# NUMERICAL ANALYSIS OF THREE-DIMENSIONAL SCATTERING PROBLEMS BY THE YASUURA METHOD

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## 1. INTRODUCTION

The Yasuura method (mode-matching method) [1][2] is a general procedure for solving boundary value problems on the electromagnetic fields. The coefficients of the truncated modal expansion of the scattered field can be determined by making to satisfy the boundary condition in the sense of least squares. It is guaranteed that the sequence of the truncated modal expansion converges to the true solution of the problem as the number of truncation tends to infinity. Up to now, this method has been applied to the various two-dimensional problems [3][4]. Of course, this method also applicable to the three-dimensional problems.

In this paper, we apply the Yasuura method to the three-dimensional scattering problems. As the modal function, we employ the multipoles which are known as the vector spherical wave functions $\mathbf{l},\mathbf{m},\mathbf{n}$ [5]. Numerical examples for the case of bodies of revolution are shown and one of them is compared with the experimental one [6]. Agreement between them is good.

## 2. FORMURATION OF PROBLEM

Let us consider a three-dimensional perfectly conducting smooth scatterer as shown in Fig.1. The point $P(r,\theta,\phi)$ is in the exterior infinite domain V, and the point $Q(r',\theta',\phi')$ is on the surface of the scatterer S. The scattered field $\mathbf{E}^s(P)$ can be approximated by the truncated modal expansion. As the modal function, we choose the spherical vector wave functions $\mathbf{m}$ and $\mathbf{n}$ [5] which satisfy the radiation condition. The truncated modal expansion is constructed as

$$\mathbf{E}_N^s(P) = \sum_{n=0}^{N} \sum_{m=0}^{n} [a_{mn}^e(N)\mathbf{m}_{emn} + a_{mn}^o(N)\mathbf{m}_{omn} + b_{mn}^e(N)\mathbf{n}_{emn} + b_{mn}^o(N)\mathbf{n}_{omn}] \qquad (1)$$

where N is the number of truncation. It is proved that if $\mathbf{E}_N^s(P)$ is made to satisfy the boundary condition in the sense of least squares, the sequence $\{\mathbf{E}_N^s(P):N=0,1,2,\ldots\}$ converges to $\mathbf{E}^s(P)$ uniformly in any subdomain of V [2]. Therefore, we determine the coefficients of the truncated modal expansion so as to minimize the normalized norm

$$\Omega(N) = \int_S |\, \boldsymbol{\nu} \times [\mathbf{E}_N^s(Q) + \mathbf{E}^i(Q)] \,|^2 dS_Q \,/\, \int_S |\, \boldsymbol{\nu} \times \mathbf{E}^i(Q) \,|^2 dS_Q \qquad (2)$$

where $\mathbf{E}^i(Q)$ is the incident plane wave on the scatterer and the vector $\boldsymbol{\nu}$ is the inward normal to the surface.

For the bodies of revolution, the integral about $\phi'$ of Eq.(2) can be done analytically and we have

$$\Omega(N) = \sum_{m=0}^{N} [\, \Omega_m^{(1)}(N) + \Omega_m^{(2)}(N) \,] \qquad (3)$$

with

$$\Omega_m^{(1)}(N) = \frac{\int_0^\pi |\sum_{n=m}^{N}[a_{mn}^e(N)\Phi_{mn}^{(1)}(\theta') - b_{mn}^o(N)\Psi_{mn}^{(1)}(\theta')] - \mathbf{f}_m^{(1)}(\theta')|^2 d\theta'}{\int_0^\pi [\, |\mathbf{f}_m^{(1)}(\theta')|^2 + |\mathbf{f}_m^{(2)}(\theta')|^2 \,]\, d\theta'} \qquad (3\text{-a})$$

$$\Omega_m^{(2)}(N) = \frac{\int_0^\pi \left| \sum_{n=m}^{N} [a^o_{mn}(N)\Phi^{(2)}_{mn}(\theta') + b^e_{mn}(N)\Psi^{(2)}_{mn}(\theta')] - \mathbf{f}^{(2)}_m(\theta') \right|^2 d\theta'}{\int_0^\pi [\ |\mathbf{f}^{(1)}_m(\theta')|^2 + |\mathbf{f}^{(2)}_m(\theta')|^2\ ]\ d\theta'} \qquad (3\text{-b})$$

where $\Phi^{(i)}_{mn}(\theta')$, $\Psi^{(i)}_{mn}(\theta')$ and $\mathbf{f}^{(i)}_m(\theta')$ (i=1,2) are the known vector functions. By minimizing $\Omega^{(1)}_m(N)$ and $\Omega^{(2)}_m(N)$ for each m, $\Omega(N)$ becomes minimum. In the numerical calculation, we divide the interval $[0,\pi]$ into L segments and discretize the norms as follows [3]:

$$\tilde{\Omega}(N,L) = \sum_{m=0}^{N} [\ \tilde{\Omega}^{(1)}_m(N,L) + \tilde{\Omega}^{(2)}_m(N,L)\ ] \qquad (4)$$

with

$$\tilde{\Omega}^{(1)}_m(N,L) = \frac{\sum_{\ell=0}^{L} \varepsilon_\ell \left| \sum_{n=m}^{N} [a^e_{mn}(N,L)\Phi^{(1)}_{mn}(\theta'_\ell) - b^o_{mn}(N,L)\Psi^{(1)}_{mn}(\theta'_\ell)] - \mathbf{f}^{(1)}_m(\theta'_\ell) \right|^2}{\sum_{\ell=0}^{L} \varepsilon_\ell [\ |\mathbf{f}^{(1)}_m(\theta'_\ell)|^2 + |\mathbf{f}^{(2)}_m(\theta'_\ell)|^2\ ]} \qquad (4\text{-a})$$

$$\tilde{\Omega}^{(2)}_m(N,L) = \frac{\sum_{\ell=0}^{L} \varepsilon_\ell \left| \sum_{n=m}^{N} [a^o_{mn}(N,L)\Phi^{(2)}_{mn}(\theta'_\ell) + b^e_{mn}(N,L)\Psi^{(2)}_{mn}(\theta'_\ell)] - \mathbf{f}^{(2)}_m(\theta'_\ell) \right|^2}{\sum_{\ell=0}^{L} \varepsilon_\ell [\ |\mathbf{f}^{(1)}_m(\theta'_\ell)|^2 + |\mathbf{f}^{(2)}_m(\theta'_\ell)|^2\ ]} \qquad (4\text{-b})$$

where

$$\varepsilon_\ell = \begin{cases} 1/2 & : \ell = 0, L \\ 1 & : \text{otherwise} \end{cases} \qquad (5)$$

and $\theta'_\ell = (\pi/L)\ell$ . By using the normal equation or the orthogonal decomposition [7], the coefficients of the modal expansion $\{a^e_{mn}(N,L),\ a^o_{mn}(N,L),\ b^e_{mn}(N,L),\ b^o_{mn}(N,L)\}$ can be determined. By using these coefficients, we get the approximate scattered field $\mathbf{E}^s_N(P)$.

## 3. NUMERICAL RESULTS

Numerical computation are carried out for the bodies of revolution whose surfaces are described by

$$r'(\theta') = a(1 + \delta\cos 3\theta') \qquad (6)$$

where $a>0$ and $0<\delta<1$ (see Fig.2). Fig.3 shows the discrete norm $\tilde{\Omega}(N,L)$ and the error about the optical theorem $\varepsilon_{opt}(N,L)$ for a fixed N as a function of L. From this result, we can make a following choice for L such that $L=2(N+1)$. Fig.4 shows the convergence property of the radar cross section $\sigma_R$ as N increases. Fig.5 shows the monotonical decrease of $\tilde{\Omega}(N,L)$ and $\varepsilon_{opt}(N,L)$. These figures show the validity of this method. Fig.6 shows the comparison between the result of this method and the experimental one [6] for the radar cross section of the prolate spheroid. The agreement between them is good. As the numerical example, the radar cross section versus ka is shown in Fig.7. The norm and the error about the optical theorem are less than 1%. This result is considerably different from that of a sphere [8].

## 4. CONCLUSIONS

We apply the Yasuura method to the three-dimensional scattering problem. It is confirmed from the numerical results that the Yasuura method is effective even for the three-dimensional scattering problems.

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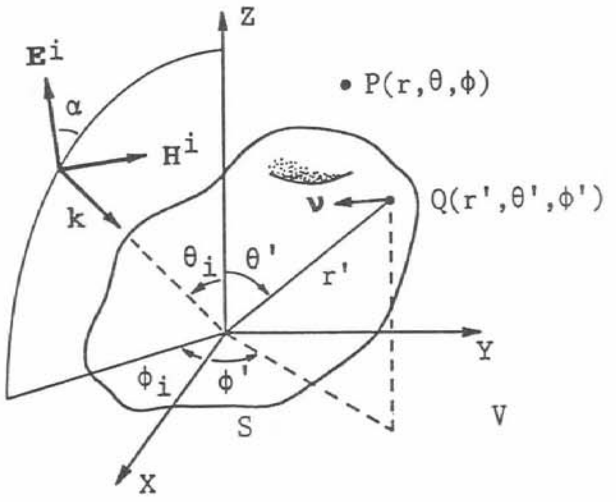


Fig.1. Geometry of the problem.

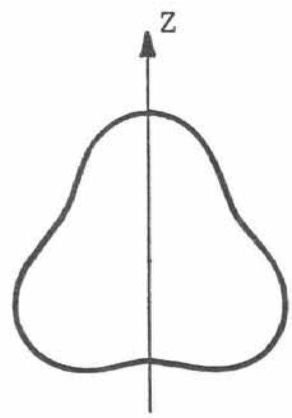


Fig.2. The cross section of the body of revolution described by Eq.(6).

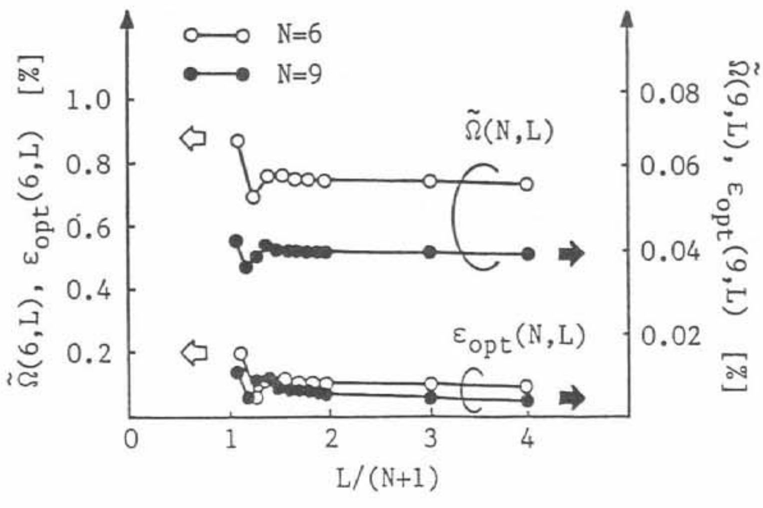


Fig.3. The norm $\tilde{\Omega}(N,L)$ and the error about the optical theorem $\varepsilon_{opt}(N,L)$ as a function of $L/(N+1)$. ($\delta=0.05$, $ka=3$, $\theta_i=30°$, $\alpha=0°$)

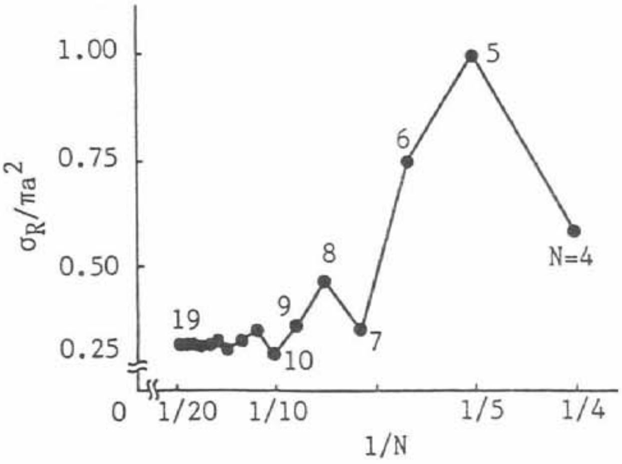


Fig.4. The radar cross section versus inverse of the number of truncation. (L=2(N+1), δ=0.15, ka=4, $\theta_i=\alpha=0°$)

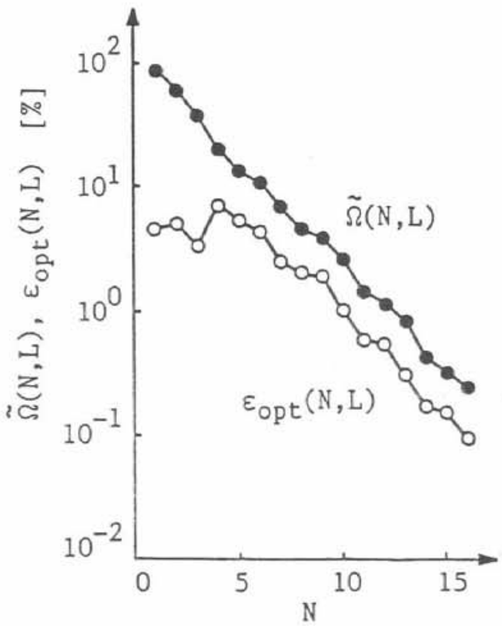


Fig.5. The norm $\tilde{\Omega}(N,L)$ and the error about the optical theorem $\varepsilon_{opt}(N,L)$ versus the number of truncation. (L=2(N+1), δ=0.15, ka=4, $\theta_i=\alpha=0°$)

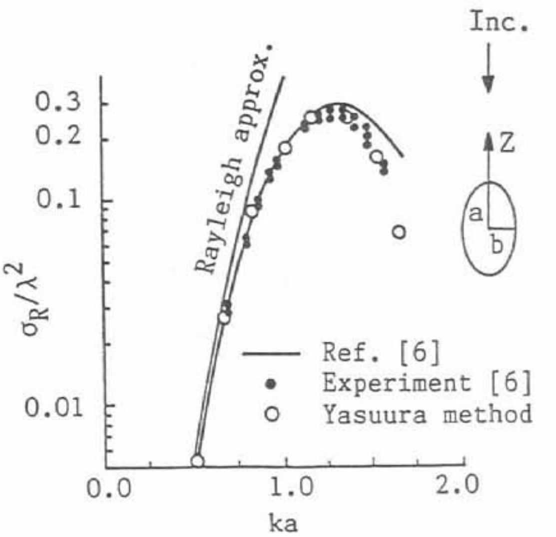


Fig.6. The radar cross section of the prolate spheroid. (L=2(N+1), a/b=1.15, $\theta_i=\alpha=0°$)

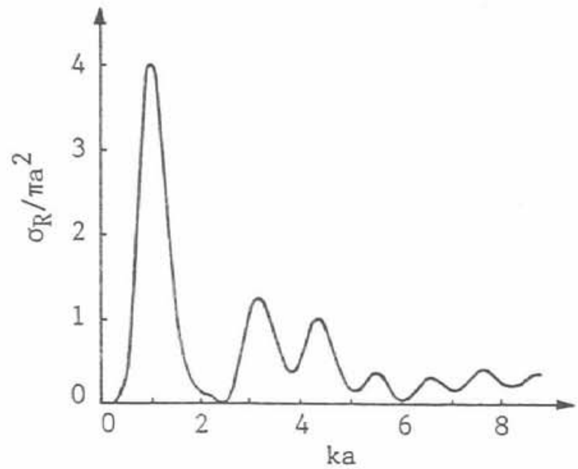


Fig.7. The radar cross section versus frequency ka. (L=2(N+1), δ=0.15, $\theta_i=\alpha=0°$)