TRANSMISSION LINE MODEL OF VARIOUS PRINTED SLOT ANTENNA SHAPES

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1 - INTRODUCTION

This article presents a model well suited for different shapes of printed slot antennas. Cohn's method is used to determined the characteristics of slot lines. The attenuation coefficient is numerically determined. Theoretical input impedance of linear and annular slots are compared successfully with experimental results given by other authors.

2 - ANALYSIS

The analysis of a radiating slot has been developped using a lossy transmission line model, which requires the computations of a propagation constant $\Gamma=\alpha+j\beta$ and a characteristic impedance Z_c [1]. β and Z_c are obtained using Cohn's method [2], and α is the solution of the numerical equation $P_i(\alpha) = P_r(\alpha)$ where $P_i(\alpha)$ equals the power delivered to the lossy line and $P_r(\alpha)$ is the radiated power from the slot antenna. It will be noticed that P_i increases with α while Pr decreases, then the solution of the equation $P_r(\alpha) = P_i(\alpha)$ is unique (figure 3). This method was applied for two shapes of slots.

2-1 LINEAR SLOT ANTENNA

- The first step is to introduce α in the expression of the electric field in the aperture (figure1) excited by a voltage Vo (the microstrip line is fed by 1 Volt (figure2)) :

$$\vec{E}^a = -\frac{V_o}{Wa}\frac{\operatorname{sh}\Gamma\left(\frac{La}{2}-|x|\right)}{\operatorname{sh}\Gamma\left(\frac{La}{2}\right)}\vec{y} \qquad -\frac{La}{2}<x<\frac{La}{2} \quad -\frac{Wa}{2}<y<\frac{Wa}{2}$$

The magnetic source M^a located in the aperture is determined using the equivalence principle :

$$\vec{M}^a = 2\vec{E}^a \wedge \vec{z}$$

This source is allowed to radiate into free space and the radiative power P_r may be computed numerically by the following formula :

$$P_r(\alpha) = \frac{\zeta|\Gamma|^2|\alpha|^2}{\eta\left|\Gamma^2+w^2\right|}\frac{k_o^2|V_o|^2}{4\pi^2\left|\operatorname{sh}\left(\Gamma\frac{La}{2}\right)\right|^2}\int_0^{\pi}\int_0^{\pi/2}\sin\theta\left\{\sin^2\varphi+\cos^2\theta\cos^2\varphi\right\}\left|\operatorname{ch}\left(\Gamma\frac{La}{2}\right)-\cos\left(w\frac{La}{2}\right)\right|^2 d\theta d\varphi$$

Where $w = k_o \sin\theta\cos\varphi$; $\eta = 120\pi$; $V_o = \dfrac{1}{1+2\frac{Z_o}{Z_c}\coth\left(\Gamma\frac{La}{2}\right)}$ and $\zeta=1+\dfrac{1}{\sqrt{\varepsilon_r}}$

ζ is an empirical coefficient taking into account the effect of the dielectric substrate behind the slot antenna (reduction of the radiated power in the half space containing the dielectric sheet).

-The second step needs the computation of the power P_i delivered to the lossy linear slot-line (figure 3). The slot is considered as two short-circuit slot-lines (with proper characteristic impedance Z_c and wave number β). The expression of P_i is given by :

$$Pi(\alpha) = \frac{1}{Zc}\frac{\operatorname{th}\left(\alpha\frac{La}{2}\right)\left[1+\operatorname{tg}^2\left(\beta\frac{La}{2}\right)\right]}{\left[2\frac{Zo}{Zc}+\operatorname{th}\left(\alpha\frac{La}{2}\right)\right]^2+\operatorname{tg}^2\left(\beta\frac{La}{2}\right)\left[1+2\frac{Zo}{Zc}\operatorname{th}\left(\alpha\frac{La}{2}\right)\right]^2}$$

$Z_o = 50\Omega$ is the generator impedance.

The input impedance at the aperture is expressed by : $Zs = \frac{Zc}{2}\,\mathrm{th}\left(\Gamma \frac{La}{2}\right)$

The effet of an offset feeding point may be easily taken into account. In this case, the antenna is considered as two short circuited slot-lines with different lengths but with the same attenuation α calculated in the centered case (first approximation).The input impedance at aperture is given by :

$$Zs = Zc\,\frac{\mathrm{th}\left(\Gamma\left(\frac{La}{2} - C\right)\right)\mathrm{th}\left(\Gamma\left(\frac{La}{2} + C\right)\right)}{\mathrm{th}\left(\Gamma\left(\frac{La}{2} - C\right)\right) + \mathrm{th}\left(\Gamma\left(\frac{La}{2} + C\right)\right)}$$

- The last step is to transform the slot impedance along the microstrip feed-line using the known transformer [3] as described in [1].

Finally the input impedance is given by : $Zin = n^2 Zs - jZ_{c1}\,\mathrm{cotg}(k^1 Ls)$

Where L_s is the length of the open stub, k^l is the wave number and Z_{cl} is the characteristic impedance of microstrip line, and n is the transformation ratio.

2-2 RESULTS

- The first example is coming from the paper of AXELROD [4] ; on figures 5(a, b, c, d) resistance and reactance versus frequency have been plotted for different offset distances C. The two groups of results show a good agreement either for the resonant frequency or for the values of resistance and reactance. As far as the feeding point is moving toward the end of the slot, the resistance decreases and 50 Ω is obtained for C = 25 mm ; for this value of C it must be noticed that the reactance keeps a positive value and no resonance occurs in this case.This inductive reactance can be compensated using a proper length of the microstrip stub.

- The second example is given in the paper of POZAR [5] ; one antenna has been built in order to check the two ways of measurement using either the transmission parameter S_{21} or the reflection parameter S_{11}. The figure 6 shows the good agreement between the measurement values and the theoretical results ; the same curve hase been obtained from S_{21} or S_{11} parameters. Some difference with the impedance curves of [5] (for the experiments ref [5]) are probably related to undesired parasitic reactances caused by connectors or quality of matched load, which appears to be a source of error for high value of resistances (here typical resistance are nearly 700 Ω).

2 -3 ANNULAR SLOT ANTENNA

The analysis of annular slot antenna (figure 7) is very similar to the previous one :

- The first step is to define the electric field along the radial direction ρ and φ angular position in the aperture :

$$\vec{E}^{a} = -\frac{Vo}{Wa}\left[e^{-\Gamma\varphi} + e^{-\Gamma(\varphi-2\pi)}\right]\vec{\rho}$$

using the equivalence principle, the magnetic source is determined.

- The radiative power may be determined by the following analytical formula :

$$Pr = P_{ro} + \frac{\zeta\pi}{\eta}\sum_{n=1}^{\infty}|An|^2\left\{\frac{2n(n+1)(k\bar{r})^{2n-2}}{(2n+1)!} - \frac{4n(n+1)(n+3)(k\bar{r})^{2n}}{(2n+3)!} + \sum_{m=0}^{\infty}\left[\frac{(-1)^m(k\bar{r})^{2n+2m+2}}{(2n+m+2)!(m+2)!(2n+2m+3)(2n+2m+5)}\,x\right.\right.$$
$$\left.\left.\left[(2n+m+1)(2n+m+2)(2n+2m+5) - 2n^2(2n+2m+3) + (m+1)(m+2)(2n+2m+5)\right]\right]\right\}$$

where $|An|^2 = \frac{|Vo|^2}{\pi^2}\,\frac{(k\bar{r})^2|\Gamma|^2\left|1-e^{-\Gamma 2\pi}\right|^2}{\left(n^2+|\Gamma|^2\right)^2}$; $Vo = \left[1+\frac{100}{Zc} + e^{-\Gamma 2\pi\bar{r}}\left(1-\frac{100}{Zc}\right)\right]^{-1}$

$\bar{r}$ = average radius of the loop

$$Pro = \frac{\zeta|Vo|^2(k\bar{r})^4}{\eta\pi}\left|\frac{1-e^{-\Gamma 2\pi}}{\Gamma}\right|^2\sum_{m=0}^{\infty}\frac{(-1)^m(k\bar{r})^{2m}}{(2m+3)(2+m)!m!}$$

- The power delivred to the lossy loop is calculated by the transmission line model and is given by :

$$Pi = \frac{th\,(\alpha\pi\bar{r})\,[1+tg^2(\beta\pi\bar{r})]}{Zc(1+\frac{100}{Zc}\,th(\alpha\pi\bar{r}))^2 + tg^2(\beta\pi\bar{r})\,[th\,(\alpha\pi\bar{r})+\frac{100}{Zc}]^2}$$

- α is determined by the same previous technique :

The numerical solution of the equation $P_i(\alpha) = P_r(\alpha)$ gives α.

- The annular slot behaves as two parallel open lines ; then the input impedance is given by the classical formula :

$$Zs = \frac{Zc}{2}\coth\left(\Gamma\frac{S}{2}\right) \quad \text{Where} \quad S = 2\pi\bar{r}$$

The final step is very similar to the description given for linear slot. Taking into account the open-circuit stub, the input impedance is given by : $Zin=n^2 Zs - jZ_{cl}\,cotg(k'Ls)$

2-4 RESULTS

The antenna design and results are described in figure 8. The quarter wave length open stub l_1 insures the coupling between the slot and the feeding line a second section of line l_2 used as a matching network.The comparaison between the theory and experimental results (figure 8) at the X band are in good agreement.

3- CONCLUSION

Theoretical results using this method show a good agreement either with previous published results and with experimental measurements obtained in the Laboratory. The effect of offset-fed linear slot has been also tested and gives very similar results as in reference [4]. Two shapes of antenna are examined,the method based on the lossy transmission line is very flexible and can be extended to calculate the impedance of different shape of slot.

4- REFERENCES

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[2] S.B. COHN"Slot-line on a dielectric substrate",IEEE MTT-17,n°10,pp. 768-778,oct.69

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[4] A. AXELROD, M. KISLIUK and J. MAOZ "Broadband Microstrip-fed Slot Radiator", Microwave Journal, June 1989,pp. 81-94.

[5] D.M. POZAR "A reciprocity method of analysis for printed slot and slot coupled microstrip antennas",IEEE TAP-34,pp. 1439-1446,Dec.1986.

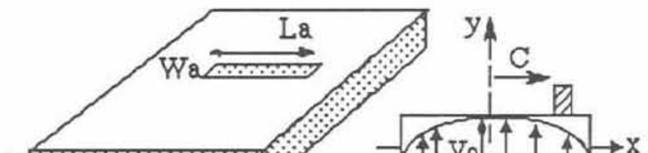

Figure 1 : Linear printed slot antenna fed by an offset line

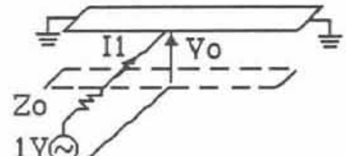

Figure 2 : equivalent circuit of the linear slot feeding by 50 Ω generator

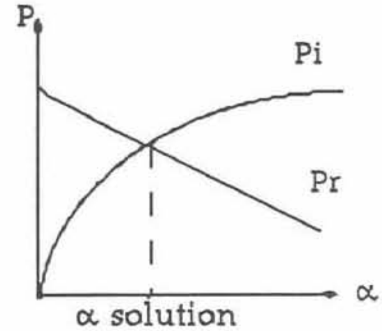

Figure 3 : α solution of Pi(α)=Pr(α)

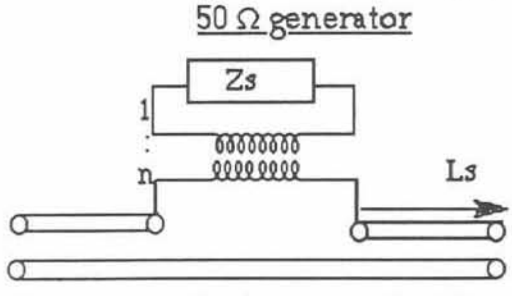

Figure 4 : equivalent circuit of the slot including the transition microstrip line/slot-line

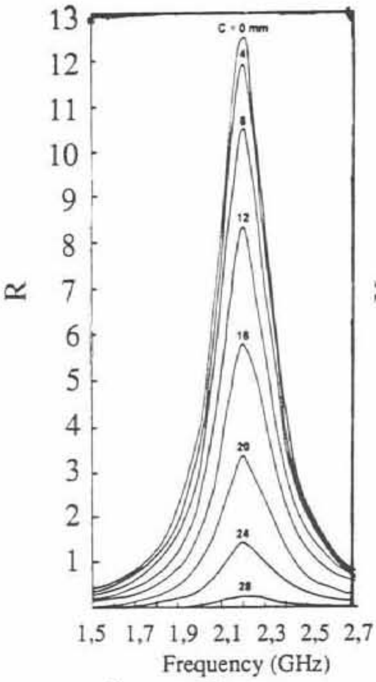

figure 5-a

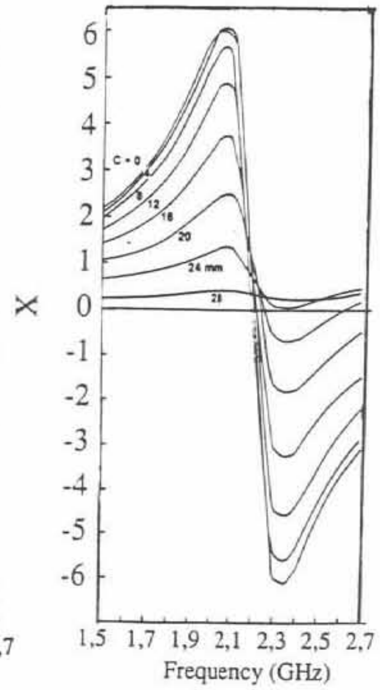

figure 5-b

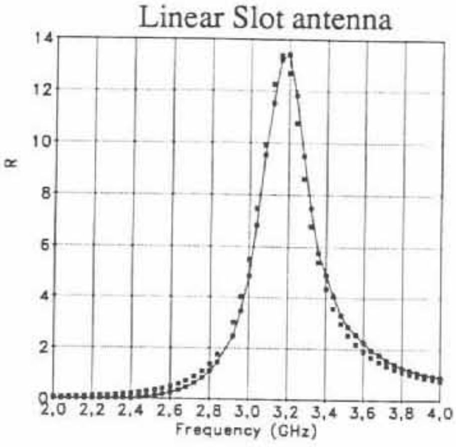

figure 6-a

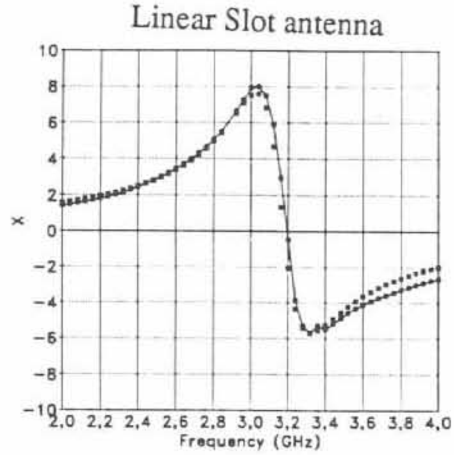

figure 6-b

figure 6: input impedance
La=40.2mm, Wa=0.7mm
εr=2.2,h=1.587mm
--●-- Experience
* * * this theory

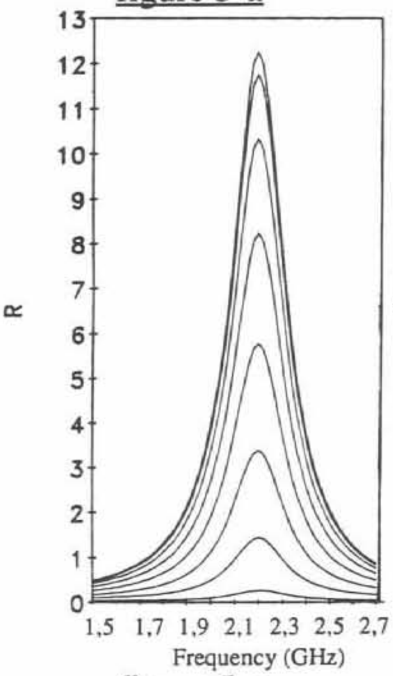

figure 5-c

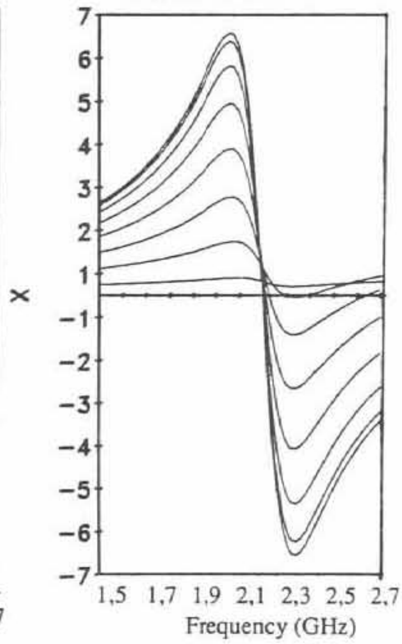

figure 5-d

figure 5:La=60mm,Wa=2mm,εr=2.2,h=1.587mm
effect of offset-fed slot on input impedance
a) and b) reference [4],c) and d) this theory[4]

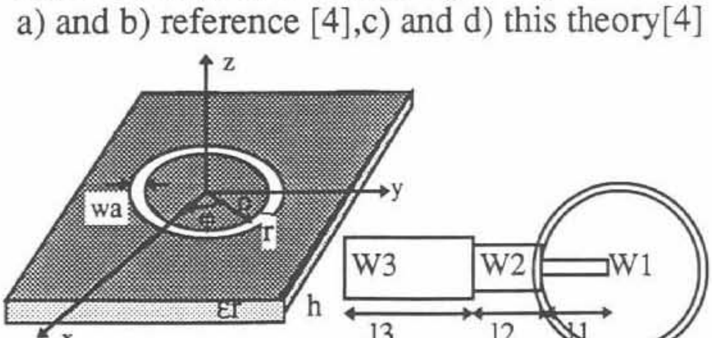

figure 7 : Printed annular slot

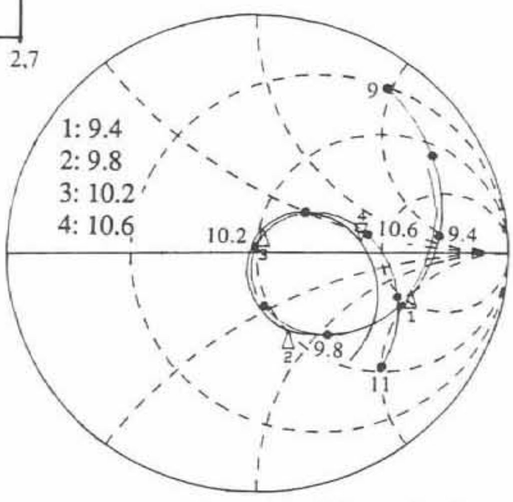

figure 8:r=4.09,wa=0.154,εr=2.17
h=0.78,w1=w2=0.373,w3=2.31
l1=6.28,l2=5,l3=21.5 (mm)
---- Experience ; -●-●- Theory