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SCATTERING OF OBLIQUELY INCIDENT WAVES BY INHOMOGENEOUS CYLINDERS

C. Yeh and P.K.C. Wang
School of Engineering and Applied Science
University of California
Los Angeles, California 90024 U.S.A.

During the past decade, a great deal of work has been carried out by various investigators on the problem of the interaction of microwaves with an inhomogeneous dielectric or plasma column[1-4]. Applications range from the reflection of waves by meteor trails, the re-entry communication problem, plasma diagnostics to the scattering of light by optical fibers. For a radially inhomogeneous cylinder, the solution of the wave equation even for the normal incidence case is a very difficult task; it usually involves infinite summations of untabulated functions, or of solutions to an infinite set of second-order differential equations. For the oblique incidence case, the wave equation reduces to two coupled second-order differential equations whose solutions are usually very difficult to obtain even with the help of computers.

A useful approximate approach for an analytical solution of electromagnetic problems involving a radially inhomogeneous column is to subdivide it into thin homogeneous layers, and to solve an easier problem in each layer[5,6]. The fields in each layer are expanded in appropriate eigenfunctions and the expansion coefficients determined by matching boundary conditions. However this straight-forward approach becomes much too tedious, and the number of simultaneous equations to be solved tends to be prohibitively large as the number of layers increases. It is therefore quite apparent that a different approach must be taken.

The purpose of this presentation is to discuss the formulation of and the solution to the problem of the scattering of an <u>obliquely incident</u> electromagnetic wave by a radially inhomogeneous dielectric or plasma cylinder.

A harmonic plane wave with its electric vector polarized in the axial direction is assumed to impinge obliquely upon a radial inhomogeneous dielectric cylinder of radius a. The axial components of the incident wave may be expressed as

$$E_z^i = E_0 \cos\theta_0 \exp(-ik_0\cos\theta_0\, y + ik_0 \sin\theta_0\, z - i\omega t) \qquad (1)$$

$$H_z^i = 0. \qquad (2)$$

where E_0 and ω are, respectively, the amplitude and the frequency of the incident wave and $k_0 = \omega(\mu_0\epsilon_0)^{\frac{1}{2}}$. θ_0 is the angle between the propagation vector and the positive y axis in the y-z plane. The axial components of the scattered wave must take the form

$$E_z^s = F_0 \sum_{n=-\infty}^{\infty} (-1)^n A_n H_n^{(1)}(k_0 r\cos\theta_0)\, e^{in\theta} \qquad (3)$$

$$H_z^s = F_0 \sum_{n=-\infty}^{\infty} (-1)^n B_n\, i\sqrt{\frac{\epsilon_0}{\mu_0}}\, H_n^{(1)}(k_0 r\cos\theta_0)\, e^{in\theta} \qquad (4)$$

$$F_0 = E_0\cos\theta_0 \exp(ik_0\sin\theta_0\, z - i\omega t)$$

where A_n and B_n are arbitrary constants. The cylindrical coordinates (r,θ,z) have been used.

Unlike the case for the scattering by homogeneous dielectric cylinders, the field components within the inhomogeneous cylinder for the present problem can not be expressed simply in terms of infinite series of

Bessel functions. Furthermore, the wave equations for E_z and H_z within the cylinder are coupled to each other. It is known, from Maxwell's equations however, that the tangential components of electric and magnetic fields within the cylinder must satisfy the following equations:

$$
\begin{aligned}
\frac{d}{d\rho} E_{zm} &= \frac{m \sin\theta_0}{i\rho\, \epsilon/\epsilon_0} \sqrt{\frac{\mu_0}{\epsilon_0}} H_{zm} + \frac{1}{i\rho}\left(1 - \frac{\sin^2\theta_0}{\epsilon/\epsilon_0}\right) \rho \sqrt{\frac{\mu_0}{\epsilon_0}} H_{\theta m} \\
\frac{d}{d\rho} \rho E_{\theta m} &= \frac{1}{i}\left(\frac{m^2}{\rho\, \epsilon/\epsilon_0} - \rho\right) \sqrt{\frac{\mu_0}{\epsilon_0}} H_{zm} + \frac{1}{i}\left(\frac{-m \sin\theta_0}{\rho\, \epsilon/\epsilon_0}\right) \rho \sqrt{\frac{\mu_0}{\epsilon_0}} H_{\theta m} \\
\frac{d}{d\rho} \sqrt{\frac{\mu_0}{\epsilon_0}} H_{zm} &= \frac{-m \sin\theta_0}{i\rho} E_{zm} + \frac{1}{i\rho}\left(\sin^2\theta_0 - \frac{\epsilon}{\epsilon_0}\right) \rho E_{\theta m} \\
\frac{d}{d\rho} \rho \sqrt{\frac{\mu_0}{\epsilon_0}} H_{\theta m} &= \frac{1}{i}\left(\rho \frac{\epsilon}{\epsilon_0} - \frac{m^2}{\rho}\right) E_{zm} + \frac{m \sin\theta_0}{i\rho} \rho E_{\theta m}
\end{aligned}
\tag{5}
$$

with

$$E_z = F_0 \sum_{m=-\infty}^{\infty} (-i)^m (E_{zm})\, e^{im\theta} \tag{6}$$

$$\sqrt{\frac{\mu_0}{\epsilon_0}} H_z = F_0 \sum_{m=-\infty}^{\infty} (-i)^m \left(H_{zm} \sqrt{\frac{\mu_0}{\epsilon_0}}\right) e^{im\theta} \tag{7}$$

$$\rho = k_0 r$$

ϵ/ϵ_0 is the inhomogeneous dielectric variation within the cylinder. The boundary conditions at $\rho=\rho_a$ with $\rho_a=k_0a$ and at $\rho=0$ for the above set of equations are, respectively,

$$
\begin{aligned}
E_{zm}(\rho_a) &= J_m(\rho_a \cos\theta_0) + A_m H_m^{(1)}(\rho_a \cos\theta_0) \\
\rho_a E_{\theta m}(\rho_a) &= \frac{1}{\cos^2\theta_0}\Big[-m \sin\theta_0\, J_m(\rho_a \cos\theta_0) - A_m m \sin\theta_0\, H_m^{(1)}(\rho_a \cos\theta_0) \\
&\qquad + B_m \cos\theta_0\, \rho_a H_m^{(1)\prime}(\rho_a \cos\theta_0)\Big] \\
\sqrt{\frac{\mu_0}{\epsilon_0}} H_{zm}(\rho_a) &= B_m i\, H_m^{(1)}(\rho_a \cos\theta_0) \\
\sqrt{\frac{\mu_0}{\epsilon_0}} \rho_a H_{\theta m}(\rho_a) &= \frac{i}{\cos^2\theta_0}\Big[\rho_a \cos\theta_0\, J_m'(\rho_a \cos\theta_0) + A_m \rho_a \cos\theta_0\, H_m^{(1)\prime}(\rho_a \cos\theta_0) \\
&\qquad - B_m \sin\theta_0\, m\, H_m^{(1)}(\rho_a \cos\theta_0)\Big]
\end{aligned}
\tag{8}
$$

and

$$\rho E_{\theta m}\big|_{\rho=0} = 0 \quad , \quad \sqrt{\frac{\mu_0}{\epsilon_0}} \rho H_{\theta m}\big|_{\rho=0} = 0 \, . \tag{9}$$

The problem now is appropriately formulated in terms of the well-known two point boundary value problem[7] with E_z, ρE_θ, $\sqrt{\mu_0/\epsilon_0}\, H_z$ and $\rho\sqrt{\mu_0/\epsilon_0}\, H_\theta$ as the dependent variables.

As a specific example, the Luneburg lens profile is chosen as the dielectric variation ϵ/ϵ_0 i.e., $\epsilon(\rho)/\epsilon_0 = 2 - (\rho/\rho_a)^2$. Numerical computations were carried out for the differential scattering cross section for various k_0a. Results are shown in Fig. 1. Detailed discussions concerning the numerical solution of the two point boundary value problem will be given in the presentation.

It is important to note that this technique may be applied easily to the problem of the interaction of obliquely incident microwaves with an inhomogeneous and anisotropic plasma column. This work is now in progress.

References

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