilde{P}(r^i,MP)$. $\underline{C}$ is the center of principal curvature of $L^r(M)=\tilde{P}(r^i,\mathbf{n})\cap S$ at M. K is the point where r^i is originated (see Fig.2). $\mathbf{e}_{MK}$ illustrates the unit vector toward K from M, etc. The length ℓ was illustrated in figure 2a. The first multiplier in (3c) is defined as

$$D(\rho,\overline{MP})=\left(\rho/(\rho-\overline{MP})\right)^{1/2} \tag{4a}$$

$$\rho=\begin{cases}\rho^D & \text{if } \overline{CM}=\overline{CP}+\overline{PM},\\ -\rho^D & \text{if } \overline{CP}=\overline{CM}+\overline{MP},\end{cases} \tag{4b}$$

by considering |2|. The upper indices go in (3a) illustrate the geometrical optics term; i.e.;

$$\left(\vec{J}_\perp^{go}(M)\quad \vec{J}_\parallel^{go}(M)\right)=2\mathbf{n}\Lambda\left(\vec{H}_\parallel^{inc}(M)\quad \vec{H}_\perp^{inc}(M)\right). \tag{5}$$

Contribution of Surface diffraction

Let r^i be tangent to S at $M_2\in S^a_{convex}$. The ray is forced to go on a curve $L^{sc}(M_2)\in S^a_{convex}$ where the plane $\tilde{P}(r^i,\mathbf{n}_2)$ is the osculating plane of the path. Hence, we define $L^{sc}(M_2)=\tilde{P}(r^i,\mathbf{n}_2)\cap S^a_{convex}$ and call it curve-surface creeping and depart from any point $\bar{M}_2\in L^{sc}(M_2)$ in direction $\mathbf{t}_2$ when there is no discontinuance on $L^{sc}(M_2)$. The contribution of tangential ray to field at P is evaluated as

$$F^c(P_5)=\bar{n}_2\Lambda J^c(\bar{M}_2)C_{os}^{(mc)}(\bar{M}_2)\Pi_s^{(m)}(\bar{M}_2,P_5;\rho^{Ds},\underline{\bar{C}}_2) \tag{6}$$

The transfer matrix $C_{os}^{(mc)}$ gives the law how surface current J^c at $\bar{M}_2$ is converted to magnetic field just at M_2 in space. The propagation matrix $\Pi_s^{(m)}$ defines the propagation law of the converted field from M_2 to P_5 (Fig.3). The transfer and propagation matrices are obtained as

$$C_{os}^{(mc)}(\overline{M}_2)=\sqrt{\frac{2}{\pi}}\begin{pmatrix}\dfrac{e^{i\pi/4}(k\rho^{sc}(\overline{M}_2))^{1/2}}{\mathring{H}^{sc}(\overline{M}_2;1)} & 0\\ 0 & \dfrac{e^{-i\pi/4}\underset{\sim}{R}^{sc}(\overline{M}_2;-1)\underset{\sim}{R}^{sc}(\overline{M}_2;+1)}{(k\rho^{sc}(\overline{M}_2))^{3/2}\underset{\sim}{\ddot{H}}^{sc}(\overline{M}_2;1)}\end{pmatrix} \tag{7a}$$

$$\Pi_s^{(m)}(\overline{M}_2,P_5;\rho^{Ds},\underline{\overline{C}}_2)=\begin{pmatrix}\overline{\overline{M}_2P_5}/\overline{\underline{\overline{C}}_2P_5} & 0\\ 0 & 1\end{pmatrix}D(\rho^{Ds}(\overline{M}_2),\overline{\overline{M}_2\underset{\sim}{P}_5})e^{ik\overline{\overline{M}_2P_5}}(k\overline{\overline{M}_2P_5})^{-1/2} \tag{7b}$$

for dominant mode. We put followings for brevity:

$$H^{sc}(\overline{M}_2;1)=G(\nu^{sc}(\overline{M}_2;1),k\rho^{sc}(\overline{M}_2)),\ \mathring{H}^{sc}(\overline{M}_2;1)=\mathring{G}(\nu^{sc}(\overline{M}_2;1),k\rho^{sc}(\overline{M}_2)) \tag{8a}$$

$$H'^{sc}(\overline{M}_2;1)=G'(\nu^{sc}(\overline{M}_2;1),k\rho^{sc}(\overline{M}_2)),\ \mathring{H}'^{sc}(\overline{M}_2;1)=\mathring{G}'(\nu^{sc}(\overline{M}_2;1),k\rho^{sc}(\overline{M}_2)) \tag{8b}$$

$$G(\nu,x)=x^{1/2}H_\nu^{(1)}(x),\ \mathring{G}(\nu,x)=x^{1/2}\dot{H}_\nu^{(1)}(x),\ G'(\nu,x)=\frac{d}{dx}(x^{1/2}H_\nu^{(1)}(x)) \tag{8c}$$

$$\mathring{G}'(\nu,x)=\frac{d}{dx}(x^{1/2}\dot{H}_\nu^{(1)}(x)),\ \underset{\sim}{H}^{sc}(\overline{M}_2;1)=G(\tilde{\nu}^{sc}(\overline{M}_2;1),k\rho^{sc}(\overline{M}_2)) \tag{8d}$$

$$\underset{\sim}{R}^{sc}(\overline{M}_2;\pm1)=\pm k\rho^{sc}(\overline{M}_2;1)+\tilde{\nu}^{sc}(\overline{M}_2;1) \tag{8e}$$

The dot over Hankel function denotes the derivative with respect to index.

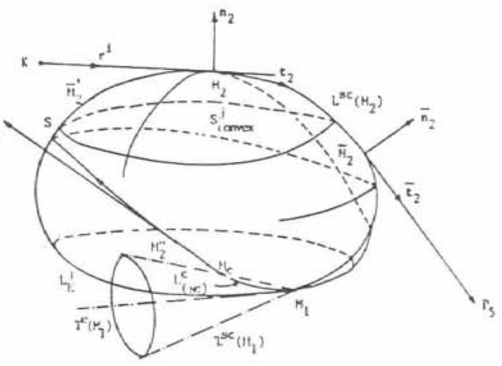


Fig.3. Surface diffraction and edge diffraction of creeping wave.

Here $\rho^{sc}(\overline{M}_2)$ is the principal curvature radius of $L^{sc}(M_2)$ at $\overline{M}_2$. $\nu^{sc}(\overline{M}_2;1)$ and $\tilde{\nu}^{sc}(M_2;1)$ are the roots closer to $k\rho^{sc}(\overline{M}_2)$ of following equations respectively:

$$G(\nu^{sc}(\overline{M}_2;n),k\rho^{sc}(\overline{M}_2))=0,\ n=1,2,3,\ldots \tag{9a}$$

$$G'(\tilde{\nu}^{sc}(\overline{M}_2;n),k\rho^{sc}(\overline{M}_2))=0,\ n=1,2,3,\ldots \tag{9b}$$

The abovesaid roots are in the first quadrant of ν-plane. Here and in what follows tilde below the letter denotes the same function with $\tilde{\nu}$. Contact points to S of tangential rays near $\overline{M}_2$ constitutes a curve $L^s(\overline{M}_2)$. The curvature of $L^s(\overline{M}_2)$ influences the divergence of creeping wave when propagating to P_5. Let oscullating plane of $L^s(\overline{M}_2)$ be $\tilde{P}^s(\overline{M}_2)$. $\rho^{Ds}(\overline{M}_2)$ is the principal curvature radius of $L^{Ds}(\overline{M}_2)=S^a_{convex}\cap\tilde{P}^S(\overline{M}_2)$ and $\underset{\sim}{P}_5$ is perpendicular projection of P_5 on $\tilde{P}^S(\overline{M}_2)$. The above terminology on curvature radius is used everywhere with similar nomenculature |3|.

CONCLUSIONS

The surface wave traveling on three dimensional curved surfaces was considered. The contributions of diffraction phenomena to surface current were obtained. Explicit expressions of the scattered field were obtained via surface current. The results handle the surface traveling wave so can be implemented in imaging of complex objects |3|.

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