3-III A5 DIFFRACTION BY A MOVING SEMI-INFINITE SCREEN IN AN ANISOTROPIC PLASMA

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Introduction

The scattering of a plane electromagnetic wave by a perfectly conducting semi-infinite screen enbedded in a homogeneous anisotropic medium was investigated previously.[1] This paper presents the theoretical analysis of the diffraction by a moving semi-infinite screen in such a medium.

Analysis

The geometry of this problem is shown in Fig.1. Two rectangular systems of Cartesian co-ordinates, S(x,y,z) and S'(x',y',z'), are introduced. The systems S and S' are stationary with respect to the medium and the screen respectively.

The entire space is filled with a homogeneous plasma and a uniform magnetic field B_0 is impressed in the z direction throughout the plasma. It is assumed that the plasma is a very simple model whose dielectric constant is tensor as follows:

$$\tilde{\varepsilon} = \begin{bmatrix} \varepsilon_1 & i\varepsilon_2 & 0 \\ -i\varepsilon_2 & \varepsilon_1 & 0 \\ 0 & 0 & \varepsilon_3 \end{bmatrix}$$

$$\varepsilon_1 = 1 - \frac{\omega_p^2}{\omega^2}\left[1 - \frac{\omega_c^2}{\omega^2}\right]^{-1}$$

$$\varepsilon_2 = \frac{\omega_p^2}{\omega^2}\left[\frac{\omega}{\omega_c} - \frac{\omega_c}{\omega}\right]^{-1}$$

$$\varepsilon_3 = 1 - \frac{\omega_p^2}{\omega^2} \qquad (1)$$

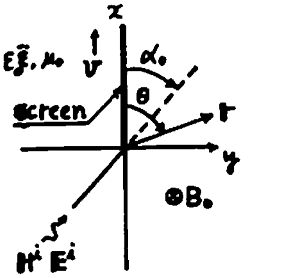

Fig.1. A moving screen in an anisotropic plasma

ω_p is the plasma frequency and ω_c is the gyromagnetic frequency. ω is the frequency of the harmonic time dependence exp($-i\omega t$) assumed for all the field components.

A perfectly conducting semi-infinite screen of zero thickness is assumed to be on the plane of z=0 and moving in the x direction with a constant velocity v.

The incident plane wave, whose direction of propagation is in a plane perpendicular to and whose magnetic vector is parallel to, the edge of the semi-infinite screen as follows:

In the S system,

$$H_z^i = H_0 \exp\{ikr\cos(\theta-\alpha_0) - i\omega t\} \qquad (2)$$

and in the S' system,

$$H_z^{i'} = H_0' \exp\{ik'r'\cos(\theta'-\alpha_0') - i\omega' t'\} \qquad (3)$$

The relations between the quantities of both systems are obtained by the covariance of Maxwell's equations and the phase invariance of plane waves.

The constitutive relations in the S' system are

$$\begin{aligned} \mathbf{D}' &= \tilde{\varepsilon}'\cdot\mathbf{E}' + \tilde{\xi}'\cdot\mathbf{H}' \\ \mathbf{B}' &= \tilde{\eta}'\cdot\mathbf{E}' + \tilde{\mu}'\cdot\mathbf{H}' \end{aligned} \qquad (4)$$

These parameters of the constitutive relations are determined by applying the covariance of Maxwell's equations.

Substituting Eq.(4) into the Maxwell's equations in the S' system, other five compornents of incident waves are deduced.

$$E_x^{i'} = -A'(\alpha_0')\,H_z^{i'}$$
$$E_y^{i'} = B'(\alpha_0')\,H_z^{i'}$$
$$E_z^{i'} = M'(\alpha_0')\,H_z^{i'}$$

$$H_x^{i'} = \frac{k' \sin\alpha_0'}{\omega' \mu_{11}'} M'(\alpha_0') H_z^{i'} \tag{5}$$
$$H_y^{i'} = -\left(\frac{n_{23}'}{\mu_{22}'} + \frac{k' \cos\alpha'}{\omega' \mu_{22}'}\right) M'(\alpha_0') H_z^{i'}$$

where $A'(\alpha_0')$, $B'(\alpha_0')$ and $M'(\alpha_0')$ are complicate factors containing the elements of the constitutive parameters, μ_{11}', n_{23}' etc.

Six components of the scattering waves are represented by means of the angular spectra of plane waves in both regions, y>0 and y<0 respectively.[2] For H_z'

$$H_z^{st'} = \int_{C_t'} P^{t'} e^{ik'r'\cos(\theta'-\alpha')} d\alpha'$$
$$H_z^{sr'} = \int_{C_r'} P^{r'} e^{ik'r'\cos(\theta'+\alpha')} d\alpha' \tag{6}$$

After changing the valiables from α' to $\lambda' = k'\cos\alpha'$ we obtain the integral equations on the angular spectra from boundary conditions.

$$x'>0, \quad \int_{-\infty}^{\infty} \frac{A'(\lambda')}{\sqrt{k'^2-\lambda'^2}} P^{t'}(\lambda') e^{ix'\lambda'} d\lambda' = -H_0' A'(\lambda_0') e^{ix'\lambda_0'}$$
$$\int_{-\infty}^{\infty} M'(\lambda') \frac{P^{t'}(\lambda')}{\sqrt{k'^2-\lambda'^2}} e^{ix'\lambda'} d\lambda' = -H_0' M'(\lambda_0') e^{ix'\lambda_0'}$$
$$\int_{-\infty}^{\infty} \frac{A^{r'}(\lambda')}{\sqrt{k'^2-\lambda'^2}} P^{r'}(\lambda') e^{ix'\lambda'} d\lambda' = -H_0' A'(\lambda_0') e^{ix'\lambda_0'}$$
$$\int_{-\infty}^{\infty} \frac{M^{r'}(\lambda')}{\sqrt{k'^2-\lambda'^2}} P^{r'}(\lambda') e^{ix'\lambda'} d\lambda' = -M'(\lambda_0') H_0' e^{ix'\lambda_0'}$$
$$x'<0 \quad \int_{-\infty}^{\infty} \left\{ \frac{P^{r'}(\lambda')}{\sqrt{k'^2-\lambda'^2}} - \frac{P^{t'}(\lambda')}{\sqrt{k'^2-\lambda'^2}} \right\} e^{ix'\lambda'} d\lambda' = 0$$
$$\int_{-\infty}^{\infty} \left\{ -M^{r'} P^{r'}(\lambda') - M'(\lambda') P^{t'}(\lambda') \right\} e^{ix'\lambda'} d\lambda' = 0$$
$$\int_{-\infty}^{\infty} \left\{ \frac{A^{r'}(\lambda')}{\sqrt{k'^2-\lambda'^2}} P^{r'}(\lambda') - \frac{A'(\lambda')}{\sqrt{k'^2-\lambda'^2}} P^{t'}(\lambda') \right\} e^{ix'\lambda'} d\lambda' = 0$$
$$\int_{-\infty}^{\infty} \left\{ \frac{M^{r'}(\lambda')}{\sqrt{k'^2-\lambda'^2}} P^{r'}(\lambda') - \frac{M'(\lambda')}{\sqrt{k'^2-\lambda'^2}} P^{t'}(\lambda') \right\} e^{ix'\lambda'} d\lambda' = 0 \tag{7}$$

$P^{t'}(\lambda')$ and $P^{r'}(\lambda')$ are determined by solving these integral equations exactly. The solutions are

$$P^{t'}(\lambda') = -\frac{H_0'}{2\pi i} \cdot \frac{\sqrt{k'+\lambda_0'}}{\lambda'-\lambda_0'} \sqrt{k'-\lambda'}\, e^{-X_+(\lambda')+X_+(\lambda_0')}$$
$$P^{r'}(\lambda') = \frac{A'(\lambda')}{A^{r'}(\lambda')} P^{t'}(\lambda') \tag{8}$$

where $\exp(-X_+)$ is the function which is regular and nonzero in the upper half plane of λ'. All the scattering fields in the S' system are obtained from Eq. (6) substituted (8). Finally the fields in the S' system are transformed inversely into the S system.

The far fields are calculated by the method of the steepest descent.

The asymptotic representations of the far fields become simple forms when θ' is near α_0'. For example

$$H_z^{st} \sim -\frac{H_0}{2}\left[1 + P(w) + Q(w) + i\{Q(w) - P(w)\}\right] e^{ikr\cos(\theta-\alpha_0)}$$
$$w = \sqrt{\frac{1}{\pi}\left[k^2 - \frac{\omega^2}{c^2} + \gamma^2\left(\beta k \cos\alpha_0 - \frac{\omega}{c}\right)^2\right]^{\frac{1}{2}} \times \{\gamma^2(x-vt)^2 + y^2\}^{\frac{1}{2}} \cos(\eta - \Gamma_0) \times \tan(\Gamma_0 - \eta)}$$
$$\cos\eta = \gamma(x-vt)/\{\gamma(x-vt)^2 + y^2\}^{\frac{1}{2}}$$
$$\cos\Gamma_0 = (\cos\alpha_0 - \beta)/(1 - \beta\cos\alpha_0)$$
$$\gamma = 1/\sqrt{1-\beta^2}, \quad \beta = v/c \tag{9}$$

The results of numerical calculations of $H_z^t = H_z^i + H_z^{st}$ are shown in Fig.2.

Conclusion

It has been shown that the scattering fields when H polarized plane wave is incident, can be solved exactly and separated into six components.

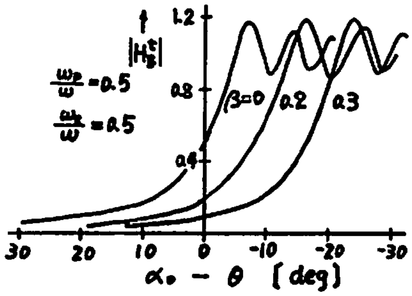

Fig.2. Absolute values of the far field H_z^t when two systems are coincident (t=0)

References

1. S.R.Seshadri and A.K.Rajagopal IEEE Trans. on AP-11, 497 (1963)

2. P.C.Clemmow, The Plane Wave Spectrum Representation of Electromagnetic Fields, Pergamon Press (1966)