# A Fast Iterative Algorithm by Using Current/Voltage Unknowns for Small Antenna Design with Finite Ground Plane

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**Abstract**

**A novel iterative algorithm using the method of moment calculation with voltage unknowns is proposed for designing antennas with a large ground plane. The proposed algorithm uses the inverse matrix obtained from the usual decomposition of a fast multipole method impedance matrix via MacLaurin's expansion. Using this algorithm, we reduced the computational time of electromagnetic performances and memory. Electromagnetic performances of a conducting plate, including antenna pattern, were calculated by using a direct algorithm with voltage unknowns, an iterative algorithm with current unknowns, and the proposed algorithm. Numerical results show an advantage when using the proposed algorithm as compared to the other algorithms. Using the proposed algorithm in a trial-and-error sequence, we determined the most suitable antenna pattern with good impedance matching in a rectangular conducting plate in twelve hours.**

## 1. Introduction

Many different wireless services, including cellar phone, wireless LAN, GPS, and so on, are currently being offered to users. Users appreciate the convenience the services that these wireless devices provide. A key component for such devices is a multi-mode antenna. The multi-mode antenna has good sensitivity at the different frequencies required for each wireless service. A planar antenna can enable multi-mode operation. Several planar antennas for these applications have been proposed [1-3]. Antennas for use in mobile wireless devices must be designed taking into account the electromagnetic effect of a circuit board as a finite ground plane. Design technology of such an antenna has been extensively studied by many authors [3-7]. The practical antenna design requires a huge number of calculations when determining a potential antenna that contains a finite ground plane. An algorithm for fast calculations of electromagnetic performance has been extensively investigated in recent years [8-13]. One of the present authors has proposed a novel method based on the method of moment (MoM) to introduce the concept of voltage unknowns instead of [current unknowns [7, 12]. This method needs to create an inverse matrix of the impedance matrix for an entire two dimensional area where conductors can exist. This inverse matrix requires not only a large memory area but also enough time due to the use of an elimination algorithm. A disadvantage of the method becomes serious when the antenna design requires a large ground plane, e.g. a LAN antenna with an LCD chassis of PC, a DMB/DVB-H antenna for a PDA, and etc. To overcome this situation, we propose an iterative algorithm by using current and voltage unknowns. The key of this algorithm is that the entire analysis area is divided into blocks, whose shapes match the shape of the antenna area, when the fast multipole method (FMM) is applied to these areas [8, 11]. The electromagnetic calculation of the MoM by voltage unknowns is introduced briefly in Section 2. The process to derive a FMM algorithm with current unknowns for a small antenna with a large ground is outlined in Section 3. Finally, by combining these two methods, we achieved a novel fast calculation algorithm, to reduce memory size and the time to design an antenna. This algorithm is explained in Section 4

## 2. Electromagnetic calculation by voltage unknowns

To applying the MoM to antenna design, the entire conducting area, including the antenna and the finite ground plane, is divided into small segments in which current unknowns are defined. We can only change the segment in the antenna area during the antenna design process. The number of unknowns on the ground is much larger than that on the antenna. In this case, the conventional MoM formulation is as follows.

$$Zi = v,$$
$$v = \left[0,\cdots 0, v_e, 0,\cdots 0\right]^t, v_e = V \tag{1}$$

where Z, is an impedance matrix, $i$ is an unknown current vector, and $v$ is a constant voltage vector. The constant voltage vector has only one non-zero component related to the segment in which the antenna is being excited by the voltage, V. Because the area of antenna is much smaller than the entire area, each Z-matrix of the potential structure to calculate the electromagnetic performances is large. A large Z-matrix requires a longer calculation time. Equation 1 uses an electrical boundary condition so that the tangential component of the electric field vanishes on the conducting segment when a current unknown exists. On the basis of the electromagnetic theory, if this segment was removed, we can consider a new unknown related to voltage, an explicit local boundary condition of zero current unknowns. This consideration is represented as by the following equation.

$$\begin{aligned}&[Z][i_1,\cdots i_{r-1},0,i_{r+1},\cdots i_n]^t\\&=[0,\cdots 0,v_e,0,\cdots 0,v_i,0,\cdots 0]^t\end{aligned} \tag{2}$$

Introducing the inverse matrix of Z, equation 2 is transformed into equation 3.

$$\begin{aligned}&[Y][0,\cdots 0,v_e,0,\cdots 0,v_i,0,\cdots 0]^t\\&=[i_1,\cdots i_{r-1},0,i_{r+1},\cdots i_n]^t, Y\equiv Z^{-1}\end{aligned} \tag{3}$$

Using this equation, an arbitrary current unknown, $i_j$, is given within a small computational time using the equation as follows.

$$\begin{aligned}&0=Y_{ie}v_e+Y_{ii}v_i\\&i_j=Y_{je}v_e+Y_{ji}v_i\end{aligned} \tag{4}$$

These procedures are verified mathematically in the appendix.

3. FMM algorithm for small antennas with large ground

The direct method can reduce the number of unknowns for the MoM calculation. However, this method includes the calculation of an inverse matrix, which requires a long computational time. To avoid calculating the inverse matrix, using an iterative algorithm to compute matrix equations of the MoM is efficient. Even if an iterative algorithm is used, the use of the original impedance matrix is not still efficient due to the large number of unknowns on the ground plane that cannot change their shape. Modifying the matrix equation is very effective to eliminate current unknowns on the ground plane. The original matrix equation for an antenna with a large ground plane is represented in the next equation.

$$\begin{bmatrix}Z_{AA} & Z_{AG}\\ Z_{GA} & Z_{GG}\end{bmatrix}\begin{bmatrix}i_A\\ i_G\end{bmatrix}=[0,\cdots 0,v_e,0,\cdots 0]^t, e\in A \tag{5}$$

,where suffix A and G indicate the "antenna" and the "ground plane". We cannot modify the shape of the ground plane when designing the antenna, therefore the number of current unknowns, $i_G$, does not change. On the basis of this limitation, equation 5 can be transformed into the following equations.

$$\begin{aligned}&Z_{AA}i_A+Z_{AG}i_G=[0,\cdots 0,v_e,0,\cdots 0]^t,\\&Z_{GA}i_A+Z_{GG}i_G=[0,\cdots 0]^t,\\&(Z_{AA}-Z_{AG}Z_{GG}^{-1}Z_{GA})i_A=[0,\cdots 0,v_e,0,\cdots 0]^t\end{aligned} \tag{6}$$

The high density of the original impedance matrix of MoM makes it inefficient to apply an iterative algorithm to the third equation in equation 6. To accelerate the iterative calculation of this equation, we introduce the FMM [7, 12]. When the FMM is implemented, $Z_{GG}$ is decomposed into two parts:

$$Z_{GG}=Z_{GG}^{near}+Z_{GG}^{far}. \tag{7}$$

The first term of the right hand side in equation 7 includes only the reaction of the same group and of the adjacent group, therefore this part is a diagonal band matrix. The other term is not a diagonal band matrix, however using multipole series expansion of Green's function and orthonormality of harmonic functions in integral expression of the expanding function [9], the typical FMM procedure transforms this term into a sparse matrix form:

$$Z_{GG}^{far}\cong V_G T_G U_G, \tag{8}$$

where matrix V and U are the de-aggregation matrix and the aggregation matrix. These matrixes are both diagonal matrices. Matrix T is the translation matrix. This matrix is not diagonal but sparse [7]. Using equation 7 and 8, we can derive the following series form of $Z_{GG}^{-1}$.

$$\begin{aligned}&(Z_{GG})^{-1}\cong\left(Z_{GG}^{near}+V_GT_GU_G\right)^{-1}\\&=\left\{Z_{GG}^{near}\left(E+\left[Z_{GG}^{near}\right]^{-1}V_GT_GU_G\right)\right\}^{-1}\\&=\{E-\left[Z_{GG}^{near}\right]^{-1}V_GT_GU_G\\&+\left(\left[Z_{GG}^{near}\right]^{-1}V_GT_GU_G\right)^2-\cdots\}\left[Z_{GG}^{near}\right]^{-1}\end{aligned} \tag{9}$$

Using the same decomposition shown in equation 7 for $Z_{AA}$, the sparse matrix form of the third equation in equation 6 is finally obtained by

$$\begin{aligned}&(Z_{AA}^{near}+V_AT_AV_A\\&-Z_{AG}\left[Z_{GG}^{near}\right]^{-1}\left\{E-V_GT_GU_G\left[Z_{GG}^{near}\right]^{-1}+\cdots\right\}Z_{GA})i_A\\&=[0,\cdots 0,v_e,0,\cdots 0]^t\end{aligned} \tag{10}$$

Equation 10 utilizes considerably less current unknowns related to the antenna area than equation 5, therefore successfully reducing the calculation time and memory size required for computation.

4. FMM algorithm by using voltage unknowns

The discussion in the previous section assumes that the area of the antenna is clearly separated from the area of the ground plane. However, setting a border between these areas is difficult. If the antenna structure is characterized by a gap and not by a conductor, this border is unnecessary. By combining the two methods in sections 2 and 3, we derive a novel FMM algorithm by using voltage unknowns. Equation 5 is directly transformed into a conventional FMM form as follows.

$$\begin{aligned}[Z][i] &= (Z_{near} + Z_{far})[i] \cong (Z_{near} + VTU)[i] \\ &= [0,\cdots 0, v_e, 0,\cdots 0]^t,\end{aligned} \qquad (11)$$

where $Z_{near}$, $Z_{far}$, V, T, and U are similar to those terms in equations 7 and 8. Applying the series expansion in equation 9, equation 11 can be converted into an equation as follows.

$$\begin{aligned}&[i_1, i_2, \cdots] \\ &= \{[Z_{near}]^{-1} - [Z_{near}]^{-1} VTU [Z_{near}]^{-1} \\ &+ [Z_{near}]^{-1}(VTU[Z_{near}]^{-1})^2 - \cdots\}[0,\cdots 0, v_e, 0,\cdots 0]^t\end{aligned} \qquad (12)$$

Current unknowns existing in the antenna area can be set to zero. Responding to an unknown equal to zero, the novel voltage unknown must be set in the column vector in the right hand term of equation 12. Assuming D is a subset of indexes, which indicate the current unknowns are set to zero. The matrix equation for an iterative algorithm is expressed by

$$\begin{aligned}&\{[Z_{near}]^{-1} - [Z_{near}]^{-1} VTU [Z_{near}]^{-1} \\ &+ [Z_{near}]^{-1}(VTU[Z_{near}]^{-1})^2 - \cdots\}_{p\in D,e}^{q\in D}[v_{i\in D}, v_e]^t \\ &= [i_{i\in D}] = [0], v_e = V\end{aligned} \qquad (13)$$

where $\{\bullet\}_m^n$ is the matrix consisting of n-columns and m-rows from matrix M. After determining $v_i$,, the current unknowns, which are not explicite zeros, are directly given by substituting $v_i$,, into the following equation.

$$\begin{aligned}&[i_{i\notin D}] \\ &= \{[Z_{near}]^{-1} - [Z_{near}]^{-1} VTU [Z_{near}]^{-1} \\ &+ [Z_{near}]^{-1}(VTU[Z_{near}]^{-1})^2 - \cdots\}_{p\in D,e}^{q\notin D}[v_{i\in D}, v_e]^t, \\ &v_e = V\end{aligned} \qquad (14)$$

## 5. Numerical results

To check the validity of the proposed method, we designed a sample antenna using the three different numerical algorithms described in sections 2, 3, and 4. The target area is the antenna implanted conducting plate, as shown in figure 1.

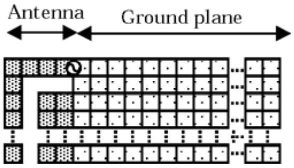


Fig. 1 Antenna implanted conducting plane

The whole target area consists of 20x40 segments.

The area of the ground plane was divided into seven groups and had the same number of segments and the same shape as the antenna area. In this case, the number of FMM localized groups was 8 and each group formed in the same shape had 100 segments. Because our MoM formulation used two-dimensional current unknowns, which were perpendicular to each other, the number of current unknown putting was 195. Only one group had 176 unknowns because a segment pair could not be made on the edge of the entire structure. We calculated the input impedance of the target area in figure 1 by using Gauss's elimination algorithm with voltage unknowns, an FMM iterative algorithm with current unknowns in the antenna area, and our proposed iterative algorithm with voltage unknowns. We used Matlab on a single PC that had an Intel Pentium 4 CPU, a 3.6-GHz clock and 2.0 GB of RAM [14]. The numerical results are listed in table 1. In this case proposed method kept good accuracy by using only first and second terms in equation 13 and 14.

Table 1 calculation time for input impedance of planer antenna

| algorithm | Elimination | Iteration | Proposed |
|---|---|---|---|
| Non-zero components | $38.0\times10^3$ | $38.0\times10^3$ | $25.1\times10^3$ |
| Calculation time [sec] | 0.02 | 0.04 | 0.02 |

The proposed algorithm successfully reduced the computational time required to design an antenna with a large ground plane. The overall design process is improved because the time to calculate the antenna design based on electromagnetic performance is reduced. We applied the proposed algorithm to develop an auto antenna design tool [12]. A sample antenna design is shown figure 2. Our tool took twelve hours to achieve this design.

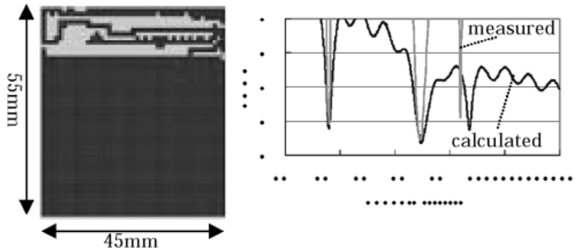


Fig. 2 Numerical results of antenna design

## 6. Conclusion

The MoM calculation of electromagnetic performance by using voltage unknowns drastically reduced the number of unknowns when designing antennas with a large ground plane. Therefore, this method effectively shortens the calculation time. However this method needs to create an inverse matrix from a huge impedance matrix, which includes unknowns in the antenna and in the ground areas. To reduce the calculation time of the inverse matrix, we proposed a novel FMM algorithm by using an approximate inverse matrix. This inverse matrix contained the usual decomposition of a FMM impedance matrix via MacLaurin expansion. Our algorithm reduced not only calculation time but also memory area. Numerical experiment for a simple conducting plate implanting antenna pattern verified the effectiveness of our algorithm. We used the algorithm to find a suitable antenna pattern in a rectangular conducting plate using a numerical trial-and-error sequence. The potential pattern satisfying good impedance matching was obtained in twelve hours. The time was much less than when a conventional algorithm, i.e. the FMM with current unknowns and a direct algorithm with an inverse matrix of the impedance matrix, is used.

In this paper, we used a simple FMM. Our objective to calculate electromagnetic performances can be limited to simple rectangular regions. A multi level FMM (MLFMM) can be applied in this situation. We will research MLFMM as well as further reduce calculation time.

## Appendix

We set the next matrix equation of the MoM.

$$\begin{bmatrix} Z_{BB} & z_{Bj} \\ z_{jB} & z_{jj} \end{bmatrix} \begin{bmatrix} i_B \\ i_j \end{bmatrix} = [0,\cdots 0, v_e, 0, \cdots 0]^t, \quad \text{(A1)}$$

$$v_e = V, e \in B, e \neq j.$$

When segment j is removed, we obtain $i_k$ by solving the following equation.

$$[Z_{BB}][i_B] = [0,\cdots 0, v_e, 0, \cdots 0]^t, \quad \text{(A2)}$$
$$v_e = V, k \in B.$$

Introducing the inverse matrix, $i_k$ is analytically derived by solving the following equation.

$$i_k = y_{ke} v_e = y_{ke} V, [Y] = [Z_{BB}]^{-1} \quad \text{(A3)}$$

Using Morris' law [5], the inverse matrix of the Z-matrix in equation A1 is created by both the Z-matrix in equation 2 and Y in equation 3 as indicated by the next expressions.

$$\begin{bmatrix} Z_{BB} & z_{Bj} \\ z_{jB} & z_{jj} \end{bmatrix}^{-1} = \begin{bmatrix} Y + Yz_{Bj} z_{jB} Y\delta & -Yz_{Bj}\delta \\ -\delta z_{jB} Y & \delta \end{bmatrix}, \quad \text{(A4)}$$

$$\delta = (z_{jj} - z_{jB} Y z_{Bj})^{-1}$$

On the basis of the procedure in section 2, $i_k$ is derived by equations A1 and A4 in the same form of equation 3 as follows.

$$0 = -\delta z_{jB} y_{Be} v_e + \delta v_j$$
$$i_k = (y_{ke} + y_{kB} z_{Bj} z_{jB} y_{Be} \delta) v_e - y_{kB} z_{Bj} \delta v_j = y_{ke} v_e \quad \text{(A5)}$$

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