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SEMI-EMPIRICAL RELATIONS FOR THE PREDICTION OF RAIN DEPOLARIZATION STATISTICS: THEIR THEORETICAL AND EXPERIMENTAL BASIS

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Introduction: With the introduction of satellite and terrestrial communications systems employing orthogonal-polarization frequency-reuse to increase capacity, it has become necessary to determine the statistics of depolarization resulting from hydrometeors (e.g., rain, sleet, snow, ice crystals) and refractive layers along the propagation path. Although several extensive measurement programs are being conducted to obtain long-term statistics, the data are applicable for a limited number of frequencies, elevation angles, polarizations, and climatic conditions. In order to predict depolarization statistics for conditions other than those used experimentally, a general model is required that fits the experimental data and utilizes readily obtained statistics of meteorological or other propagation quantities.

In the case of depolarization due to rain, a general theory has been developed by Oguchi [1] that can be used as a basis for simple, approximate prediction equations employing path attenuation or point rainrate statistics. The theoretical basis for the approximate relation between the cross-polar discrimination (XPD) and co-polar attenuation (CPA)

$$\mathrm{XPD} = U_1 - V_1 \log(\mathrm{CPA}) \qquad (1)$$

has recently been analyzed by Nowland et al. [2] and expressions given for U_1 and V_1. Another relation in terms of the rainrate R and effective path lengh ℓ

$$\mathrm{XPD} = U_2 - V_2 \log R - W_2 \log \ell \qquad (2)$$

results from the same approximations [2,3]. In the utilization of these equations for predictions, the coefficients U_1, V_1, etc., can be obtained from direct tabulations [3], from their equations and tabulations of constituent parameters [2,3], from graphs [3], or from semi-empirical equations. Although slightly less accurate than the other techniques, the use of semi-empirical equations is adequate for most design purposes. Consequently, such equations for the coefficients in (1) based on the numerical data in [2,3] have recently been adopted at the Final Meeting of C.C.I.R. Study Group 5 [4,5]. This paper summarizes the theoretical and experimental basis of these equations, and presents slightly more general forms, including a semi-empirical form of (1).

Theoretical Background: As shown in [2,3], Oguchi's general theory [1] can be approximated by the relations

$$\mathrm{XPD} = -20 \log(m\ell c R^{d} \cos^2\varepsilon \ \sin 2|\phi-\tau|/2) \ (\mathrm{dB}), \qquad \mathrm{CPA} = aR^{b}\ell \ (\mathrm{dB}) \qquad (3)$$

where ε is the path elevation angle, τ is the polarization tilt angle of the incident wave relative to the horizontal, ϕ is the effective canting angle of the raindrops, and m is a "canting-angle distribution factor". The latter relation is the familiar approximate form for the path attenuation [6], but here generalized for a medium of non-spherical raindrops with

$$a \simeq [(a_x + a_y) + m(a_x - a_y)\cos^2\varepsilon \ \cos 2(\phi-\tau)]/2$$

$$b \simeq [(a_x b_x + a_y b_y) + m(a_x b_x - a_y b_y)\cos^2\varepsilon \cos 2(\phi-\tau)]/2a \tag{4}$$

It is based on the constituent forms $A_x \simeq a_x R^{b_x}$ and $A_y = a_y R^{b_y}$, where A_x and A_y are the specific attenuations of waves polarized along the principal planes of a medium of "equi-oriented" raindrops [7]. Eqn. (3) for XPD is a new power-law form which contains the constituent form $(\Delta\alpha^2+\Delta\beta^2)^{\frac{1}{2}} = cR^d$, where $\Delta\alpha$ is the differential attenuation and $\Delta\beta$ the differential phase shift per unit distance between these waves. As discussed in [2], m can be written as $m = \exp(-2\sigma^2)$, where σ (in radians) is the effective standard deviation for the raindrop canting angle distribution.

Eqns. (3) can be combined to yield

$$XPD = U_3 - V_3 \log(CPA) - W_3 \log R \tag{5}$$

$$U_3 = -20 \log(m \sin 2|\phi-\tau|) - 20 \log(c \cos^2\varepsilon/2a), \quad V_3 = 20, \quad W_3 = 20(d-b)$$

Although (5) could be used for prediction purposes, it is complicated by the additional term involving R. However, this term can be usefully combined with that involving CPA to give eqn. (1) since it is small in comparison and since CPA and R are correlated. The additional approximation required, $\ell \simeq uR^v$, results in

$$U_1 \simeq -20 \log(cu \cos^2\varepsilon \; m \sin 2|\phi-\tau|/2) + 20[(d+v)/(b+v)] \log(au)$$

$$V_1 \simeq 20(d+v)/(b+v) \tag{6}$$

Furthermore, $d \approx b$ for a range of frequencies centered about 12 GHz (see Fig. 1) so that u and v are eliminated and eqn. (1) becomes

$$XPD = 0.0053\sigma^2 - 20 \log(\sin 2|\phi-\tau|) - 20 \log(c/2a) - 40 \log(\cos\varepsilon) - 20 \log(CPA) \tag{7}$$

Here, the factor $-20 \log(m) = 0.0053\sigma^2$ (σ in degrees) has also been separated from the improvement, $I = -20 \log(\sin 2|\phi-\tau|)$, of linear polarization over circular polarization. In the development of semi-empirical prediction equations, the coefficient a in (7) can be approximated with little error by the coefficient, $a_c = (a_x+a_y)/2$, applicable to circular polarization.

Semi-Empirical Prediction Equations: Eqns. (6) and (7) suggest the possibility of semi-empirical prediction equations of the form

$$XPD = 0.0053\sigma^2 - 20 \log(\sin 2|\phi-\tau|) + A + B \log(f) - C \log(\cos\varepsilon) - V_1 \log(CPA) \tag{8}$$

with different values for the constants A, B, C, and V_1 in each frequency band. The functional dependence on frequency f follows from (7) and the fact that a_c and c can be approximated by power-laws in f over a fairly wide range of frequencies [6,3]. Regressions of XPD on log(CPA) have been carried out using the "exact" equations and Pruppacher-Pitter scattering amplitudes of Oguchi [1], the Laws and Parsons dropsize distribution [6], and an empirical equation for the effective path length ℓ on an earth-space path [2]. With a slight rounding of coefficients, these results can be summarized by

$$XPD = 0.0053\sigma^2 - 20 \log(\sin 2|\phi-\tau|) + 30 \log(f) - 40 \log(\cos\varepsilon) - V_1 \log(CPA)$$

$$V_1 = 20, \; 8 \le f \le 15 \text{ GHz}; \qquad V_1 = 23, \; 15 < f \le 35 \text{ GHz} \tag{9}$$

A comparison of calculations based on (9) and Oguchi's "exact" equations [1] shows it to be accurate to within about 1.5 dB. Since the effect of the particular model for ℓ [2] is negligible in the lower frequency range, here the equation can be used for both earth-space and terrestrial paths.

The use of (9) for predictions is preferable when attenuation statistics are available because of its minimal sensitivity to effective path length and dropsize distribution [2,3,8]. However, for the 4-6 GHz band where accurate attenuation statistics are normally not available, or for higher frequencies, eqn. (2) can be used with $W_2 = 20$ and

$$U_2 = 0.0053\sigma^2 - 20\log(\sin 2|\phi-\tau|) + 90 - 20\log(f) - 40\log(\cos\varepsilon), 4 \leq f \leq 35 \text{ GHz}$$

$$V_2 = 25, \quad 4 \leq f \leq 15 \text{ GHz}; \quad V_2 = 27 - 0.13 f, \quad 15 < f \leq 35 \text{ GHz} \qquad (10)$$

Again, comparison of calculations based on (2) and (10) and Oguchi's "exact" equations shows this semi-empirical form to be accurate to within 2.0 dB.

Effective Values of the Canting-Angle Parameters: The main factor affecting the accuracy of the prediction equations is the limited information available on the effective values of the parameters ϕ and σ of the canting-angle distribution. Consequently, the choice of values for use in prediction must be on the conservative side at the present time. A comparison of the theory and the results of circular polarization measurements [2] shows $\sigma=0°$ to be a reasonably conservative choice for the effective standard deviation. This is the value implicit in the equations adopted by the C.C.I.R. [4,5].

It is more difficult to obtain a suitably conservative value for ϕ because of the great sensitivity of XPD to this parameter in systems using "near-horizontal" and "near-vertical" polarizations. For use in planning dual vertically and horizontally polarized terrestrial links, a value of I=12 dB for the improvement of linear over circular polarization was adopted by the C.C.I.R. [5], primarily on the basis of measurements carried out in Japan [9]. This corresponds to $|\phi|=7.3°$. For the planning of earth-space systems in which τ is geographically dependent, $|\phi|=0°$ was adopted along with a minimum value of I=9.3 dB corresponding to $\tau=10°$ [4]. A choice more consistent with that adopted for terrestrial links would be to set $|\phi-\tau|=|\tau|+7.3°\cos\varepsilon$, where the factor $\cos\varepsilon$ accounts approximately for the dependence of ϕ on elevation angle [2].

With the analysis of data from the increasing number of long-term cross-polarization experiments being carried out, more information on the meteorological factors governing the values of ϕ and σ will become available. Particularly important are experiments utilizing more than one polarization state that will allow values for ϕ and σ to be separated. Results of switched linear and circular polarization measurements at 11.6 GHz during three events on an 18 km terrestrial path [10,11] are replotted on an XPD versus log(CPA) scale in Fig. 2. Also shown for comparison are curves based on (9) with I=12 dB (i.e., $|\phi|=7.3°$), and both $\sigma=0°$ and $\sigma=25°$. Cumulative distributions of XPD and CPA for both polarizations will enable more accurate values of $|\phi|$ and σ to be obtained for various percentages of time.

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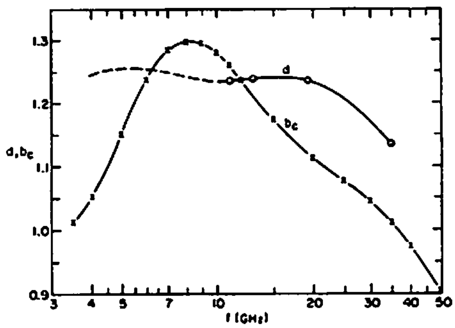

Fig. 1. Frequency dependence of the regression coefficients d and b_c in the R=1-50 mm/h range. Laws and Parsons dropsize distribution, 20°C rain temperature. Values of b_c approximated by values for spherical drops [6]. --- curve for Pruppacher-Pitter raindrops extrapolated on the basis of curve for spheroidal drops [3].

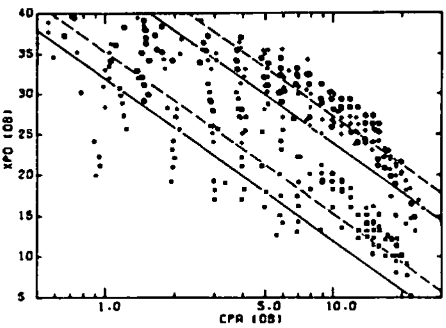

Fig. 2. Comparison of measurements of Dilworth and Evans [10,11] with curves based on eqn. (9).
x circular polarization, o horizontal polarization, + vertical polarization, —— (σ=0°), --- (σ=25°); $|\phi|$=7.3° for linear polarizations.

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