# TLM MODELLING OF ELECTROMAGNETIC SCATTERING IN FORWARD AND INVERSE TIME SEQUENCE 

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#### Abstract

This paper reports on recent progress made in the TLM modeling of electromagnetic scattering, particularly on the possibility to reverse the time sequence of the TLM process and to reconstruct the geometry of a scatterer from its frequency domain response by alternate forward and backward simulation of the scattering process. Time domain synthesis of a simple metallic scatterer will be presented as an example.


## 1 Introduction

Like all other time domain numerical methods, the TLM method has been used exclusively in the past to model physically realistic processes, i.e. processes that evolve forward in time. However, unlike real experiments, computer simulations can be reversed in time without any difficulty, allowing reverse scattering to be performed in the time domain. The principles of inverse scattering based on time reversal have recently been discussed by Sorrentino et al. [1]. In this paper, we will describe how this new concept can be used to synthesize the geometry of a scatterer from its known frequency domain behaviour, and apply it to a simple example to demonstrate the procedure.

## 2 TLM Algorithm Reversed in Time

The Transmission Line Matrix (TLM) algorithm was first introduced by Johns and Beurle [2]. Several schemes for 2D and 3D space discretization have been developed as described in [3]. While 3D modeling is required for solving the majority of realistic problems, 2 D situations are easier to present on paper and to execute on small and medium-sized computers. The application presented here is therefore two-dimensional, i.e. typical for TE or TM-type field problems. However, all principles and conclusions apply equally to the 3D TLM procedures.

The well-known two-dimensional TLM shunt node is represented schematically in Fig. 1a, while the three-dimensional symmetrical condensed node is shown in Fig. 1b.

The impulse scattering matrix of the 2D shunt node in Fig. 1a relates the voltage impulses $V_{n}^{r}$ emerging from the node at time step $k$, to the voltages $V_{n}^{i}$ incident on each line at the previous time step $k-1$ :


Fig. 1 TLM discretization schemes (a) Two-dimensional shunt node (b) Three-dimensional symmetrical condensed node

$$
\left[\begin{array}{l}
V_{1}  \tag{1}\\
V_{2} \\
V_{3} \\
V_{4}
\end{array}\right]^{r}=\frac{1}{2}\left[\begin{array}{cccc}
-1 & 1 & 1 & 1 \\
1 & -1 & 1 & 1 \\
1 & 1 & -1 & 1 \\
1 & 1 & 1 & -1
\end{array}\right]_{k-1}\left[\begin{array}{l}
V_{1} \\
V_{2} \\
V_{3} \\
V_{4}
\end{array}\right]^{i}
$$

However, this scattering matrix is identical to its inverse, so that we can also write

$$
\left[\begin{array}{l}
V_{1}  \tag{2}\\
V_{2} \\
V_{3} \\
V_{4}
\end{array}\right]^{i}=\frac{1}{2}\left[\begin{array}{cccc}
-1 & 1 & 1 & 1 \\
1 & -1 & 1 & 1 \\
1 & 1 & -1 & 1 \\
1 & 1 & 1 & -1
\end{array}\right]_{k} \quad\left[\begin{array}{l}
V_{1} \\
V_{2} \\
V_{3} \\
V_{4}
\end{array}\right]^{r}
$$

This means that the time direction of the process can be reversed without changing the scattering algorithm. Thus, one can easily retrace the time evolution of a scattering process by running TLM backwards in time, starting from a known field solution.

## 3 Technique of Geometrical Reconstruction by Time Reversal

The essence of the time reversal synthesis will be explained and demonstrated for the reconstruction of a simple inductive discontinuity in a parallel plate waveguide (conducting septum). This configuration can propagate a TEM wave, and possesses a complete spectrum of well-defined high order modes.

Fig. 2 shows the parallel plate waveguide without (a) and with (b) the obstacle. Both structures are terminated at both ends by an absorbing boundary and excited in the linear excitation zone by a Gaussian impulse, a signal with a limited bandwidth such that the travelling waveform is not noticeably distorted due to velocity dispersion. The following procedure consists of three stages :


Fig. 2 Arrangement for computing the homogeneous (a) and the total field solution (b) in a parallel plate waveguide with a scattering obstacle. The propagation and scattering of a Gaussian pulse in these structures is shown for two instances on the right-hand side
(1) Forward analysis of the empty waveguide to obtain the homogeneous field solution in the empty waveguide, (2) forward analysis of the loaded waveguide to obtain the total field solution which is the sum of the homogeneous solution and the scattered field (particular solution, and (3) backward synthesis to reconstruct the topology of the scattering obstacle.

At every computational step of the synthesis, the absolute value of the Poynting vector is displayed at each node inside the structure according to a color map and updated if the current value is larger than the maximum previous value. From the spatial distribution of the maximum of the Poynting vector, the shape of the obstacle can easily be reconstructed, as shown in Fig. 3. However, a synthesis does not typically start with an analysis of the desired structure as in the previous demontration, but rather from a desired frequency response. However, such a frequency response usually does not contain information for frequencies considerably higher than the upper band limit of interest. For example, the specifications for an inductive iris are given only for the dominant mode, and there is no information about the distribution of energy in higher order modes. In other words, there is no information about the transversal distribution of the fields in the reference planes. Nevertheless, this information is essential for the reconstruction of the obstacle.

To obtain this information a forward TLM analysis is first performed of an obstacle having approximately the desired dimensions. These can be found in many cases from closedform expressions, given in the literature, which link the dimensions of discontinuities to their equivalent lumped element circuits. The dominant mode content of this first response is then extracted and replaced by the desired dominant mode content in the frequency domain. The modified total reponse is then converted back into the time


Fig. 3 Reconstruction of a metallic septum in a parallel plate waveguide. (a) Distribution of the maximum magnitude of the Poynting vector after the injection of the reversed impulse response. (b) Geometry and position of the septum extracted from (a).
domain and, reduced by the homogeneous response of the empty waveguide, reinjected into the computational domain in the inverse time sequence. The resulting synthesized scatterer geometry is then already closer to the desired goal. The cycle beginning with a forward analysis, followed by low frequency characteristic substitution and synthesis may be repeated again until the result is fully satisfactory. The performances of this new method depends mainly on the mesh density and the quality of the initial guess for the dimensions of the obstacle.

## Conclusion

A new technique for the time domain synthesis of microwave structures has been presented. Based on the principles introduced by Sorrentino, So and Hoefer [1], this new efficient algorithm based on the Transmission-Line Matrix method has yielded good results for simple scatterers inside a waveguide. It has been shown that the required information on higher order modes (which is not provided by the design specifications) can be generated by a forward analysis of an approximated structure. It is at this point that the designer can make a choice among several acceptable solutions. For example, an inductive obstacle could be realised in many possible forms, such as rectangular or circular posts, longitudinal or tranverse septa, irises, etc. This distinguishes design from the inverse scattering where a specific unkown scatterer must be recontructed. Future research will be aimed at the development of more general procedures for the synthesis of more complex geometries such as filters, couplers and hybrid junctions.

## References

[1] R. Sorrentino, P.P.M. So, W.J.R. Hoefer, "Numerical Microwave Synthesis By Inversion of The TLM process", in $21^{s t}$ European Microwave Conference Digest, pp 1273-1277, Stuttgart, Germany, Sept. 1991.
[2] P.B. Johns, R.L. Beurle, "Numerical Solution of 2-Dimensional Scattering Problems using a Transmission-Line Matrix", Proc. Inst. Electr. Eng., vol. 118, pp. 1203-1208. Sept. 1971.
[3] W.J.R Hoefer, "The Transmission Line Matrix (TLM) Method", Chapter 8 of "Numerical Techniques for Passive Microwave and Millimeter-Wave Structures," edited by T. Itoh, John Wiley \& Sons, New York 1989, pp. 496-591.

