3-III A2 ON THE SCATTERING OF WAVES FROM TWO INCLINED HALF PLANES

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There exist numerous investigations on the scattering of electromagnetic or acoustic waves from a slit on a conducting plane, or two coplanar half-planes. The scattering or radiation of waves from two parallel half-planes, or the open-ended parallel-plate waveguides have also been studied by various workers. However, there are relatively few studies on the scattering of waves by two inclined half-planes, or an aperture on a conducting wedge. Only recently has this problem been studied by Skal'skaya[1], Millar[2] and Tan[3]. Skal'skaya obtained the leading terms of the asymptotic expression for the current on the inclined half-planes induced by a normally incident plane waves. The problem has also been formulated in some general terms by Millar. No solution is given there. Tan studied this problem theoretically, using the Kirchhoff's approximation, the variation technique and Kontorovich-Lebedev transform, and experimentally.

The viewpoint adopted in the present work is similar to those used by Keller,[4] Karp and Russek,[5] Seshadri,[6] and other works on the scattering of waves from two coplanar half-planes.

The geometry of the problem is shown in Fig. 1. The coordinate system (x,y, z) is so chosen that the conducting half-planes S_1, S_2 are perpendicular to the x-y plane. The incident electric field is polarized in the z-direction.

The nonvanishing components of the fields are E_z, H_x and H_y. The problem is completely specified by the wave equation for E_z, the radiation condition, the edge condition and the boundary conditions that $E_z=0$ on S_1 and S_2. Two coupled integral equations are obtained by expressing E_z in terms of induced current densities K_1, K_2 on S_1 and S_2, and then imposing the boundary conditions.

According to Keller's geometrical theory of diffraction,[4] the principal contribution to high frequency diffraction comes from the regions near the knife edges and the discontinuities of the curvatures of the scatterers. For the problem under consideration, the crucial regions are those near the edges of S_1 and S_2. Furthermore, at high frequencies, $a \gg \lambda$, therefore the kernels of the integral equations may be approximated asymptotically. When this is done, the resulting integral equations can be solved by Weiner-Hopf technique. Knowing K_1 and K_2, the expressions for electric fields can be obtained immediately:

$$E_z(\vec{r})=E_z^i(\vec{r})+\sum_{j=1}^{2}[E_j^{(o)}(\vec{r})+\frac{1}{\sqrt{ka}}E_j^{(1)}(\vec{r})$$

$$+\frac{1}{ka}E_j^{(2)}(\vec{r})] + O((ka)^{-3/2})$$

where

$$E_z^i=e^{ikr\cos(\theta-\theta_i)}$$

and in the transmitted region

$$E_1^{(0)}(\vec{r})= \frac{e^{i3\pi/4}e^{ik_-a'}}{\sqrt{\pi}}[e^{ikr_1\cos(\theta_1-\theta_i+\Omega)}$$

$$F(-\sqrt{2kr_1}\sin\frac{\theta_1-\theta_i+\Omega}{2})$$

$$+e^{ikr_1\cos(\theta_1+\theta_i-\Omega)}F(-\sqrt{2kr_1}$$

$$\sin\frac{\theta_1+\theta_i-\Omega}{2})]$$

$$E_1^{(1)}(\vec{r})= -\frac{\alpha}{\sqrt{\pi}}e^{-ik_+a'+i\pi/4}$$

$$\frac{\cos\Omega/2\ \sin(\theta_i+\Omega)/2}{\cos(\theta_i+\Omega)-\cos\Omega}$$

$$[e^{ikr_1\cos(\theta_1-\Omega)}F(-\sqrt{2kr_1}\sin\frac{\theta_1-\Omega}{2})+e^{ikr_1\cos(\theta_1+\Omega)}F(-\sqrt{2kr_1}\sin\frac{\theta_1+\Omega}{2})]$$

$$E_1^{(2)}(\vec{r}) = -\frac{\alpha^2}{2\sqrt{\pi}}e^{ik_-a'+i3\pi/4}\frac{\cos^3\Omega/2\,\cos(\theta_1-\Omega)/2}{\cos\Omega[\cos(\theta_1-\Omega)+\cos\Omega]}[e^{ikr_1\cos(\theta_1-\Omega)}F(-\sqrt{2kr_1}\sin\frac{\theta_1-\Omega}{2})+e^{ikr_1\cos(\theta_1+\Omega)}F(-\sqrt{2kr_1}\sin\frac{\theta_1+\Omega}{2})]$$

In the equations given above, r_1 and θ_1 are defined in Fig. 1, and

$$F(x)=\int_{-\infty}^{x}e^{ix^2}dx;\quad \alpha=\frac{e^{i(2ka-\pi/4)}}{\sqrt{\pi}}$$

$$k_{\pm}=k\cos(\theta_1\pm\Omega);\, k_1 = k\cos\Omega$$

Similar expressions for $E_2^{(o)}$, $E_2^{(1)}$, and $E_2^{(2)}$ are also obtained. The interpretation of these terms are simple. E_2^i is the incident wave. $E_1^{(o)}$ and $E_2^{(o)}$ are the singly diffracted waves due to S_1 and S_2. $E_1^{(1)}$ is the wave first scattered by S_2 and then scattered again by S_1. The other terms can be interpreted in the same fashion. In the regions $\Omega < \theta < 2\Omega$, and $\pi - 2\Omega > \theta > \pi - \Omega$, the multiply scattered waves contain both the diffraction due to the edges and the reflection from the conducting screens.

Fig. 2 shows the theoretical results for fields along the y-axis ($a=3\lambda$, $\Omega = \pi/4$, normal incidence), together with the corresponding experimental data by Tan. It is seen that the theoretical results agrees well with the experimental data, except for large y. There are small ripples in the experimental data, while the theoretical curve is rather smooth. The effects of the angle of inclination and the angle of incidence on the fields patterns are also studied.

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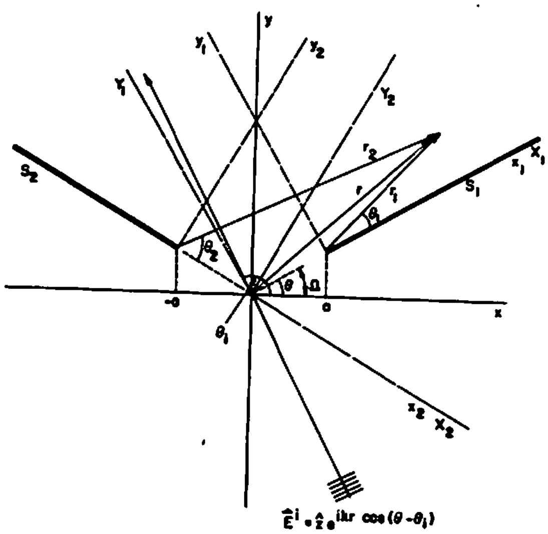

FIGURE 1. GEOMETRY OF THE PROBLEM.

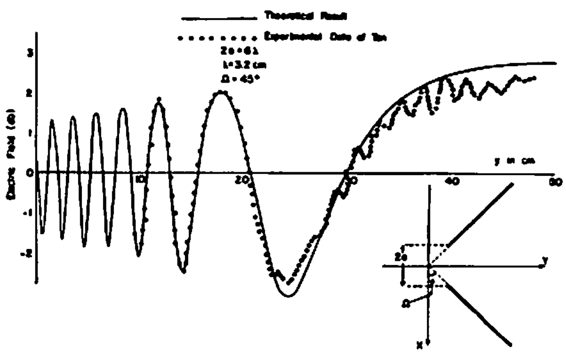

FIGURE 2. COMPARISON OF THEORETICAL AND EXPERIMENTAL RESULTS