

MOM AND FDTD ANALYSIS OF TM WAVE SCATTERING BY FINITE HOLLOW CYLINDER

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1. Introduction

The Finite Difference Time Domain (FDTD) method [1],[2] has been widely used for solving electromagnetic problems. This method is straightforward and efficient for modeling the electromagnetic fields in complex geometries within a practical level of the accuracy. However, when one requires a quite accurate result, a great amount of computer resource is often needed even the geometry is relatively simple.

A technique using quasi-static approximation was introduced by the authors [3],[4] for improving the accuracy in the FDTD method for various types of linear antenna analyses. This technique incorporated a quasi-static field behavior into the FDTD update equations. The validity of this approach was confirmed numerically for dipole and rectangular loop antennas. In this paper, in order to verify the strength of the quasi-static approximation technique, the application of this quasi-static approximation technique to the FDTD analyses of scattering problems [5] is proposed. The scattering analysis of a hollow cylinder using the Method of Moment (MoM) [6], quasi-static approximation technique, and contour-path FDTD (CPFDTD) method [7],[8] are performed and the results are compared.

2. Scattered-field analysis for cylindrical conductor

The geometry of the problem is shown in Fig. 1. We assume that the length of a cylinder is finite, and the thickness of the cylinder is infinitesimal. Assuming a plane wave is incident from y -axis, the incident electromagnetic fields are expressed by

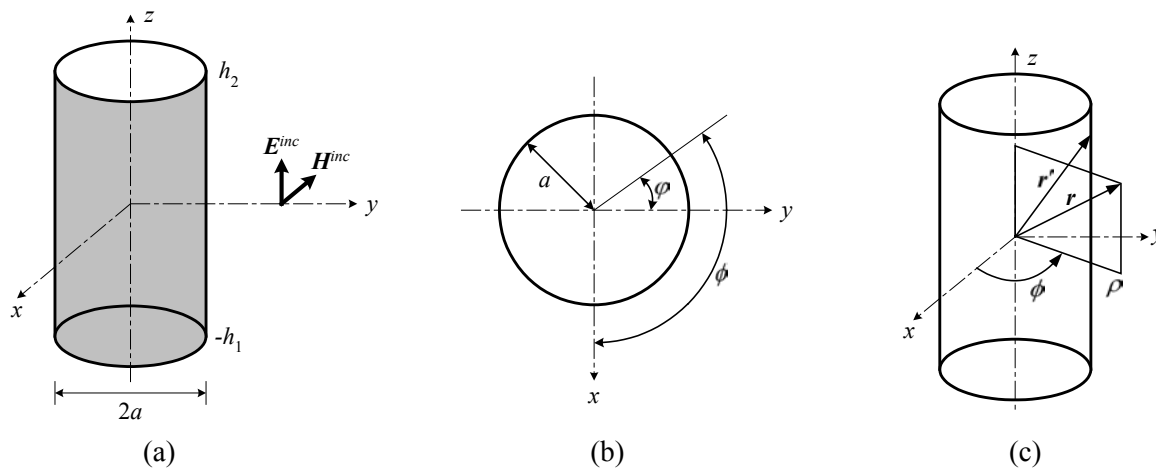


Fig. 1 Cylindrical conductor model.

$$\begin{cases} \mathbf{E}^{inc}(\mathbf{r}) = E_0 e^{jk_0 y} \hat{z} = E_0 e^{jk_0 \rho \sin \phi} \hat{z} \\ \mathbf{H}^{inc}(\mathbf{r}) = -\frac{E_0}{Z_0} e^{jk_0 y} \hat{x} = -\frac{E_0}{Z_0} e^{jk_0 \rho \sin \phi} (\cos \phi \hat{\rho} - \sin \phi \hat{\phi}) \end{cases} \quad (1)$$

A. Method of Moment

An electric field integral equation;

$$\hat{\rho} \times \left[+j\omega\mu_0 \int_S \overline{\overline{\mathbf{G}}}_0(\mathbf{r}, \mathbf{r}') \cdot \mathbf{J}(\mathbf{r}') dS' \right] = \hat{\rho} \times \mathbf{E}^{inc}(\mathbf{r}) \quad (2)$$

is satisfied on the cylindrical surface. The current density $\mathbf{J}(\mathbf{r}') = J_z(z', \phi') \hat{z}$ is expanded by a piecewise-sinusoidal function along z' and by $\cos p\phi = \cos p(\phi - \pi/2)$ for ϕ direction, that is

$$\mathbf{J}(\mathbf{r}') = \sum_{p=1}^{N_p} \sum_{n=1}^N I_n^{(p)} \mathbf{f}_n^{(p)}(z', \phi') \quad (3)$$

where

$$\mathbf{f}_n^{(p)}(z', \phi') = \frac{1}{2\pi a} f_n^{(p)}(z', \phi') \hat{z} \quad (4)$$

$$f_n^{(p)}(z', \phi') = \begin{cases} \cos(p-1)(\phi' - \frac{\pi}{2}) f_n(z') & z_{n-1} \leq z' \leq z_{n+1} \\ 0 & \text{others} \end{cases} \quad (5)$$

$$f_n(z') = \begin{cases} \frac{\sin k_0(z' - z_{n-1})}{\sin k_0 \Delta h_n} & z_{n-1} \leq z' \leq z_n \\ \frac{\sin k_0(z_{n+1} - z')}{\sin k_0 \Delta h_{n+1}} & z_n \leq z' \leq z_{n+1} \end{cases} \quad (6)$$

Substituting eq. (3) and carrying with the conventional MoM process, we can obtain expansion coefficients $I_n^{(p)}$ for all n and p . The scattering cross section can be obtained using $I_n^{(p)}$ as

$$\frac{\sigma(\theta, \phi)}{\pi a^2} = \left| \frac{1}{\pi} \sum_{p=1}^{N_p} \sum_{m=1}^N \tilde{I}_m^{(p)} \tilde{Q}_p(\theta, \phi) F_m(\theta) \right|^2 \quad (7)$$

where,

$$\tilde{I}_m^{(p)} = \frac{\varepsilon_p}{(-1)^{p-1}} \frac{Z_0}{8\pi^2} \frac{k_0}{E_0} I_m^{(p)} \quad (8)$$

$$\tilde{Q}_p(\theta, \phi) = \frac{(-1)^{p-1} e^{j(p-1)\phi} + e^{-j(p-1)\phi}}{2\varepsilon_p} \frac{J_{p-1}(k_0 a \sin \theta)}{k_0 a \sin \theta} \quad (9)$$

$$F_m(\theta) = \frac{e^{jk_0 \cos \theta z_{m-1}} - \cos k_0 \Delta h_m e^{jk_0 \cos \theta z_m}}{\sin k_0 \Delta h_m} + \frac{e^{jk_0 \cos \theta z_{m+1}} - \cos k_0 \Delta h_{m+1} e^{jk_0 \cos \theta z_m}}{\sin k_0 \Delta h_{m+1}} \quad (10)$$

and $J_n(x)$ is the Bessel function.

B. FDTD analysis using quasi-static approximation technique

Since the current density is assumed to be in z direction only as expressed for the Method of Moment, the same quasi-static current distribution as used in dipole antenna analysis [3] is applied to a

cylindrical conductor as

$$J_z^{st}(z) = \begin{cases} \frac{I}{2\pi a} \left(1 + \frac{z}{h_1}\right), & -h_1 \leq z \leq 0 \\ \frac{I}{2\pi a} \left(1 - \frac{z}{h_2}\right), & 0 \leq z \leq h_2 \end{cases} \quad (11)$$

Therefore, the quasi-static expressions of both electric and magnetic fields inside and outside a cylindrical conductor can be derived from a static vector potential as follows.

$$\mathbf{E}^{st}(\mathbf{r}) = \frac{1}{j\omega\mu_0\epsilon_0} \nabla \nabla \cdot \mathbf{A}^{st}(\mathbf{r}) - j\omega \mathbf{A}^{st}(\mathbf{r}) \approx \frac{1}{j\omega\mu_0\epsilon_0} \left\{ \frac{\partial F(\mathbf{r})}{\partial x} \hat{x} + \frac{\partial F(\mathbf{r})}{\partial y} \hat{y} + \frac{\partial F(\mathbf{r})}{\partial z} \hat{z} \right\} \quad (12)$$

$$\mathbf{H}^{st}(\mathbf{r}) = \frac{1}{\mu_0} \nabla \times \mathbf{A}^{st}(\mathbf{r}) = \frac{1}{\mu_0} \left[\frac{\partial A_z^{st}(\mathbf{r})}{\partial y} \hat{x} - \frac{\partial A_z^{st}(\mathbf{r})}{\partial x} \hat{y} \right] \quad (13)$$

Where,

$$\mathbf{A}^{st}(\mathbf{r}) = \frac{\mu_0}{4\pi} \int_V \frac{\mathbf{J}^{st}(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} dV' \quad (14)$$

$$F(\mathbf{r}) = \frac{\partial A_z^{st}(\mathbf{r})}{\partial z} \quad (15)$$

Moreover, since the radius of cylindrical conductor used in scattered-field analysis is comparatively larger than a dipole antenna, the modified FDTD update equations are applied to the FDTD and CPFDTD calculations both inside and outside the cylindrical conductor.

3. Numerical results

Parameters used for calculations are shown in Table 1. For the FDTD analysis, the Gaussian pulse TM plane wave is applied and the frequency was set $f = 2$ GHz. The quasi-static approximation technique substituting the modified FDTD update equation to 2 FDTD cells away from the cylindrical conductor both inside and outside and the conventional CPFDTD method are compared.

Fig. 2 illustrates the scattering cross section of a cylindrical conductor at $\theta = 90$. For the results of a smaller radius $a = 0.08\lambda$ cylindrical conductor shown in Fig. 2 (a), FDTD analysis using quasi-static field behavior can achieve higher accuracy agree very well with the MoM comparing with the conventional CPFDTD method. Moreover, for a larger radius cylindrical conductor $a = 0.8\lambda$ shown in Fig. 2 (b), quasi-static approximation technique gives higher accuracy results comparing with the CPFDTD method, but do not agree well with the MoM as in smaller radius cylindrical conductor.

Table 1 Parameters set in scattered-field simulations of cylindrical conductor.

Conductor height	$h_1 = h_2 = 2\lambda$
Conductor radius	$a = 0.08\lambda$ $a = 0.8\lambda$
N_p, N (MoM)	$N_p = 40$ $N = 80$
Cell size (FDTD)	$\Delta x = \Delta y = \Delta z = 2\lambda/75$

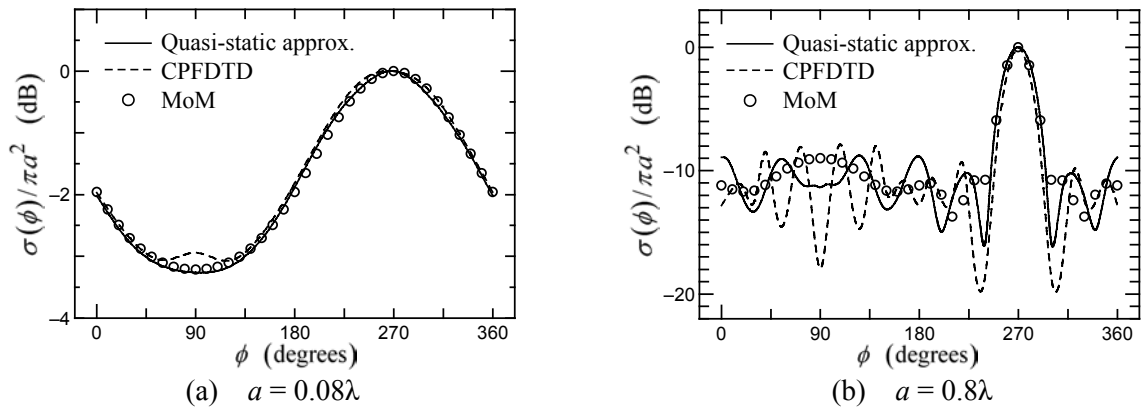


Fig. 2 Scattering cross section of cylindrical conductor

4. Conclusion

This paper has expressed the validity of the quasi-static approximation technique to the scattering problem. The scattered field analysis of a cylindrical conductor is performed, where the same quasi-static current distribution as dipole antenna analysis is used and the modified FDTD update equations derived from the quasi-static field distribution is applied to the FDTD cells both inside and outside the cylindrical conductor. The simulation results indicated that the quasi-static approximation technique could also achieve higher accuracy for the FDTD analysis of scattering problem of a cylindrical conductor.

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