THE PROPAGATION OF VLF RADIO WAVES INFLUENCED BY A LARGE-SCALE THREE-DIMENSIONAL IONOSPHERIC DEPRESSION

O.V.Soloviev and V.V.Agapov
Radiophysics Department, University of St. Petersburg
Ulyanovskaya 1, St. Petersburg 198904, Russia

1. Introduction

This paper presents a further development of analytical-numerical approach to the three-dimensional problem of wave propagation in the presence of localized irregularity. The word "localized" means that it is limited in extent in both directions - longitudinal and transverse to propagation path. If the irregularity is small in comparison with wavelength, then its influence may be evaluated by using first-order perturbation technique or directly numerically. In the case of an inhomogeneity of infinite lateral extent the problem may be treated with the aid of the well known two-dimensional mixed-path approximation. Quite different the case of the large-scale inhomogeneity which is limited in transverse direction. Then the three-dimensional effects are significant and an efficient and accurate approach is required for their proper account.

In order to illustrate the productivity of the developed technique we focus here on a particular application of the problem and consider the VLF radio wave propagation influenced by a localized ionospheric disturbance. For some recent years there were numerous papers dealing with the modelling of VLF propagation in the irregular Earth-ionosphere waveguide (e.g. [1], [2]). There were also many earlier attempts to develop an efficient numerical procedure to evaluate quantitatively the effect of a localized inhomogeneity of terrestrial wavegude on both subionospheric and ground wave propagation. The ample list of relevant references may be found in our paper [3] and their detailed survey provided us with the motivation for the current study. Unlike the most of earlier investigations our theory proceeds from more realistic model of the irregularity as we characterize the non-uniform boundaries of the guide by inhomogeneous surface impedance function. Another distinctive feature of our approach is the employment the asymptotic technique which allows to facilitate the computations and retains in account the finite lateral extent of an inhomogeneity.

2. Model and formulation

Let us consider the three-dimensional domain $\mathcal{D}$ bounded by two impedance surfaces S_e and S_i one of which is non-uniform. The surface S_e is defined as $z = 0$ and the surface S_i is formed by the plane $z = h$ and by the truncated cylinder of the height h_p standing on this plane and protruding inside the cavity of the guide. The lateral surface of cylinder S_l and its base S_p are characterized by the impedance values δ_l and δ_p, respectively.

The harmonic $(e^{-i\omega t})$ point source which excites the waveguide cavity $\mathcal{D}$ is assumed to be a vertical electric dipole. Therefore, in the scalar approximation the electromagnetic field may be characterized by the vertical component of Hertz's vector $\Pi(x, y, z)$. The unknown function $\Pi(x, y, z)$ obeys Helmholtz's equation and the following impedance boundary conditions:

$$\frac{1}{\Pi}\frac{\partial \Pi}{\partial n} = ik\delta(M)|_{M\in S_e, S_i}$$

where: $k = \omega\sqrt{\epsilon_0\mu_0}$, $\delta(M) = \delta_e$ if $M \in S_e$, $\delta(M) = \delta_i$ if $M \in S_i\setminus(S_p \cup S_l)$, $\delta(M) = \delta_p$ if $M \in S_p$, and $\delta(M) = \delta_l$ if $M \in S_l$. We point out here that in practice for such field polarization: $|\delta_{e,i,p}| < 1$ and $|\delta_l| \gg 1$.

3. Asymptotic procedure

The slowly varying attenuation function V is defined as follows:

$$\Pi(r, \varphi, z) = \frac{P_0}{2\pi\epsilon_0}\frac{e^{ikr}}{r}V(r, \varphi, z)$$

Using the second Green's formula we may relate the unknown attenuation function $V(\mathbf{R})$ and the attenuation function for uniform waveguide $V_0(\mathbf{R})$ by the following integral equation:

$$V(\mathbf{R}) = V_0(\mathbf{R}) + \frac{ikr}{2\pi}\iint_{S_p}\left\{\delta_p - \frac{1}{V_0(\mathbf{R},\mathbf{R}')}\frac{\partial V_0(\mathbf{R},\mathbf{R}')}{ik\,\partial z'}\right\} V(\mathbf{R}')V_0(\mathbf{R},\mathbf{R}')\frac{e^{ik(r'+r_1-r)}}{r'r_1}dS' +$$
$$+\frac{ikr}{2\pi}\iint_{S_l}\left\{V_0(\mathbf{R},\mathbf{R}') - \frac{1}{\delta_l}\frac{\partial r_1}{\partial n'}\left[\frac{\partial V_0(\mathbf{R},\mathbf{R}')}{ik\,\partial r_1} + \left(1-\frac{1}{ikr_1}\right)V_0(\mathbf{R},\mathbf{R}')\right]\right\}\times$$
$$\times\left\{\frac{\partial V(\mathbf{R}')}{ik\,\partial n'} + \left(1-\frac{1}{ikr'}\right)\frac{\partial r'}{\partial n'}V(\mathbf{R}')\right\}\frac{e^{ik(r'+r_1-r)}}{r'r_1}dS' \quad (1)$$

where: $\mathbf{R}(r,\varphi,z) \notin (S_p \cup S_l)$ - observation point, $\mathbf{R}'(r',\varphi',z') \in (S_p \cup S_l)$ - integration point, n' - normal to S_l directed outside from the cavity of the guide, and $r_1 = \sqrt{r^2 + r'^2 - 2rr'cos(\varphi - \varphi')}$. In the limit case $\mathbf{R} \in (S_p \cup S_l)$ an additional item $V(\mathbf{R})/2$ arises in the formula (1) due to the discontinuity of the normal derivative of Green's function $\partial\Pi_0(\mathbf{R},\mathbf{R}')/\partial z'$ or $\partial\Pi_0(\mathbf{R},\mathbf{R}')/\partial n'$.

The key point of the theory is the asymptotic evaluation of integral over S_p with the aid of stationary phase method. We introduce the elliptic coordinate system on the surface S_p [4] and substitute the variables of integration:

$$\begin{array}{lll} r' = \frac{r}{2}[ch\,u + cos\,v] & r_1 = \frac{r}{2}[ch\,u - cos\,v] & -\infty < u < +\infty \\ x' = \frac{r}{2}\,ch\,u\,cos\,v & y' = \frac{r}{2}\,sh\,u\,sin\,v & 0 \le v \le \pi \end{array}$$

Taking in account that $dS' = r_1 r' du dv$, we may rewrite the first integral in (1) as follows:

$$\iint_{S_p} f(\mathbf{R},\mathbf{R}')\frac{e^{ik(r'+r_1-r)}}{r'r_1}dS' = \iint_{S_p} f(u,v)e^{ikr(ch\,u-1)}du dv = \int_{v_<}^{v_>}\left(\int_{u_<(v)}^{u_>(v)} f(u,v)e^{ikr(ch\,u-1)}du\right)dv$$

where the functions $u_<(v)$ and $u_>(v)$ specify the boundary of perturbed area in elliptic coordinate system. Performing some more preliminaries:

$$\int_{u_<(v)}^{u_>(v)}\ldots du = \int_{u_<(v)}^{\infty}\ldots du - \int_{u_>(v)}^{\infty}\ldots du$$

we assume $kr \gg 1$ to be the large parameter of the problem and indicate the slowly varying part of the integrand $f(u,v)$. Following [5] or [6], we write out the complete expressions for the first two terms of asymptotic expansion [7]:

$$\int_{u_b(v)}^{\infty} f(u,v)e^{ikr(ch\,u-1)}du = \frac{exp\{ikr[ch\,u_b(v)-1]\}}{i\sqrt{2kr}}G[u_b(v),v] + O[(kr)^{-3/2}]$$

where:

$$G[u(v),v] = \sqrt{\pi}e^{i3\pi/4}f(0,v)w(e^{i\pi/4}p) + \frac{1}{p}\left[f(0,v) - \frac{f[u(v),v]}{ch[u(v)/2]}\right],$$

$p = \sqrt{2kr}sh[u(v)/2]$, and the function $w(x) = e^{-x^2}\left(1 + \frac{2i}{\sqrt{\pi}}\int_0^x e^{t^2}dt\right)$ is the conventional probability integral of a complex variable.

Using the formulas above we can deduce that the remained integral with respect to v may be represented as contour integral along the boundary of inhomogeneous area:

$$\iint_{S_p} f(\mathbf{R},\mathbf{R}')\frac{e^{ik(r'+r_1-r)}}{r'r_1}dS' = \frac{1}{i\sqrt{2kr}}\oint_{\partial S_p} exp\{ikr[ch\,u(v)-1]\}G[u(v),v]dv + O[(kr)^{-3/2}] \quad (2)$$

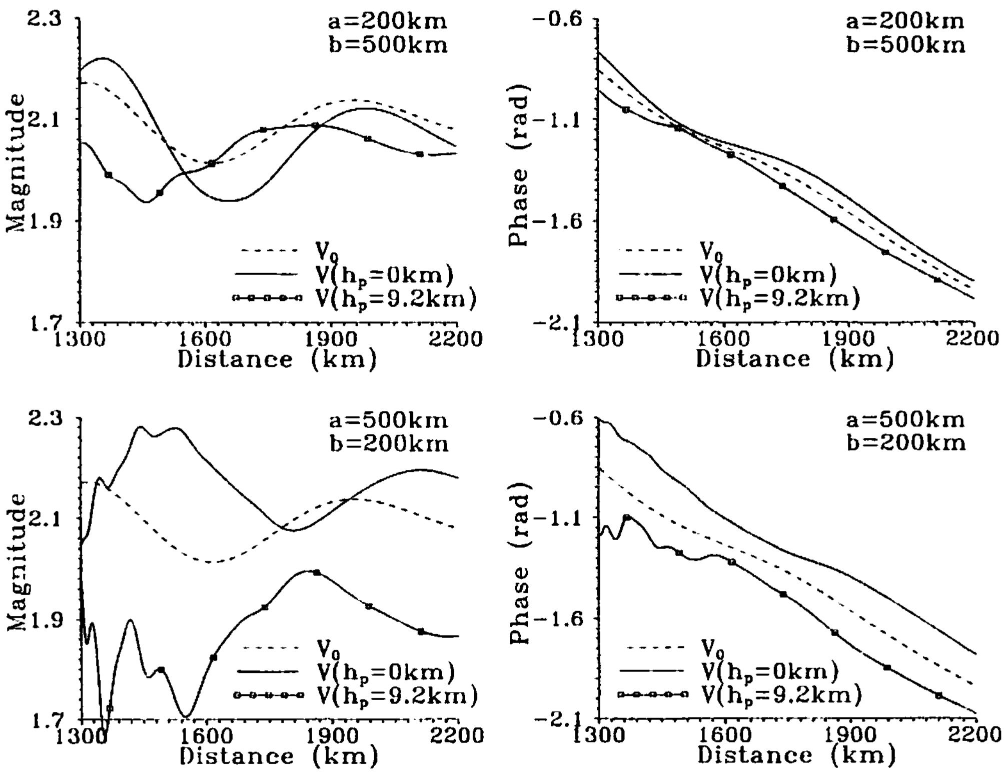
The asymptotic formula (2) is more accurate than that from [4], as it completely incorporates the second term of asymptotic expansion $O(1/kr)$. The proposed contour integral representation proves to be much more convenient for numerical implementation.

4. Numerical algorithm

In order to gain the most benefit from the performed asymptotic integration, we use the algorithm which combines both inversion and iteration procedures. Let us represent the integral operator in our asymptotic integral equation as the sum of three items $V = V_0 + \mathcal{A}V + \mathcal{B}V + \mathcal{C}V$, where $\mathcal{A}$ is the integral operator which acts on the function $f(0, v)$, $\mathcal{B}$ is the integral operator which acts on the function $f[u_b(v), v]$, and $\mathcal{C}$ is the second item from (1). We point out that $\mathcal{A}$ is the Volterra integral operator and it can be easily inverted with the aid of conventional stepwise procedure. Moreover, it contains the first, dominant term of asymptotic expansion. In comparison with $\mathcal{B}$ its contribution to solution is expected to be as more significant as large the value of parameter kr. Furthermore the numerical calculations performed for the case of small irregularity [8] exhibited the insignificant contribution of the second integral over S_l in (1) to the diffracted field in comparison with the influence of S_p. Thus we find the following algorithm to be the most judicious [9], [10]:

$$V^{(0)} = V_0 , \quad V^{(n)} = V_0 + \mathcal{A}V^{(n)} + \mathcal{B}V^{(n-1)} + \mathcal{C}V^{(n-1)} \qquad n = 1, 2, 3, \ldots$$

where n is the iteration number. The Volterra integral operator $\mathcal{A}$ is inverted directly during each iteration, and the remained operators $\mathcal{B}$ and $\mathcal{C}$ are inverted by successive approximations.



5. Results and discussion

We simulated VLF (12.1 kHz) radio wave propagation in terrestrial wavegude using the parallel-plane model introduced above. The spatial behavior of the modulus and argument of the attenuation function V (longitudinal diffraction patterns) is illustrated at the figures. Both transmitter and receiver are assumed to be on the Earth's surface, where their Cartesian coordinates are : $\{x_t = 0, y_t = 0\}$, $\{1300 < x_r < 2200\ km, y_r = 0\}$. The elliptic cross section of the cylinder $[(x-x_p)/a]^2+[y/b]^2 \leq 1$ is characterized by the values $x_p = 800\ km$, $a = 200\ km$, $b = 500\ km$ (upper plots), and $a = 500\ km$, $b = 200\ km$ (lower plots). The waveguide height $h = 59.8km$ and the impedance values $\delta_i = 0.4965 + 0.2215i$ and $\delta_p = 0.2364 + 0.2698i$ were calculated using the ionosphere height profiles taken from [11]. Other parameters of the model are: $h_p = 0\ km$ or $h_p = 9.2\ km$ as indicated on the figures, $\delta_e = 2.9(1-i)10^{-4}$, $\delta_l = 1.0(1-i)10^2$.

The presented diagrams allow to conclude that the localized irregularity causes the complex scattering mechanism. One may also observe that the convexity of the irregularity ($h_p \neq 0$) substantially affects field behavior.

References

[1] J.R.Wait, "VLF radio wave mode conversion for ionospheric depressions", *Radio Sci.*, 26(5), 1261-1265, 1991.

[2] W.L.Poulsen, T.F.Bell, and U.S.Inan, "Three-dimensional modeling of subionospheric VLF propagation in the presence of localized D region perturbations associated with lightning", *J. Geophys. Res.*, 95(A3), 2355-2366, 1990.

[3] V.V.Agapov and O.V.Soloviev, "An Asymptotic 3D Approach To The Problem Of Environmental VLF and LF Radio Wave Propagation In The Presence of Localized Irregularity", *Proc. of the EUROEM-94, Int.Symp. on Electromagnetic Environments and Consequences, Bordeaux, France, May 30-June 4, 1994*, Bordeaux, France, 1994.

[4] R.J.King and W.I.Tsukamoto, "Groundwave propagation across semi-infinite strips and islands on a flat earth", *Radio Sci.*, 1(7), 775-788, 1966.

[5] V.P.Peresada, "On the calculation of finite limit integral of a function with rapidly changing phase", *Radiotekhnika*, 12(9), 12-19, 1957.

[6] L.B.Felsen and N.Marcuvitz, *Radiation and Scattering of Waves*, Prentice Hall, Englewood Cliffs, N.J., 1973.

[7] O.V.Soloviev, "On the solution to the propagation problem for the locally irregular waveguide", *Izv. VUZ Radiofiz.*, 33(9), 1078-1087, 1990.

[8] O.V.Soloviev, "Field calculation for a plane impedance waveguide with a local geometrical irregularity", *Izv. VUZ, Radiofiz.*, 34(8), 908-918, 1991.

[9] V.Agapov and O.Soloviev, "An asymptotic three-dimensional approach to the problem of the point source field diffracted by localized impedance discontinuity", *Proc. of the Int. Conf. on Mathematical Methods in Electromagnetic Theory, Kharkov, Ukraine, 7-10 September, 1994*, pp.1-6. Kharkov, 1994.

[10] O.V.Soloviev and V.V.Agapov, "An Asymptotic 3D Approach to the Spherical Wave Diffraction by a Convex Irregularity in Plane Impedance Waveguide", *Proc. of the URSI Int. Symp. on Electromagnetic Theory, St. Petersburg, Russia, 23-26 May, 1995*, p.401-403. St. Petersburg, 1995.

[11] M.A.Itkina, D.S.Kotik, Z.N.Krotova, S.V.Polyakov, and V.O.Rapoport, "The heating of the lower ionosphere by HF radio waves", *Preprint NIRFI*, 162, Gorki, 1983.

Acknowledgment. This work was supported by the Russian Basic Research Foundation under grant 93-02-17048.