THE ELECTROMAGNETIC WAVE DIFFRACTION FROM A DIELECTRIC MULTILAYER-COATED FOURIER GRATING IN CONICAL MOUNTING

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1 Introduction

A multilayer-coated grating consists of multiple coated dielectric on metallic substrate, the diffraction efficiency can be controlled by changing profile or the permittivity of a multilayer-coated dielectric media. The multilayer-coated grating, which is alternately laminated by set of a low and a high refractive index on the relief metallic grating, is used as a beam-sampling mirror for the high-power lasers.

The resonance absorption anomaly occurs on grating surface composed of the metal in which the power of incident wave is absorbed at the specific incident angle. This also has large influence on the incident angle and intensity of the absorption so that the profile and the permittivity of a multilayer-coated dielectric on the grating rule the resonance anomaly. There are detailed analysis by the mode matching method[1] in analyzing the diffraction problem of the plane wave which is illuminated arbitrary incident angle at the diffraction grating which has also periodic structure the problem of conical mounting in the one direction for the polarization conversion, and also analysis by the differential method and the T-matrix method[2]. Moreover, it argues about polarization conversion using differential method, and the diffraction problem by exposing the plane wave of arbitrary incidence at the multilayer-coated dielectric diffraction grating is also analyzed by the differential method[3] with R-matrix expression.

In this paper, the plane electromagnetic wave diffraction problem was analyzed as a quasi-two-dimensional problem such as the multilayer-coated Fourier grating in conical mounting. The feature of the formulation by the concept of space harmonics was maintained at the incident medium and the substrate using the T-matrix method. And expression of R-matrix based on extinction theorem in the coating layer is presented by using recurrence matrix calculation. As a numerical example, -1th diffraction efficiency (Rittrow mounting) in the Fourier grating which is laminated by the dielectric of layers on the aluminum substrate versus normalized layer thickness and the incident angle with azimuth angle are shown in figures.

2 Analysis

2.1 Geometry of the problem

In Fig.1, the cross section and geometry of the multilayer-coated Fourier grating are shown. The grating profile is assumed periodically in the x direction, uniform in the y direction. We consider three regions of the problem which is a dielectric multilayer, an incident medium and a substrate in half space, respectively.

The incident and substrate region of the grating are filled by material of homogenous isotropic medium (permittivity ε_0 and ε_{I+1} , the permeability μ_0 and μ_{I+1}), respectively, and also multilayer media (ε_i and $\mu_i (i = 1, 2, .., I)$) are considered.

This problem with the wave number k_{0y} has a uniform structure in the y direction, the fields E_{iy} and H_{iy} in y direction yield, the transverse fields $\boldsymbol{E}_{it}$, $\boldsymbol{H}_{it}$ in x and z direction are obtained from the following equations, i.e., quasi-two-dimensional problem can be treated.

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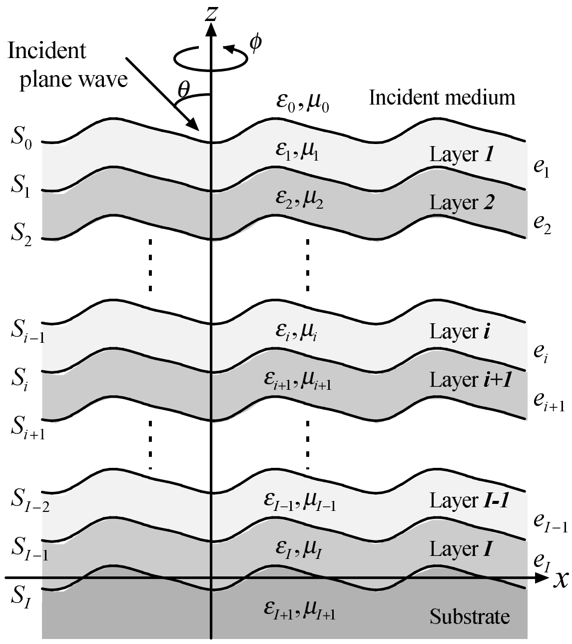


Figure 1: Geometry of the Problem and a multilayer-coated Fourier grating

The electromagnetic fields in the y direction are presented for the each region, by applying Huygens principle and Green's theory on the extinction theorem, the formulation can be expressed in the each region.

By applying boundary condition in the surface of each region, the position vector of the secondary source on the surface must be noted, those are implicated a vector on the surface at the left hand and a vector on the surface at the right hand, respectively. The electromagnetic fields in the y direction is continuously on the boundary and satisfy radiation condition for the z direction and periodic properties for the x direction.

2.2 T-matrix formulation

In order to find T-matrix formulation by using R-matrix expression, the immediate coefficients are eliminated in the form of the recombined matrix. The reflected coefficients $\boldsymbol{b}_m$ and transmitted coefficients $\boldsymbol{A}_m$ are finally represented for the incident wave coefficients $\boldsymbol{a}_m$ and $\boldsymbol{B}_m$ satisfying radiation condition in the substrate. The total T-matrix formulation $[T_{mn}]$ with R-matrix expression is finally obtained, that is

$$\begin{bmatrix} \boldsymbol{b}_m \\ \boldsymbol{A}_m \end{bmatrix} = [T_{mn}] \begin{bmatrix} \boldsymbol{a}_m \\ \boldsymbol{B}_m \end{bmatrix} \tag{1}$$

where

$$[T_{mn}] = \begin{bmatrix} X_{11}R_{11}^{(I)} + X_{12} & X_{11}R_{12}^{(I)} \\ Y_{21}R_{21}^{(I)} & Y_{21}R_{22}^{(I)} + Y_{22} \end{bmatrix} \begin{bmatrix} X_{21}R_{11}^{(I)} + X_{22} & X_{21}R_{12}^{(I)} \\ Y_{11}R_{21}^{(I)} & Y_{11}R_{22}^{(I)} + Y_{12} \end{bmatrix}^{-1}, \tag{2}$$

matrix X consists of elements $Q_D^{\pm}(k_0, f_0)$ in free space, matrix Y also consists of elements $Q_D^{\pm}(k_{I+1}, f_I)$ in substrate media. Furthermore, R–matrix $R^{(I)}$ is obtained from the recurrence matrix $r^{(i)}$ composed of elements $Q_D^{\pm}(k_i, f_l)$, and also $\zeta_i^{\pm} = \exp\{\pm jk_{izm}(d)\}$.

The grating profiles are expressed by profile functions $f_l(x')$ of the Fourier grating and assumed to be given by

$$f_l(x) = -h\left\{\cos\left(\frac{2\pi}{P}x\right) + \gamma\cos\left(\frac{4\pi}{P}x + \delta\right)\right\} \tag{3}$$

where the P, h are the period and amplitude of the basis grating, $h\gamma$ and δ denote amplitude and phase of the second harmonic wave, respectively. The elements of the Dirichlet matrices $Q_D^{\pm}$, Neumann matrices $Q_N^{\pm}$ and hybrid matrices $Q_h^{\pm}$ can be analytically expressed by the Bessel functions

$$Q_D^{\pm}(k_i, f_l) = \frac{-1}{\sqrt{k_{izm}}} \sum_{l=-\infty}^{\infty} \exp\left\{\mp jl(\frac{\pi}{2}+\delta\right\} (\mp j)^{|n-m\pm 2l|} J_{|n-m\pm 2l|}(k_{izm}h) J_l(k_{izm}h\gamma) \quad (4a)$$

$$Q_N^{\pm}(k_i, f_l) = \left[\frac{1-\alpha_{in}\alpha_{im}}{\pm\beta_{im}}\right] Q_D^{\pm}(k_i, f_l) \quad (4b)$$

$$Q_h^{\pm}(k_i, f_l) = \alpha_{in} Q_D^{\pm}(k_i, f_l) \quad (4c)$$

This is Fourier grating whose profile is expressed by summation of two sinusoidal functions, for which are rigorously formulated in this paper. Also, the expression of the sinusoidal grating is obtained by setting $\gamma = 0$ in Fourier grating profile, that is, analysis of the electromagnetic diffraction from a dielectric coated sinusoidal grating ($\gamma = 0$) has already been done[1].

2.3 Diffraction efficiencies

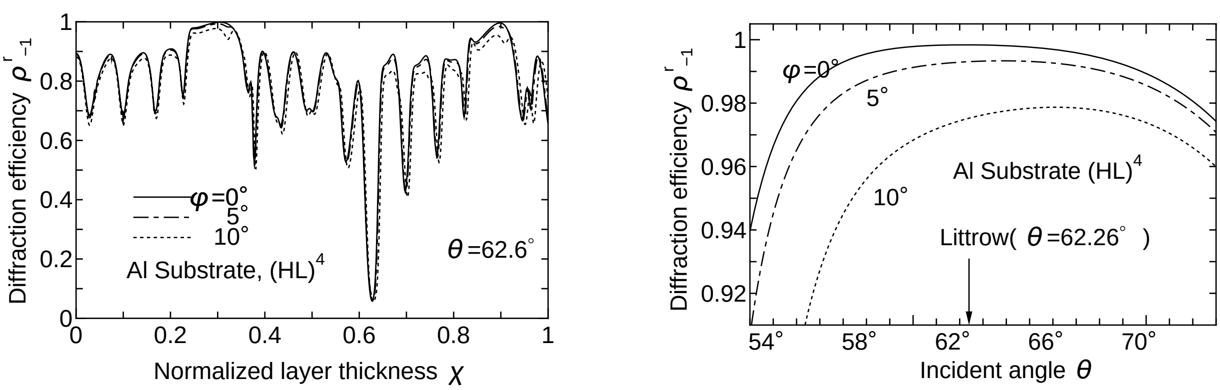
The diffraction efficiency ρ_m^r of the reflected wave in TE mode (mode in which electric field does not exist in the direction of z) and the diffraction efficiency ρ_m^t of the transmitted wave in TM mode (mode in which a magnetic field does not exist in the direction of z) in the total power can be defined by using the time averages of Poynting vector by normalizing those incident electromagnetic waves. Then, the reflective diffraction efficiency ρ_m^r and the transmission diffraction efficiency ρ_m^t in each diffraction mode ($= 0, \pm1, \pm2, \pm3, ...$) are obtained for by the following formulation

$$\rho_m^r = \frac{|b_m^{(e)}|^2 + |\eta_0 b_m^{(h)}|^2}{|e_{0y}|^2 + |\eta_0 h_{0y}|^2} \quad (5a)$$

$$\rho_m^t = \frac{k_{0\perp}}{k_{(I+1)\perp}} \frac{\varepsilon_{I+1}}{\varepsilon_0} \frac{|A_m^{(e)}|^2 + |\eta_{I+1} A_m^{(h)}|^2}{|e_{0y}|^2 + |\eta_0 h_{0y}|^2} \quad (5b)$$

The total reflected ρ_{total}^r and transmitted ρ_{total}^t diffraction efficiencies are also finally obtained by summing of the propagation mode.

3 Numerical example and discussions



(a) versus normalized layer thickness χ

(b) versus incident angle θ

Figure 2: The diffraction efficiency ρ_{-1}^r of multilayer-coated Fourier grating.

This numerical example is chosen to be the dielectric multilayer-coated film of a low refractive index ($n_L = 1.39$, M_gF_2) and a high refractive index ($n_H = 2.45$, T_iO_2) on the Fourier grating substrate of aluminum ($n_{sub} = 0.997 - j6.94$, $\lambda = 590nm$). The multilayer-coated film (eight layers) $(HL)^4$ Fourier grating laminated 4 times, however a shape parameter set $P = 333.3nm$, $h = 60nm$, $\gamma = 0.2$ and $\delta = \pi/2$. It is the case where some azimuth angle ϕ was changed and an incident plane wave by polarization angle $\tau = 90°$. When normalized thickness χ is defined by

$$\chi = e_i n_i / \lambda \tag{6}$$

where H; $i =$ odd and L; $i =$ even, thus a set of the thickness e_H and the thickness e_L mean a thickness of the dielectric film, λ is also the wave number in free space.

The diffraction efficiency characteristics of the -1th mode versus χ for an azimuth angle ϕ is shown in Fig.2(a) and -1th diffraction efficiency (Littrow angle $= 62.26°$) characteristics versus incident angle θ for thickness $\chi = 0.304$ is also shown in Fig.2(b), respectively.

In the case of a sine wave grating ($\gamma = 0$), this is checking that it is in good agreement with a numerical result[3] in an azimuth angle ($\phi = 0$).

4 Conclusion

The electromagnetic plane wave diffraction characteristic in arbitrary incidence and polarization on the multilayer-coated Fourier grating was exactly analyzed by the T-matrix method using the R-matrix expression within the coating layer.

As a result, numerical singularity of the matrix calculation by evanescent mode is suppressed and the matrix element is analytically obtained by using the Bessel function.

This formulation is applied almost regardless of the layer's number or the layer's thickness for the coating. This result is excellent compared with the conventional calculating method in the accuracy and the efficiency of numerical calculation. But, this formulation exists restrictions of the conventional T-matrix method in the groove depth of the grating.

As an example of numerical computation, the characteristics of the thickness and the incident angle for the diffraction efficiency of the Fourier grating with laminating the dielectric layers to the aluminum substrate were shown.

Moreover, grating is due to applying this method for the analysis in the arbitrary incidence in that is called doubly grating that has periodic structure in the two directions, or the diffraction grating which is made them stratified, and arbitrary polarization in the future.

References

[1] T. Matsuda, D. Zhou, Y. Okuno, "Numerical analysis of TE-TM mode conversion in a metal grating placed in conical mounting," IEICE Trans. Electronics(C–I), vol.J82–C–I, no.2, pp.42–49, Feb. 1999(Japanese).

[2] S. L. Chuang, and J. A. Kong,"Wave scattering from a periodic dielectric surface for a general angle of incidence," Radio Science, vol.17, no.3, pp.545–557, May–June 1982.

[3] L. Li, "Multilayer-coated diffraction gratings: differential method of Chandezon et al. revisited," *J. Opt. Soc. Am. A*, Vol.11, No.11, pp.2816–2828, Nov. 1994.

[4] M. Ohki, T. Kurihara, and S. Kozaki, "Analysis of electromagnetic wave diffraction from a metallic Fourier grating by using the T-matrix method," *J. Electromagnetic Waves and Applications*, Vol.11, No.9,pp.1257–1272, 1997.