A COMPACT FORM OF UTD FOR HYPERBOLOIDAL AND ELLIPSOIDAL ANTENNAS

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Abstract- Radiation patterns of axially symmetric reflectors are computed using the Uniform Theory of Diffraction (UTD). Using UTD, presented formulation gives a compact, short and easy way for the results of scattered fields of the mentioned reflectors.

INTRODUCTION

Because of their wide usage in single and dual reflector antenna systems, analysis and computation of far field radiation patterns of hyperboloidal and ellipsoidal reflector antennas are very important. The uniform Theory of Diffraction is mostly applied to find the radiation patterns of reflector antennas [1]-[3]. Using the geometry of hyperboloidal and ellipsoidal reflector antennas, UTD formulation can be put in a compact and easy form in order to reduce the computation time.

ANALYSIS

Geometry of considered hyperboloidal and ellipsoidal reflector antennas are shown in Fig.1 and 2. The chosen feed pattern at O' can be written in a general form as below

$$\bar{E}^i(r,\psi,\zeta) = [D_\psi(\psi)\sin\zeta\hat{\psi} + D_\zeta(\psi)\cos\zeta\hat{\zeta}]\,\frac{e^{-jkr}}{r} \qquad (1)$$

where $D_\psi(\psi)$, $D_\zeta(\psi)$ are the field patterns of the feeder in $\zeta=90^o$ and $\zeta=0^o$ planes respectively. At the far zone the incident field component in terms of $(R,\Theta,\emptyset)$ are given by

$$\begin{bmatrix} E_\Theta \\ E_\emptyset \end{bmatrix}^i = \begin{bmatrix} D_\psi(\pi-\Theta)\sin\emptyset \\ -D_\zeta(\pi-\Theta)\cos\emptyset \end{bmatrix} e^{jkF\cos\Theta}\,\frac{e^{-jkR}}{R} \qquad (2)$$

The reflected field components for the chosen feed can be found from

$$\begin{bmatrix} E_\Theta \\ E_\emptyset \end{bmatrix}^r = \left|\frac{e^2-1}{q^2}\right| \begin{bmatrix} \pm D_\psi(\psi)\sin\phi \\ D_\zeta(\psi)\cos\phi \end{bmatrix} e^{-jk\frac{F}{e}}\ \frac{e^{-jkR}}{R}\ ;\ |e| \gtrless 1 \qquad (3)$$

where

$$q^2 = e^2+1 \pm 2e\cos\Theta \qquad (4)$$

Here e denotes the reflector eccentricity

$$e = \sin\frac{\psi_o+\Theta_y}{2} \Big/ \sin\frac{\Theta_y-\psi_o}{2} \qquad (5)$$

and

$$\theta_y = \begin{cases} \theta_o & ; |e| > 1 \\ 2\pi - \theta_o ; & |e| < 1 \end{cases} \tag{6}$$

The diffracted field from the edge of the reflector at any point can be found using the formulation of the uniform theory of diffraction (UTD) as below [1].

$$\begin{bmatrix} E_{\theta\pm} \\ E_{\phi\pm} \end{bmatrix}^d_{Q_\pm} = \begin{bmatrix} D_h^{\pm} & 0 \\ 0 & D_s^{\pm} \end{bmatrix} \begin{bmatrix} E_{\theta\pm} \\ E_{\phi\pm} \end{bmatrix}^i_{Q_\pm} \sqrt{\frac{\rho c_\pm}{s_\pm(s_\pm + \rho c_\pm)}} \, e^{-jks_\pm} \tag{7}$$

where

$$\sqrt{\frac{\rho c_\pm}{s_\pm(s_\pm + c_\pm)}} = \frac{\sqrt{\rho c_\pm}}{s_\pm} \tag{8}$$

$$\rho c_\pm = \pm \frac{a}{\sin\theta} \tag{9}$$

$$D^{\pm}_{\substack{h \\ s}} = \frac{-e^{-j\pi/4}}{2\sqrt{2\pi k \sin\beta_o}} \left[\frac{F(|Z_i^{\pm} e^{j\pi/4}|)}{\cos(\frac{\gamma_d^{\pm} - \gamma_i^{\pm}}{2})} \pm \frac{F(Z_r^{\pm} e^{j\pi/4})}{\cos(\frac{\gamma_d^{\pm} + \gamma_i^{\pm}}{2})} \right] \tag{10}$$

$$Z^{\pm}_{\substack{i \\ r}} = \sqrt{2kL^{\pm}_{\substack{i \\ r}}} \left| \cos \frac{\gamma_d^{\pm} \mp \gamma_i^{\pm}}{2} \right| \tag{11}$$

Using the geometries given in Fig.1 and 2.

$$\cos(\frac{\gamma_d^{\pm} + \gamma_i^{\pm}}{2}) = I.\sin(\frac{\theta_y \mp \theta}{2}) \tag{12}$$

and

$$\cos(\frac{\gamma_d^{\pm} - \gamma_i^{\pm}}{2}) = \cos(\frac{\theta \pm \psi_o}{2}) \tag{13}$$

From Fig.1 and 2. $r_o + s_\pm$ can be expressed in different forms such that ($I = \pm 1$ for $|e| \gtrless 1$)

$$\begin{aligned} r_o + s_+ &= R - F\cos\theta + 2r_o\cos^2(\frac{\theta + \psi_o}{2}) \\ &= R + \frac{F}{e} + I.2\rho_o\sin^2(\frac{\theta - \theta_y}{2}) \end{aligned} \tag{14}$$

and

$$\begin{aligned} r_o + s_- &= R - F\cos\theta + 2r_o\cos^2(\frac{\theta - \psi_o}{2}) \\ &= R + \frac{F}{e} + I.2\rho_o\sin^2(\frac{\theta + \theta_y}{2}) \end{aligned} \tag{15}$$

Considering Eqns.(8)-(15) in Eqn.(7), Using the properties of F(x) given in [4] and after long manipulations, one can obtain the diffracted field from $Q_{\pm}$ as below

$$\begin{bmatrix} E_\Theta \\ E_\phi \end{bmatrix}^d_{Q_\pm} = \mathrm{Sgn}(z_i^\pm)Q(|z_i^\pm|)\begin{bmatrix} E_\Theta \\ E_\phi \end{bmatrix}^i_{Q_\pm}\sqrt{\frac{\sin\psi_0}{\sin\Theta}} + \mathrm{Sgn}(z_r^\pm)Q(|z_r^\pm|)\begin{bmatrix} E_\Theta \\ E_\phi \end{bmatrix}^r_{Q_\pm} \cdot$$

$$\sqrt{\left|\frac{\sin\Theta_0}{\sin\Theta}\right|}\cdot A(\Theta)\,\frac{e^{-jkR}}{R}\;;\; 0<\Theta<\pi \quad (16)$$

where

$$A(\Theta) = \begin{cases} e^{j\frac{\pi}{2}U[I.(\Theta\text{-}\pi)]} & \text{for } Q_+ \\ e^{j\frac{\pi}{2}U[I.(\pi-\Theta)]} & \text{for } Q_- \end{cases} \quad (17)$$

$$Q(x) = \frac{e^{j\pi/4}}{\sqrt{\pi}}\int_x^\infty e^{-jt^2}\,dt \quad (18)$$

and U(x)=1 for $x\geq 0$ and 0 for $x<0$. In the analysis of ellipsoidal reflectors complex conjugate of Q(x) is seen in Eqn.(16) as in [4].

Total field at any point $P(R,\Theta,\phi)$ can be written as

$$\bar{E}(P) = \bar{E}^i(P) + \bar{E}^r(P) + \bar{E}^d_{Q_+}(P) + \bar{E}^d_{Q_-}(P) \quad (19)$$

Also, the above formulations can be used for spheroidal reflector antennas.

RESULTS

Total electric fields, obtained using Eqn.(19) are shown in Fig.1 and 2. for hyperboloidal and ellipsoidal antennas. In these fiqures $D=15\lambda$, n=4, $\psi_0=20^\circ$, $\Theta_0=60^\circ$. The region defined by $0\leq\Theta\leq\Theta_0$ $(\pi-\psi_0\leq\Theta\leq\pi)$ is the reflection (shadow) region, diffraction from $Q_\pm$ causes the oscillations on the reflect (direct) fields. In the neighborhood of axial points, the above formulations can't give the correct field as known from UTD [1] , [2]. As it is seen that the formulation given above simple and in a compact form for the calculation of radiation fields of mentioned reflector antennas.

REFERENCES

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FIGURES

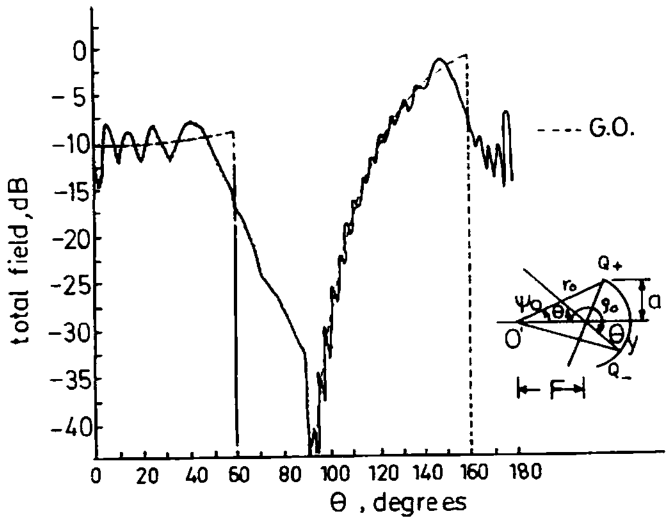

Fig.1. E-plane radiation pattern of an ellipsoidal antenna.

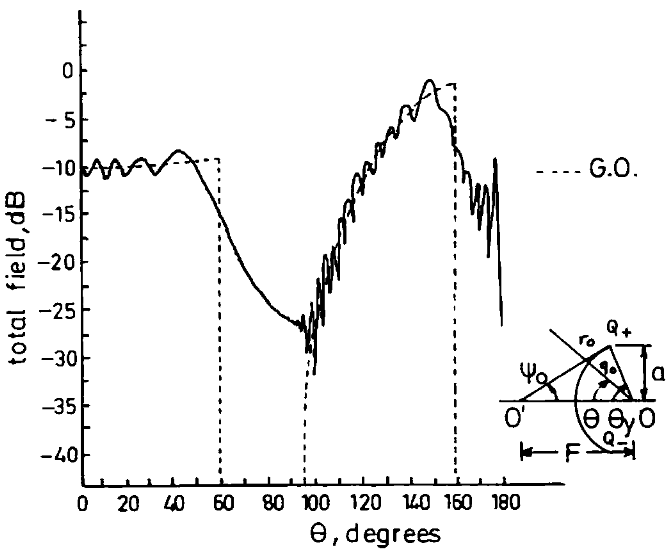

Fig.2. H-plane radiation pattern of a hyperboloidal antenna.