SIDELOBE PEAK ENVELOPE FUNCTION FOR CIRCULAR APERTURE WITH CENTER BLOCKING

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1. Introduction

Radiation pattern of the circular aperture can be calculated by integrating the aperture distribution. And various distributions for suppressed sidelobe have been proposed, for example, Taylor distribution(1). The previous studies focus on the maximum sidelobe level, that is, first sidelobe level.

On the practical point of view, however, it is important to suppress the sidelobe level in the offaxis region away from the first sidelobe, because, for example, CCIR recommends the 3dB lower sidelobe (29-25logθ) in one to 20 degree offaxis region than the other region for satellite communication earth station antennas. And the aperture distribution with the suppressed first sidelobe does not necessarily exhibit the low sidelobe in the wide angle region.

This paper presents the approximate expression for the sidelobe peak envelope for circular aperture with center blocking. The comparison with the results from the direct integration is shown for various distribution and shows the validity of the expression. Although similar expression may be known empirically, this paper will give the theoretical background and general expressions for arbitrarily distribution and provide the measure for the design of the aperture antenns.

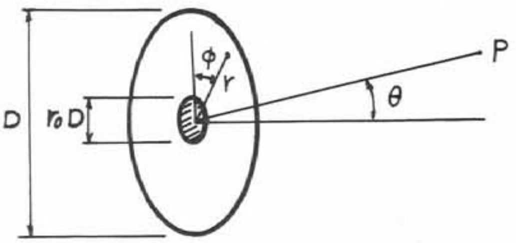

Fig.1 Aperture with blocking

2. Approximation of the Integral

Let the point on the aperture be (r,ϕ) as shown in Fig.1, then the radiation pattern at the θ offaxis point for the aperture with the diameter D and the center blocking diameter $r_0 D$ $(0 \leq r_0 < 1)$ can be calculated by the following integral.

$$g(u) = \frac{1}{\pi}\int_0^{2\pi}\int_{r_0}^{1} f(r)\, e^{jur\cos\phi}\, r\, dr\, d\phi = 2\int_{r_0}^{1} f(r)\, J_0(u)\, r\, dr \tag{1}$$

Where,

$$u = (\pi D/\lambda)\sin\theta \tag{2}$$

and $f(r)$, $(0 \leq r \leq 1)$ is the aperture electric distribution function and is supposed to be normalized so that averaged electric field on the aperture may be unity.

$$\left| 2\int_{r_0}^{1} f(r)\, r\, dr \right| = 1 \tag{3}$$

The following integral formula is applied to Eq.(1).

$$\int r^n J_{n-1}(ur)\, dr = \frac{1}{u} r^n J_n(ur) \tag{4}$$

And on the assumption that u is large, the integral in Eq.(1) is expanded by partial integral,

$$\int_{r_0}^{1} f(r) J_0(ur) r\,dr = \frac{1}{u} J_1(u) f(1) - \frac{1}{u} r_0 J_1(ur_0) f(r_0) - \frac{1}{u}\int_{r_0}^{1} r J_1(ur) f'(r)\,dr$$
$$= \frac{1}{u} J_1(u) f(1) - \frac{1}{u} r_0 J_1(ur_0) f(r_0) - \frac{1}{u^2} J_2(u) f'(1) + \frac{1}{u^2} r_0^2 J_2(ur_0) \frac{f'(r_0)}{r_0}$$
$$+ \frac{1}{u^2}\int_{r_0}^{1} r^2 J_2(ur) \frac{d}{dr}\left(\frac{f'(r)}{r}\right) dr$$
$$\approx \frac{1}{u} J_1(u) f(1) - \frac{(-1)^{n+1}}{u^{n+1}} J_{n+1}(u) f^{(n)}(1) - \frac{1}{u} r_0 J_1(ur_0) f(r_0) \qquad (5)$$

Where, n is the order for the lowest non zero derivative, that is,

$$f^{(1)}(1) = f^{(2)}(1) = \cdots\cdots = f^{(n-1)}(1) = 0, \quad f^{(n)}(1) \neq 0$$

Applying the asymptotic expansion for Bessel function

$$J_n(u) \approx \sqrt{\frac{2}{\pi u}} \cos\left(u - \left(\frac{n}{2} + \frac{1}{4}\right)\pi\right) \qquad (u : large) \qquad (6)$$

and empirical approximate expression

$$J_1(u) \doteqdot (1 - 0.8 e^{-2ur_0}) \sqrt{\frac{2}{\pi u r_0}} \cos\left(ur_0 - \frac{3}{4}\pi + \frac{\pi}{4} e^{-0.55ur_0}\right) \qquad (0 < ur_0) \qquad (7)$$

to Eq.(5) and substituting it into Eq.(1), we obtain

$$g(u) = 2\int_{r_0}^{1} f(r) J_0(ur) r\,dr = M \cos\left(u - \frac{3}{4}\pi + \alpha\right) - N \cos\left(ur_0 - \frac{3}{4}\pi + \frac{\pi}{4} e^{-0.55ur_0}\right) \qquad (8)$$

where,

$$M = \frac{1}{u}\sqrt{\frac{8}{\pi u}}\left\{[f(1)]^2 + \left[\frac{f^{(n)}(1)}{u^n}\right]^2 + 2f(1)\frac{f^{(n)}(1)}{u^n}(-1)^n \cos\left(\frac{2(n+1)^2 - 1}{8u} - \frac{n\pi}{2}\right)\right\}^{1/2} \qquad (9)$$

$$N = f(r_0)\frac{\sqrt{r_0}}{u}\sqrt{\frac{8}{\pi u}}(1 - 0.8 e^{-2ur_0}) \qquad (10)$$

3. Peak Envelope Function

From Eq.(8), peak envelope function $g_e(u)$ can be written as follows.

$$g_e(u) = M + N\left|\cos\left(ur_0 - \frac{3}{4}\pi + \frac{\pi}{4} e^{-0.55ur_0}\right)\right| \qquad (11)$$

Taking 20log of the above expression, we obtain the following equation. Let θ be the angle in degrees,

I) When $f(1) \neq 0$ and $f'(1) \neq 0$, ($n = 1$)

$$20\log g_e \doteqdot 41.9 - 30\log\frac{D}{\lambda} + 20\log f(1) - 30\log\theta$$
$$+ 20\log\left\{\sqrt{1 + \left[\frac{f'(1)}{f(1)}\frac{1}{u}\right]^2}\right.$$
$$\left. + \frac{f(r_0)}{f(1)}\sqrt{r_0}(1 - 0.8 e^{-2ur_0})\left|\cos\left(ur_0 - \frac{3}{4}\pi + \frac{\pi}{2} e^{-0.55ur_0}\right)\right|\right\} \qquad \text{(I-a)}$$
$$\approx 41.9 - 30\log\frac{D}{\lambda} + 20\log f(1) - 30\log\theta$$
$$+ 20\log\left[1 + \frac{f(r_0)}{f(1)}\sqrt{r_0}\left|\cos\left(ur_0 - \frac{3}{4}\pi\right)\right|\right], \qquad (u \to large) \qquad \text{(I-b)}$$

II) When $f(1) \neq 0$, $f'(1) = 0$ and $f''(1) \neq 0$, ($n = 2$)

$$20\log g_e \doteqdot 41.9 - 30\log\frac{D}{\lambda} + 20\log f(1) - 30\log\theta$$
$$+ 20\log\left\{\left|1 - \frac{f''(1)}{f(1)}\frac{1}{u^2}\right|\right.$$
$$\left. + \frac{f(r_0)}{f(1)}\sqrt{r_0}(1 - 0.8 e^{-2ur_0})\left|\cos\left(ur_0 - \frac{3}{4}\pi + \frac{\pi}{2} e^{-0.55ur_0}\right)\right|\right\} \qquad \text{(II-a)}$$
$$\approx 41.9 - 30\log\frac{D}{\lambda} + 20\log f(1) - 30\log\theta$$
$$+ 20\log\left[1 + \frac{f(r_0)}{f(1)}\sqrt{r_0}\left|\cos\left(ur_0 - \frac{3}{4}\pi\right)\right|\right], \qquad (u \to large) \qquad \text{(II-b)}$$

III) When $f(1)=0$, $f'(1)\neq 0$, ($n=1$)

$$20\log g_e \doteqdot 41.9 - 30\log\frac{D}{\lambda} - 30\log\theta$$
$$+20\log\Bigl\{\frac{f'(1)}{u}$$
$$+f(r_0)\sqrt{r_0}\,(1-0.8e^{-2ur_0})\left|\cos\left(ur_0-\frac{3}{4}\pi+\frac{\pi}{4}e^{-0.55ur_0}\right)\right|\Bigr\} \qquad \text{(III-a)}$$
$$\approx 41.9 - 30\log\frac{D}{\lambda} - 30\log\theta + 20\log\left[f(r_0)\sqrt{r_0}\left|\cos\left(ur_0-\frac{3}{4}\pi\right)\right|\right] \quad \binom{r_0\neq 0}{u:\text{large}} \text{(III-b)}$$

IV) When $f(1)=f'(1)=0$, $f''(1)\neq 0$, ($n=2$)

$$20\log g_e \doteqdot 41.9 - 30\log\frac{D}{\lambda} - 30\log\theta$$
$$+20\log\Bigl\{\frac{f''(1)}{u^2}$$
$$+f(r_0)\sqrt{r_0}\,(1-0.8e^{-2ur_0})\left|\cos\left(ur_0-\frac{3}{4}\pi+\frac{\pi}{4}e^{-0.55ur_0}\right)\right|\Bigr\} \qquad \text{(IV-a)}$$
$$\approx 41.9 - 30\log\frac{D}{\lambda} - 30\log\theta + 20\log\left[f(r_0)\sqrt{r_0}\left|\cos\left(ur_0-\frac{3}{4}\pi\right)\right|\right] \quad \binom{r_0\neq 0}{u:\text{large}} \text{(IV-b)}$$

From the approximate function derived above, the following general characteristics are obtained.

(1) Fundamental angular dependence of the peak envelope is in the wide angle region is

$$-30\log\theta, \quad (\theta \text{ in degrees})$$

(2) The peak envelope has uniform factor corresponding to the normalized aperture edge level (pedestal level)

$$20\log f(1)$$

Therefore, if the edge level is decreased, the peak envelope is decreased by the same amount uniformly (in the case without center blocking).

(3) In the near axis region peak envelope is raised by the factor

$$f'(1)/u \quad \text{or} \quad f''(1)/u^2$$

corresponding the derivatives of the distribution function.

Therefore, in order to improve the near axis sidelobe, small value of the derivative is effective.

4. Comparison with the Direct Integration

Fig.2 to Fig 6 show the comparison of approximation function with the result from the direct integration Eq.(1) for various distribution.

Fig. 7 shows the case for actual aperture distribution calculated for the Cassegrain antenna including the subreflector scattering effect. In this case the peak envelope function expresses the outline of the actual sidelobe.

5. Conclusion

The peak envelope approximate function is derived analytically. For analytically expressed distribution function, the envelope function coincidents well with the peak envelope calculated by the direct integration, except the first sidelobe. For the actual distribution, the envelope function well expresses the outline of the actual peak envelope.

The peak envelope approximate function presented here will provide the simplified means for the estimation of the sidelobe performance and the measure for the design of the aperture antenna.

Reference

(1)T.T.Taylor:"Design of circular apertures for narrow beamwidth and low sidelobes",IRE Trans., **AP-18**,1,pp.17-22 (Jan.1960)

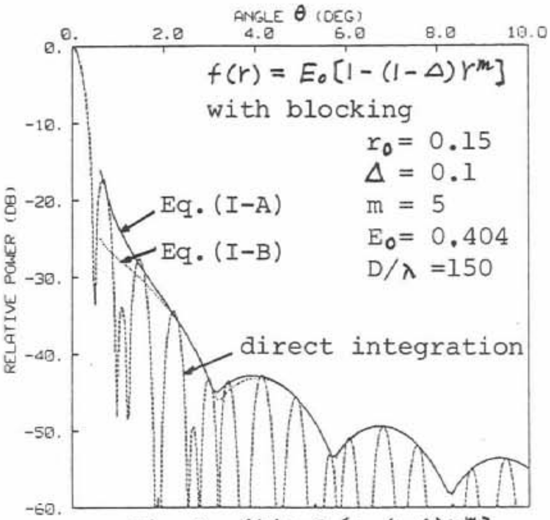

Fig. 2 $f(r) = E_0[1-(1-\Delta)r^m]$

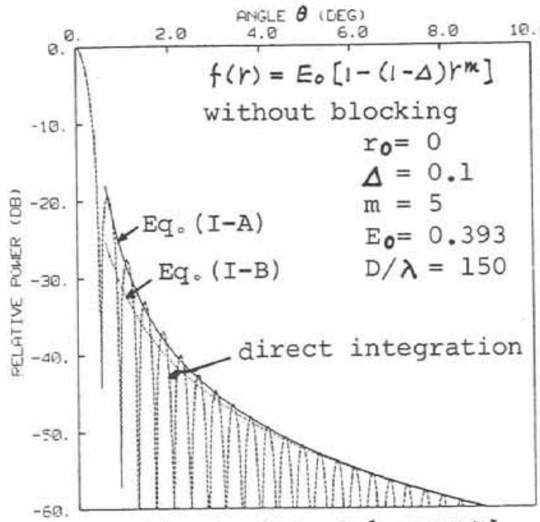

Fig. 3 $f(r) = E_0[1-(1-\Delta)r^m]$

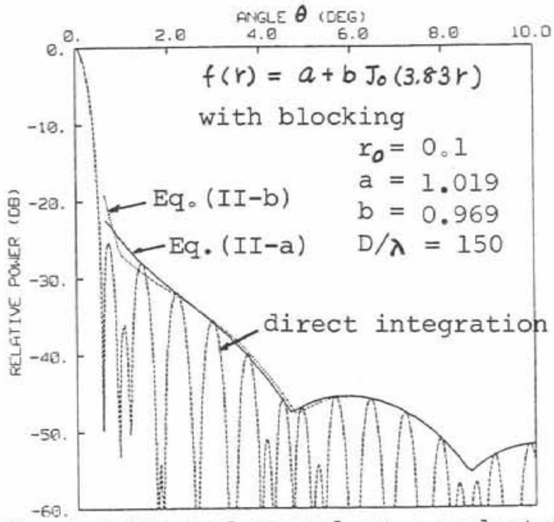

Fig.4 Inflected Bessel on a pedestal

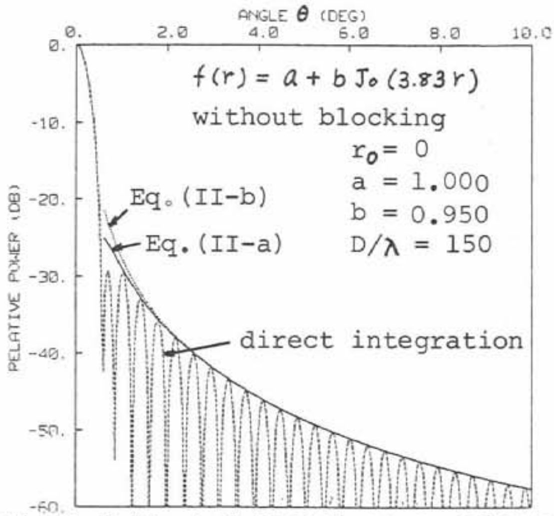

Fig. 5 Inflected Bessel on a pedestal

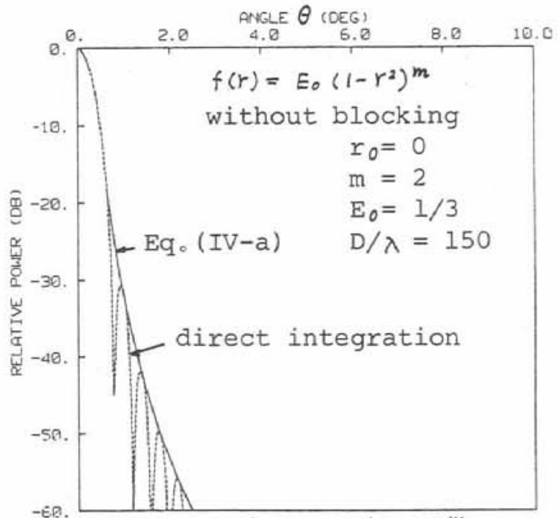

Fig. 6 $f(r) = E_0(1-r^2)^m$

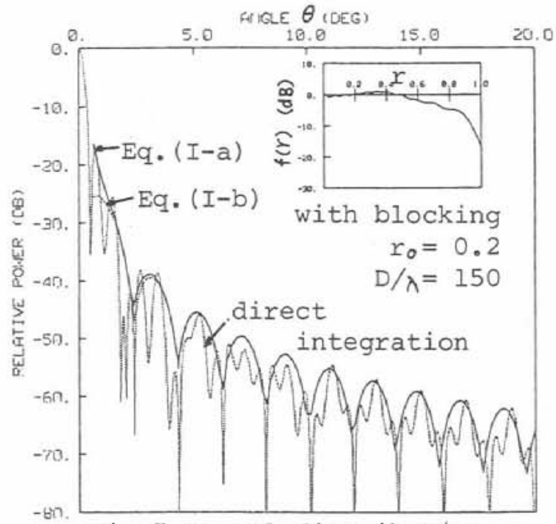

Fig. 7 Actual distribution