B–11–1 SOME RECENT APPROACHES TO ANALYSIS AND SYNTHESIS OF REFLECTOR ANTENNA SYSTEMS

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I. INTRODUCTION

Increasing demands on the performance of satellite communication antennas have generated recent interest in efficient methods for systematically analyzing and synthesizing reflector antennas and associated feed systems. In this paper, we will briefly review a number of analytical and numerical techniques developed by the author and his co-workers for investigating the characteristic of reflector antennas.

Scanning through the literature on reflector antennas, one finds that the greatest needs for development exist in the following areas: (i) efficient computation of secondary pattern for prime-focus and Cassegrainian reflectors; (ii) investigating the scan properties of these antennas as functions of feed displacement; (iii) synthesizing and analyzing shaped dual reflectors for achieving pattern and sidelobe control. A brief discussion of each of these topics will be presented in the paper.

II. EFFICIENT COMPUTATION OF RADIATION CHARACTERISTICS OF REFLECTOR ANTENNAS

The conventional approach [1] to computing the radiation pattern of reflector antennas entails the repeated evaluation of the two-dimensional radiation integral

$$E_1(u,v) = \int_0^a \int_0^{2\pi} f(r,\phi') \, e^{jP(u,v.r,\phi')} r dr d\phi' \qquad (1)$$

E_1 in (1) represents one of the scalar components of the radiated field, $f(r,\phi')$ is the so-called "effective aperture distribution" which is directly obtainable (without approximation) from the induced surface current distribution on the reflector surface, $\exp\{jP\}$ is a known kernel function, and (u,v) are the observation variables ($u = \sin\theta \cos\phi$; $v = \sin\theta \sin\phi$).

It is not difficult to see that the computation of (1) for a two-dimensional grid of observation coordinates is extremely time-consuming if performed by a two-dimensional numerical quadrature, even when an adaptive integration procedure is employed. The problem becomes particularly acute for multiple-beam and contour beam applications since many such beams must be computed accurately, in both amplitude and phase, in order to estimate the composite sidelobe level and other such characteristics of the superimposed beams. In the method developed by the author and his co-workers [2] the far-field radiation integral is expressed in a rapidly converging series whose coefficients are independent of observation angles and whose leading term is of the type

$$\frac{J_1 \; ka\sqrt{(u-u_M)^2 + (v-v_M)^2}}{ka\sqrt{(u-u_M)^2 + (v-v_M)^2}} \qquad (2)$$

i.e., the Airy function, in the direction of the beam maximum ($u = u_M$, $v = v_M$). The derivation of the series expansion is based on several key steps. The first is to show that the secondary pattern can be expressed as a Fourier transform of the effective aperture distribution. The second is to demonstrate that the small-angle representation can be extended into the wide-angle regime in an analytically continuous manner so that the pattern computation for the entire range of observation angles can be carried out in an efficient manner. The computational efficiency results from the use of a series of Jacobi polynomials in r and a Fourier series in ϕ' to express the effective aperture distribution $f(r,\phi')$ in (1). The choice of the Jacobi polynomials is found to be unique not only from the point of analytical integrability when the series for $f(r,\phi')$ is substituted in (1), but also because of their recursion property that allows the integral in (1) to be conveniently expressed in a rapidly bi-convergent series form for all observation angles, both near and far from the beam maximum. Extensive numerical computations have been carried out using the series expansion and the numerical efficiency is found to be comparable to that of the FFT performed on a planar aperture, although the integration here is on the curved reflector surface.

It should also be emphasized that the coefficients of the series need be computed only once for calculation of the secondary pattern at an arbitrary observation angle.

Although not obvious at first, the paper demonstrates that the unique features of the Jacobi-polynomial series method carry over is virtually unchanged, even when the antenna is an offset paraboloid [3]. Once again, the series approach is considerably more efficient than the conventional methods that become extremely time-consuming because of lack of azimuthal symmetry in the offset geometry. Shaped reflectors, which are extremely difficult to analyze, can still be handled using the series approach; however, not unexpectedly, some loss of efficiency results in this case because the recursion formula can no longer be employed.

As an example of the usefulness of an efficient pattern computation scheme, the paper shows its applitcation to the synthesis of contour beam covering the eastern time zone of the United States.

III. SYNTHESIS OF DUAL REFLECTORS

The problem of synthesizing dual Cassegrainian antennas for controlling the beam shape and sidelobe characteristics of reflector antennas is a challenging one and is the subject of discussion of the second part of the paper. It begins by addressing some important issues related to the questions of existence and uniqueness of the solution. Next, a systematic method for synthesizing dual reflector systems with arbitrarily specified phase and amplitude distributions in the aperture is described and its scope and limitations are discussed. The method is a generalization of the synthesis prodecure for the circularly symmetric system, originally developed by Galindo [4] and Kinber [5]. The offset problem is considerably more involved than the corresponding circularly symmetric case and its solution has remained elusive for many years. The development of the three-dimensional synthesis procedure is based on a simultaneous enforcement of four conditions, viz., (i) the path-length condition that guarantees that the ray path between the phase center of the illumination and the exit aperture plane equals that specified by the aperture phase condition; (ii) the energy condition requiring that the product of the specified amplitude in the exit aperture plane, and the elemental area in a ray pencil crossing this plane, equals the energy in the corresponding ray pencil incident on the subreflector from the illuminating source with a specific amplitude taper; (iii) Snell's law at the subreflector; and, finally (iv) the smoothness or consistency condition which requires that the points on the main reflector and subreflector surfaces vary in a manner consistent with the slopes determined from Snell's law and path length condition. The last condition represents an important departure from the circularly symmetric case where the smoothness condition is redundant. This condition is also very significant from the point of view of guaranteeing that the mathematical or numerical solutions for the synthesis problems are meaningful from a physical point of view, since violating the smoothness criterion renders the goemetrical optics (GO) solution invalid. Numerical results obtained from the application of the methods are included in the paper to illustrate its scope and to demonstrate the inherent difficulties encountered in the synthesis problem when the specified amplitude taper of the feed illumination is not small.

IV. SUBREFLECTOR ANALYSIS

The last topic to be discussed in the paper is the important problem of computing the geometrical optics and edge diffracted fields scattered from a subreflector illuminated by a feed source which has a specified amplitude and phase pattern that can be related to a single phase center. In the conventional GO and GTD (geometrical theory of diffraction) analysis the determination of scattered fields requires the searching for the stationary or specular points. This step, which requires a non-linear search for each new combination of source and observation points, can be very time consuming, particularly when the surface is specified only numerically as is the situation for the subreflector surfaces derived from an application of the synthesis procedure. In this paper, we describe an alternative approach that obtains the scattered field in a more direct manner obviating the need for a search of the specular point. The method can be used only when the coordinates, as well as the direction of the normal to the surface are specified at a set of grid points on the surface. However, this is typically the situation when the subreflector surface has been derived from the solution of the synthesis procedure.

The method begins by computing the reflected rays off the surface at the prescribed grid points and defines a "mean" ray associated with each pencil comprising a cluster of three rays. The GO field at a given observation point is obtained by simple, linear, algebraic manipulations requiring only the orientation and perpendicular distance of the observation point to the mean ray. Once again, numerical calculations illustrating the method of computation are included in the paper which also compares the speed of the direct approach relative to the specular point method.

V. CONCLUSIONS

In this paper we have briefly summarized the results of some recent research carried on by the author and his co-workers on the subject of analyzing and synthesizing reflector antennas. Whenever appropriate, the methods of analysis and synthesis have been compared with those reported by other works in recognized journals and some conclusions have been derived.

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