

## DIRECTIONAL SCANNING OF COMPLEX ELECTROMAGNETIC ENVIRONMENTS

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### INTRODUCTION

As rf and microwave sources (both intentional and inadvertent) multiply, the electromagnetic (EM) environment in which electronic devices (and people) must function becomes increasingly complicated, while at the same time its characterization becomes more important. In order to completely characterize an EM environment without knowledge of the radiating sources, the sampling theorem requires that systematic measurements of the amplitude and phase of the field be made throughout the volume at spacings of no more than one-half wavelength (of the highest frequency present). This is often impossible and seldom convenient. There is a need for practical techniques which would determine useful properties of an EM environment from relatively few measurements [1]. One recent suggestion for such a technique [2] is to use directional measurements at a single point in conjunction with a plane-wave expansion of the field. This paper reports the formulation of the technique and the results of a simulation using it.

### FORMULATION

We restrict ourselves to a volume which is free of sources (both primary and induced), and expand the electric field within the volume as

$$E(\bar{x}, t) = E(\bar{x}) e^{-i\omega t} = \lambda^{-2} e^{-i\omega t} \int d\phi_k d\cos\theta_k e^{i\bar{k}\cdot\bar{x}} \bar{e}(\theta_k, \phi_k), \quad (1)$$

where  $|\bar{k}| = 2\pi/\lambda$ , and we have assumed a single frequency. With a perfectly directional probe the response for a given probe orientation  $\theta_0, \phi_0$  would directly measure (a component of) the plane-wave coefficient  $\bar{e}(\theta_0, \phi_0)$ . Real antennas integrate over a finite solid angle--allowing us to cover the entire  $4\pi$  solid angle with a finite number of measurements, but requiring a deconvolution to obtain the plane-wave coefficients. The response of the antenna for some orientation  $\theta_0, \phi_0, \chi_0$  (angle of rotation about longitudinal axis) is

$$R(\theta_0, \phi_0, \chi_0) = \int d\Omega [A_\theta(\theta_0, \phi_0, \chi_0; \theta, \phi) e_\theta(\theta, \phi) + A_\phi(\theta_0, \phi_0, \chi_0; \theta, \phi) e_\phi(\theta, \phi)], \quad (2)$$

where the  $\theta, \phi$  subscripts denote vector components in spherical coordinates.

$A_\theta(\theta_0, \phi_0, \chi_0; \theta, \phi)$  is the acceptance of the antenna for the  $\theta$  component of the electric field when the probe is at angles  $\theta_0, \phi_0, \chi_0$  in the lab and the plane wave is incident from direction  $\theta, \phi$  in the lab.  $A_\theta$  and  $A_\phi$  can be obtained from the known acceptance of the probe in a probe-fixed coordinate system by an exercise in rotations and coordinate transformations.

One then makes measurements at various  $\theta_j, \phi_j$  ( $j = 1, N_d$ ), with two antenna orientations ( $\chi = 0, \pi/2$ ) for each direction to measure both polarizations. For computational convenience, we divide the surface of a sphere centered at the origin into  $N_d$  equal-surface-area (and therefore equal-solid-angle) patches, and choose the measurement directions  $\theta_j, \phi_j$  to coincide with the centers of the patches. The measurements then yield a set of  $2N_d$  equations of the form of Eq. (2), which can be deconvoluted to obtain  $e_\theta$  and  $e_\phi$ . To effect the deconvolution we perform an angular pulse expansion of the plane-wave coefficients  $e_\theta$  and  $e_\phi$ ,

$$e_a(\cos \theta, \phi) \approx \sum_{j=1}^{N_d} e_a^j \Pi^j(\cos \theta, \phi), \quad (3)$$

where  $a = \theta, \phi$  and the  $\Pi^j$ 's are equal to one within the solid angle defined by the  $j$  patch and zero elsewhere. If Eq. (3) is inserted into Eq. (2), we obtain

$$R(\theta_i, \phi_i, \chi_\alpha) = \sum_{j=1}^{N_d} [e_\theta^j A_\theta^j(\theta_i, \phi_i, \chi_\alpha) + e_\phi^j A_\phi^j(\theta_i, \phi_i, \chi_\alpha)], \quad (4)$$

$$A_a^j(\theta_i, \phi_i, \chi_\alpha) \equiv \int d\Omega \Pi^j(\cos \theta, \phi) A_a(\theta_i, \phi_i, \chi_\alpha; \theta, \phi),$$

where  $i = 1, N_d$  and  $\alpha = 1, 2$ . Equation (4) can be put in the form of a matrix equation, solution of which yields the quantities  $e_a^j$ . These determine the approximation to the plane-wave coefficients (3), which in turn determine the approximate electric field throughout the volume (1).

## RESULTS

To test this method we have simulated the measurement process by taking a known incident field and feeding it into Eq. (2) to produce sets of measurements which were then analyzed. The antenna was assumed to be sensitive to only one component of polarization and to have a gaussian acceptance ( $\bar{A} \propto e^{-\theta^2/W^2}$  in probe coordinate system). The incident field was composed of a specified number of plane waves (1 through 15), whose directions, phases, and relative amplitudes were usually generated randomly. We investigated the dependence of the results on the width (W) of the acceptance pattern of the probe, the number of measurements, and the complexity of the incident field (the number of plane waves).

We found that for a given width W there is an optimal number of measurements one should make. If  $N_d$  is too small, gaps are left between measurements; if it is too large it leads to instabilities in the inversion process. The optimal number occurs when patch dimensions are comparable to the probe's width. Assuming one uses the number of measurements appropriate to the probe width, the quality of the results is insensitive to the number of incident plane waves, even for a width as large as 1.0, from which we infer that the method does not require simple incident field configurations. The results improve for smaller W (larger  $N_d$ ), but are quite acceptable even for  $W = 1.0$ ,  $N_d = 10$ , as we shall see below.

The reconstruction of the spatial dependence of the field is not very successful unless a plane wave happens to have incidence angles which coincide with a measurement direction. Nevertheless, the method does provide useful information--the rms field within the volume and an upper bound on the maximum field strength. Assuming the volume is large enough (many wavelengths), the rms electric field is given by

$$E_{rms} \equiv [ \langle |E|^2 \rangle_{vol} ]^{1/2} \approx [ \sum_j |\bar{e}^j|^2 ]^{1/2} \frac{4\pi}{N_d}. \quad (5)$$

The bound on the maximum field strength is

$$|E(\bar{x})|_{max} < E(u.b.),$$

$$E(u.b.) \approx \{ ( \sum_j |e_x^j|^2 ) + ( \sum_j |e_y^j|^2 ) + ( \sum_j |e_z^j|^2 ) \}^{1/2} \frac{4\pi}{N_d}. \quad (6)$$

Equations (5) and (6) refer to the measured quantities, of course. To test the accuracy of the technique the results from Eqs. (5) and (6) must be compared to the true results for  $E_{rms}$  and  $E(u.b.)$ , which are obtained from Eqs. (5) and (6) by converting the sums over measurement directions to sums over incident plane waves. We have done this, and the results for  $W = 1.0$ ,  $N_d = 10$  and various numbers of incident plane waves are displayed in the table. For each number of plane waves 25 or 50 different random configurations were generated, and that sample was used to compute the rms fractional errors in  $E_{rms}$  and  $E(u.b.)$  for that number of incident plane waves.

The results are quite encouraging. In particular, note that a 14 percent error in  $E$  corresponds to an error of 1.2 dB in the average intensity within the volume. Thus, although we cannot determine the spatial variation of the field, we can measure  $E_{rms}$  and in addition obtain an indication of the possible range of electric fields within the volume. Problems remain, such as inclusion of measurement errors and allowance for highly directional sources, but directional scanning does hold promise of providing useful information [ $\langle |E|^2 \rangle$ ,  $E(u.b.)$ ] from a reasonable number of measurements at one point.

#### REFERENCES

1. Three possible techniques are outlined in: Kanda, M.; Randa, J.; Nahman, N. S., "Possible Estimation Methodologies for Electromagnetic Field Distributions in Complex Environments," to appear as an Nat. Bur. Stand. (U.S.) Tech. Note.
2. Chang, D. C., and Maley, S., private communication.

Table: RMS fractional errors.

#Plane waves	1	2	3	5	10	15
#Different configurations of incident field	50	25	25	50	25	50
Error in $E_{rms}$	0.145	0.134	0.136	0.145	0.158	0.141
Error in $E(u.b.)$	0.464	0.368	0.286	0.192	0.158	0.256