APPLICATION OF A MULTIGRID OPTIMIZATION METHOD TO ELECTROMAGNETIC INVERSE SCATTERING PROBLEM

Mitsuru TANAKA, Ryouji KUROGI, Hiroyuki YOSHIDA, and Atsushi KUSUNOKI
Department of Electrical and Electronic Engineering, Oita University
700 Dannoharu, Oita-shi, Oita 870-1192, Japan
E-mail: mtanaka@cc.oita-u.ac.jp

1. Introduction

An electromagnetic inverse scattering of unknown objects continues to be of great interest and practical importance due to a variety of applications in biomedical imaging, detection of underground objects, and nondestructive testing of materials [1]-[12]. As is well known [2], the electromagnetic inverse scattering problem requires us to solve a nonlinear integral equation for a contrast function, which is related to the relative permittivity of the object. In a frequency-domain analysis, the method of moments with pulse-basis functions and point matching [13] has been employed to discretize the integral equation. Then the electromagnetic inverse scattering problem is reduced to solving a nonlinear matrix equation for the expansion coefficients of the contrast function [8]. The matrix equation can be solved in the least-squares sense using one of the descent methods [4]-[9]. However, it takes much computational cost to obtain a convergent solution of the matrix equation [1].

It is the purpose of this paper to provide an iterative inversion algorithm of accelerating the reconstruction of the relative permittivity of a cylindrical dielectric object based on a multigrid optimization method. The object situated in a homogeneous background medium is illuminated with single-frequency or multifrequency cylindrical electromagnetic waves. We define a cost functional as the norm of a discrepancy between the scattered electric fields measured and calculated for an estimated contrast function. Thus the electromagnetic inverse scattering problem may be treated as an optimization problem where the cost functional is minimized to determine the contrast function. We apply the conjugate gradient (CG) method [4] and the frequency-hopping (F-H) technique [3],[12] to the minmization of the cost functional, and also employ the multigrid method with a V-cycle [14]-[16] to accelerate the rate of convergence. Some numerical results are presented for lossless and homogeneous dielectric circular cylinders. The convergence behaviors of the proposed metod and the conventional CG method are compared with each other.

2. Theory

Consider a cylindrical dielectric object of relative permittivity $\epsilon_s(\boldsymbol{\rho})$ illuminated by TM cylindrical electromagnetic waves with electric field $\mathbf{E}_p^i$ $(= \mathbf{u}_z E_p^i(\theta;\, \boldsymbol{\rho}))$ for the frequency of f_p, where $\mathbf{u}_z$ is the unit vector in the z-direction and $p = 1,\, 2,\, \cdots, P$. The object with cross section Ω, which is infinitely long along the z-axis, is assumed to be located in a homogeneous background medium of relative permittivity ϵ_b. Line sources generating the incident waves are placed at points with polar coordinates $(\rho,\, \theta + \pi)$. Measurements of the scattered electric field $\mathbf{E}_p^s$ $(= \mathbf{u}_z E_p^s(\theta;\, \boldsymbol{\rho}))$ for each illumination are performed at the observation points with polar coordinates $(\rho,\, \phi)$. The geometry of the problem is shown in Fig. 1. A contrast function, which characterizes the object, is defined as

$$c(\boldsymbol{\rho}) = \epsilon_s(\boldsymbol{\rho}) - \epsilon_b. \tag{1}$$

The electromagnetic inverse scattering problem can be formulated as the solution to the

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following nonlinear integral equation for the contrast function:

$$E_p^s(c;\,\theta;\,\boldsymbol{\rho}) = k_p^2 \iint_\Omega c(\boldsymbol{\rho}')\,E_p^t(c;\,\theta;\,\boldsymbol{\rho}')G_p(\boldsymbol{\rho};\,\boldsymbol{\rho}')\,d\boldsymbol{\rho}', \quad \boldsymbol{\rho} \in \overline{\Omega}, \tag{2}$$

where $\overline{\Omega}$ is a domain outside of Ω, k_p a free-space wavenumber for the frequency of f_p, and $G_p(\boldsymbol{\rho};\,\boldsymbol{\rho}')$ is the two-dimensional Green's function for the background medium. In Eq. (2), $E_p^t(c;\,\theta;\,\boldsymbol{\rho})$ is the total electric field inside the object, which may be obtained by solving a well-known linear integral equation [2]. Note that $E_p^t(c;\,\theta;\,\boldsymbol{\rho})$ is expressed as a sum of $E_p^i(\theta;\,\boldsymbol{\rho})$ and $E_p^s(c;\,\theta;\,\boldsymbol{\rho})$.

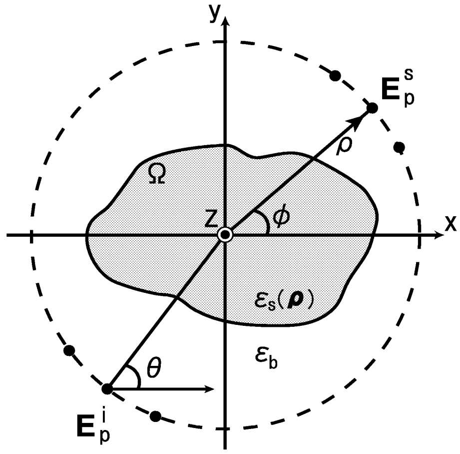

Fig. 1 Geometry of the problem.

Now the object is successively illuminated by L incident waves for one frequency. The incident waves are generated from the line sources placed at the discrete positions with polar angles $\theta = \theta_l$, where $l = 1, 2, \cdots, L$. For each incident wave, measurements of the scattered electric field are made at M discrete observation points with polar angles $\phi = \phi_m$, where $m = 1, 2, \cdots, M$. The square region containing the object and the background medium is subdivided into $N \times N$ elementary square cells. The method of moments with pulse-basis functions and point matching [13] is employed to discretize Eq. (2) and the linear integral equation for $E_p^t(c;\,\theta;\,\boldsymbol{\rho})$ inside the object. Note that $c(\boldsymbol{\rho})$ and $E_p^t(c;\,\theta;\,\boldsymbol{\rho})$ are expanded in terms of the same pulse-basis functions. The unknown expansion coefficients of the contrast function may be determined by the least-squares sense,

$$F(c) = \sum_{l=1}^{L} \sum_{m=1}^{M} \left|\, E_p^s(c;\,\theta_l;\,\phi_m) - \tilde{E}_p^s(\theta_l;\,\phi_m) \,\right|^2 \rightarrow minimum, \tag{3}$$

where $\tilde{E}_p^s(\theta_l;\,\phi_m)$ and $E_p^s(c;\,\theta_l;\,\phi_m)$ are the scattered electric fields measured and calculated for an estimated contrast function, respectively. Note that the simulated data for the scattered electric field based on the FFT-CG method [17] are used in place of the real measured data.

Introducing the cost functional $F(c)$, the inverse scattering problem is reduced to an optimization problem where $F(c)$ is minimized to find $c(\boldsymbol{\rho})$. We apply the CG method [4] and the F-H technique [3],[12] to the minimization of $F(c)$. In the CG method, the Polak-Ribière-Polyak method [18] is employed to obtain the direction vector, and the step size is determined by using a univariate search technique proposed by Davies et al. [18]. Furthermore, the multigrid method with a V-cycle [14]-[16] is employed to accelerate the rate of convergence of the CG method. Note that the V-cycle is constructed from two fine grides and a coarse grid. The reconstruction scheme is called the multigrid optimization (MO) method. The algorithm based on the MO method at a fixed frequency is summarized as follows:

Step 1 At the first fine grid h, apply n_1 iterations to the minimization of the cost functional using the CG method: $min[F^{(h)}(c^{(h)})] \rightarrow \hat{c}^{(h)}$.

Step 2 Restrict $\hat{c}^{(h)}$ to $c^{(H)}$ at the coarse grid H: $\Pi_h^H\,\hat{c}^{(h)} \rightarrow c^{(H)}$.

Step 3 At the coarse grid H, apply n_2 iterations to the minimization of the cost functional using the CG method: $min[F^{(H)}(c^{(H)})] \rightarrow \hat{c}^{(H)}$.

Step 4 Correct $\hat{c}^{(h)}$ using the difference between $\hat{c}^{(H)}$ and $c^{(H)}$: $\hat{c}^{(h)} + \Pi_H^h\,[\hat{c}^{(H)} - c^{(H)}] \rightarrow c^{(h)}$.

Step 5 At the second fine grid h, apply n_3 iterations to the minimization of the cost functional using the CG method: $min[F^{(h)}(c^{(h)})] \rightarrow \hat{c}^{(h)}$.

Step 6 Judge the convergence conditions: $F^{(h)}(c^{(h)}) < \epsilon \rightarrow$ end or $F^{(h)}(c^{(h)}) > \epsilon \rightarrow$ return to *Step 1*.

In the above algorithm, Π_h^H denotes a restriction operator converting a fine resolution to a coarse resolution. On the other hand, Π_H^h is an interpolation operator converting a coarse resolution to a fine resolution. The iteration terminates if the relative residual error δ in the scattered electric field becomes less than a prescribed convergence criterion at the second fine grid. Note that δ is written as the cost functional $F^{(h)}(c^{(h)})$ normalized by the norm of the scattered electric field measured.

3. Numerical results

Numerical results are presented for some dielectric circular cylinders using the single-frequency and the multifrequency scattering data in microwave region. The number of illuminations of TM cylindrical electromagnetic waves is 36 for one frequency. For each illumination, we employ 36 measurement points equally spaced along a circle of radius 2λ, where λ is the wavelength for the highest frequency of 6GHz in the background free space. The $2\lambda \times 2\lambda$ square region containing the object and the background medium is uniformly subdivided into 48×48 or 24×24 elementary square cells corresponding to the fine grid or the coarse one. The initial guess of the contrast function is zero, and all the numbers of n_1, n_2, and n_3 are 10.

First we consider the reconstruction of an object with the relative permittivity of 1.8 and the radius of 0.8λ using the single-frequency scattering data obtained for the frequency of 6GHz. Figure 2 shows the value of δ versus the number of iterations. The solid and the dotted lines present the results based on the MO method (method I) and the conventional CG method (method II). Figure 3 illustrates the reconstructed results of the relative permittivity after 58 and 141 iterations corresponding to the method I and the method II. The convergence criterion for δ at the second fine grid is $\delta < 10^{-5}$. For reference, the true profile of the relative permittivity is also denoted by the thin solid line in Fig. 3.

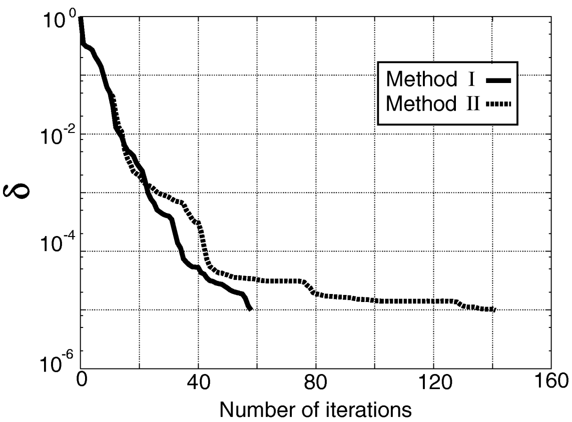

Fig. 2 Relative residual errors in the scattered electric field for single-frequency case.

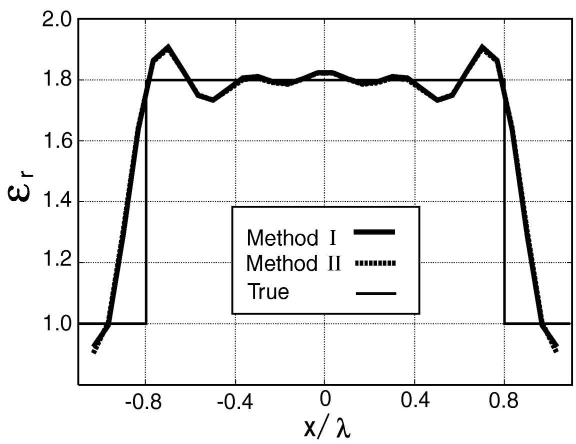

Fig. 3 Reconstructed results of the relative permittivity for single-frequency case.

Figures 4 and 5 show the results for an object with the relative permittivity of 4.0 and the radius of 0.8λ. These results are obtained from the multifrequency scattering data at the frequencies of 1GHz, 3GHz, 4GHz, and 6GHz. The four frequencies are used in the F-H technique. The current frequency is changed to the next higher-frequency when $\delta < 5 \times 10^{-3}$. The final convergent solutions in Fig. 5 for $\delta = 5 \times 10^{-4}$ are the results after 66 and 109 iterations corresponding to the method I and the method II.

It is seen from Figs. 2−5 that the rate of convergence can be remarkably accelerated by

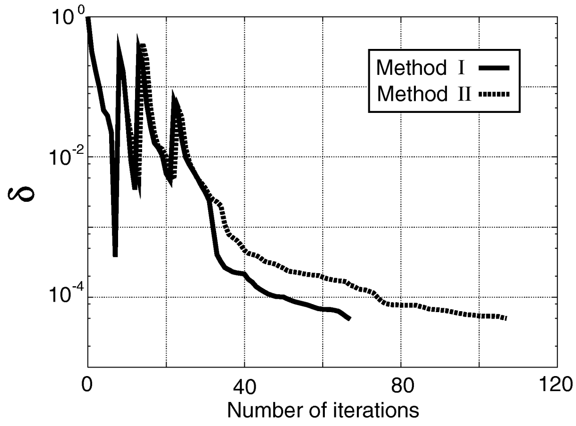

Fig. 4 Relative residual errors in the scattered electric field for multifrequency case.

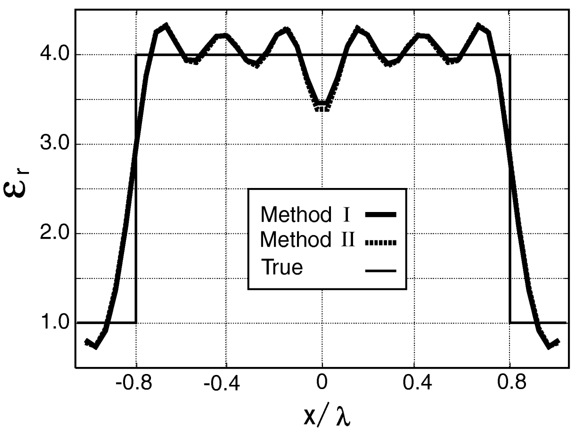

Fig. 5 Reconstructed results of the relative permittivity for multifrequency case.

using the MO metod.

4. Conclusion

A fast inversion algorithm of iteratively estimating the relative permittivity of a cylindrical dielectric object based on the MO method has been provided. The electromagnetic inverse scattering problem is treated as an optimization problem where a cost functional is minimized. The CG method and the F-H technique are applied to the minimization of the cost functional. We also employ the multigrid method with a V-cycle to accelerate the rate of convergence. Nomerical results for dielectric circular cylinders demonstrate the effectiveness of the proposed method. Research on the incorporation of a regularization into the reconstruction scheme remains a topic for further study.

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