ON-SURFACE RADIATION CONDITION THEORY TO SCATTERING BY TWO DIMENSIONAL PENETRABLE OBJECTS

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Abstract: The recent new analytical formulation of the electromagnetic wave scattering by perfectly conducting two dimensional objects using the on-surface radiation boundary condition approach is conveniently extended to the case of two dimensional homogeneous convex dielectric objects and cavity backed apertures. The existing classical solution for the scattering and penetration analysis of penetrable objects is based upon the coupled field formulation with the external and internal fields directly coupled together through the electromagnetic surface boundary conditions. It is shown here that a substantial simplification in the analysis can be obtained by applying the out-going radiation boundary condition on the surface of the convex homogeneous dielectric object and similarly at the opening of the cavity backed aperture. This analysis procedure decouples the fields in the two regions to yield explicitly a differential equation relationship between the external incident field excitation and the corresponding field distribution in the interior of dielectric object or inside the cavity backed aperture. The interior fields can be obtained by solving the differential equation using either an analytical approach or a suitable numerical method. Two dimensional scattering examples along with validations are reported showing the near surface field distributions for a homogeneous circular dielectric cylinder and an elliptic dielectric cylinder each with transverse magnetic plane wave excitation. For the case of cavity back aperture, an example of a rectangular trough is considered. The resulting surface currents and far-field distributions are compared with good agreement to those obtained from the integral equation solution.

Background: Recently a novel analytical technique based on the on-surface radiation condition (OSRC) theory [1] was introduced for modeling high-frequency electromagnetic wave scattering. For two dimensional analysis of electromagnetic scattering by convex conducting objects, the OSRC approach has demonstrated substantial simplification of the usual integral equation for the induced surface currents through the application of a radiation boundary operator [2] directly on the object surface. This method has been shown to give good results for two dimensional perfectly conducting objects having either transverse magnetic (TM) or transverse electric (TE) plane wave excitation. The main purpose of this paper is to demonstrate further an extension and application of this novel OSRC analytical technique to the analysis of electromagnetic scattering and penetration by homogeneous convex dielectric objects and for the analysis of cavity backed apertures.

Over the past several years, there has been substantial investigation in the development of expansions for local radiation boundary conditions of higher order than the Sommerfeld condition [2]. The goal has been to achieve nearly reflection-free truncations of space grids used to model wave interactions with structures via direct finite-difference [3] or finiteelement simulations of the governing partial differential field or wave equations. The boundary operators, containing mixes of both space and time partial derivatives principally exploit the asymptotic behavior of the scattered field in either cylindrical or spherical coordinate system. While the radiation boundary operators have been successfully applied to the scattered field away from the specific scatterer of interest, the OSRC concept was prompted by the observation that the outer boundaries of finite-difference time-domain space grids [3] employing the second-order radiation boundary operator, could be brought very close to a scatterer without adversely affecting the far-field results. In a limit, if the radiation boundary condition is applied directly on a two dimensional conducting scatterer surface, the original integral equation for the scattered field can be reduced to merely a line integral of known fields around the surface for the TM case, or an ordinary differential equation to be solved around the surface for TE case, whereby analytical expressions for the induced surface current distribution can be obtained [1]. Interestingly enough elaborate finite-difference numerical simulation schemes are no longer required. In order to extend the general applicability of the OSRC technique to the case of scattering and penetration involving re-entrant and other related cavity type interaction problems, a detail study is reported in this paper concerning the electromagnetic scattering and interaction analysis of homogeneous convex dielectric object [4]. In fact, the formulation is applicable in a wide frequency regimes, and specifically for the case of high frequency regimes, an asymptotic expansion in terms of the wave number k can also be used in the derivation of the near and far scattered fields.

For the analysis of homogeneous dielectric objects [4], use of the dielectric boundary conditions in the OSRC differential equation yields a relation between the electric field and its normal derivative on the surface of the scatterer. As the OSRC equation is only valid over the boundary surface, one needs an additional relation between the electric field and its normal derivative on the scatterer contour to completely solve the problem. In this study, a functional form for the interior field distribution is specified, immediately providing the required additional relation through differentiation. Specifically, the interior electric field is represented by a modal expansion with unknown coefficients. For a circular dielectric cylinder, a direct substitution of this modal expansion into the OSRC equation followed by an enforcement of the orthogonality of the angular eigenfunctions gives a solution for the unknown coefficients. However, for a general dielectric cylinder, such a procedure is not applicable because a modal series defined in an arbitrary dielectric object is not guaranteed convergence at points everywhere on the scatterer contour, so that the boundary conditions cannot be applied to this expansion. To surmount this difficulty, an analytic continuation method is used [5]. In this scheme, the interior field modal expansions are analytically continued throughout the scatterer interior in order to obtain series expansions which are valid on the contour of the dielectric scatterer. Once this is done, the OSRC differential equation can be used to apply the boundary conditions, yielding a matrix equation for the unknown expansion coefficients.

Formulation: Let us consider a two dimensional, convex, homogeneous dielectric scatterer excited normally by a TM polarized plane wave. The dielectric scatterer is assumed to be uniform in the z-coordinate direction. The cross section of the arbitrary convex cylinder is contained in region 2 and is bounded by a contour C. Outside the region 2 is region 1 representing an isotropic free space medium. It is assumed that the parameters (ε_1, μ_1) and (ε_2, μ_2) are the permittivity and permeability of the free space and dielectric region. Further

$(\bar{E}_1^s, \bar{H}_1^s)$: Electric and magnetic scattered fields in the free space region

$(\bar{E}_2^s, \bar{H}_2^s)$: Electric and magnetic scattered fields in the dielectric region

$(\bar{E}^{i}_{1}, \bar{H}^{i}_{1})$: Electric and magnetic incident fields in the free space region

In the classical approach, the expressions for the scattered fields in the two regions are obtained in terms of the equivalent magnetic and electric currents on the scatterer contour C, by invoking the electromagnetic equivalence principle. In the application of OSRC method for the electromagnetic scattering by the dielectric object, higher order radiation boundary condition is enforced on the contour C of the convex scatterer itself. According to this approach, for the case of a TM excited two dimensional smooth convex cylinder, the z-component of the scattered electric field in the free space should satisfy the following radiation boundary condition [1]:

$$B_2 E_1^s = O(R^{-2m-1/2}) \tag{1a}$$

$$B_2 = \frac{\partial}{\partial v} + \frac{\xi}{2} + jk - \frac{1}{2(\xi + jk)}\left(\frac{\partial^2}{\partial s^2} + \frac{\xi^2}{4}\right) \tag{1b}$$

where R is the radius of large circle, k is the propagation factor in the free space region, ω is the frequency, $\partial/\partial v$ is the normal derivative, ξ and s are the curvature and arc length respectively along the contour C. Further, across the boundary contour C of the homogeneous dielectric cylinder, the z component of the electric fields and the transverse component of the magnetic fields are continuous, yielding the regular surface boundary conditions:

$$E_1^s + E_1^i = E_2^s \tag{2a}$$

$$\frac{1}{\mu_1}\left(\frac{\partial E_1^i}{\partial v} + \frac{\partial E_1^s}{\partial v}\right) = \frac{1}{\mu_2}\frac{\partial E_2^s}{\partial v} \tag{2b}$$

On substituting the boundary conditions (2a) and (2b) into the OSRC boundary operator (1)

$$\frac{\mu_1}{\mu_2}\frac{\partial E_2^s}{\partial v} + AE_2^s + B\frac{\partial^2 E_2^s}{\partial s^2} = \frac{\partial E_1^i}{\partial v} + AE_1^i + B\frac{\partial^2 E_1^i}{\partial s^2} \tag{3a}$$

$$A = jk + \frac{\xi}{2} + \frac{j\xi^2}{8(k - j\xi)} \tag{3b}$$

$$B = \frac{j}{2(k - j\xi)} \tag{3c}$$

The right hand side of the expression (3a) is completely known. Further, the total electric field evaluated on the contour C is proportional to the equivalent magnetic surface current on C, and that the normal derivative of the electric field on C is proportional to the equivalent electric surface current on C. Thus, the expression (3a) provides a relation between the equivalent electric and magnetic currents on the surface of the dielectric scatterer. Since the expression (3a) is a relation between two unknowns, it alone is not enough to solve the scattering problem;

an additional relation between the total electric field and its normal derivative on C is also necessary. If a functional form is assumed for the interior scattered electric field, the second required relation is immediately provided through differentiation. This single expansion, when substituted into the relationship (3a), is enough to yield the unknown scattering coefficients. Once the electric field is known along the contour C, the corresponding scattered magnetic field along the contour C can be obtained by taking the normal derivative of the scattered electric field.

Results: In this paper, the application of the OSRC method to the case of homogeneous circular dielectric object is presented. A brief outline of the analytic continuation formulation, which is required only inside the scatterer, is then presented for the case of a homogeneous dielectric elliptic cylinder. Further an analysis procedure is also presented for the case of cavity back apertures based on the same above ideas. Numerical results for the surface electric and magnetic current distributions are presented for both the dielectric circular and elliptic cylinders having TM plane wave excitation. The numerical results for a rectangular trough is also presented to demonstrate application to cavity backed apertures. The TE excitation cases are not reported here since it forms a trivial dual case. For the canonical penetrable case studies reported, good validations have been obtained based on the regular eigenfunction solution and the combined field coupled integral equation numerical solution.

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