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RESONANT SOLUTIONS INVOLVED IN THE INTEGRAL EQUATION APPROACH TO SCATTERING FROM DIELECTRIC CYLINDERS

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Abstract

The possibility of obtaining erroneous solutions caused by resonances involved in the integral equation approach to scattering from a dielectric cylinder is checked through numerical computation of matrix condition numbers and scattering cross sections. The scatterer treated here is a dielectric square cylinder with side-width/wavelength ratio being less than about 1.27. As a method of removing the resonant solutions, the method taking some lower order equations of the so-called "EBC method" into consideration is proposed.

I. Introduction

The problem of scattering from dielectric cylinders can be analyzed by using the surface integral equations greatly simpler than the Müller-type equations[1]-[3]. These surface integral equations are worth noticing because they are simple and fundamental. Unfortunately, however, these simple equations do not necessarily give the unique solution, that is, they may include some interior resonant solutions in certain parameter ranges[4]. Resonant solutions for the problem of scattering from conductors and some methods of elimination of their erroneous solutions were extensively discussed by Mittra and Klein[5]. As for the dielectric cylinders, the possibility of appearance of resonant solutions and the removal of these solutions were briefly mentioned by the author[9].

This paper presents the more detailed data of condition numbers and scattering cross sections for the example of a dielectric square cylinder, in order to make clear the degree of unstable situations and the parameter ranges of their occurrence. The method extending the boundary conditions into some interior points can eliminate the resonant solutions. In the past methods, however, the location of these points was able to be arbitrary except that these points must not be on nodal lines of a modal field. This makes the above method somewhat awkward since the arbitrariness causes the vagueness. A method having no such vagueness is proposed in this paper.

II. Theory

Here, the explanation of the problem is made only for the case of transverse magnetic(TM) excitation. When a TM plane wave

$$E_z^{inc} = e^{-jk_0 x} \qquad (1)$$

is incident from the negative x axis upon a homogeneous dielectric cylinder with κ as refractive index, the following integral equations can be obtained by using the boundary conditions on the surface of the cylinder[6]-[9]:

$$\frac{1}{2}\phi = -\frac{j}{4}\int_c \left(\frac{\partial \phi}{\partial n'} \psi_d - \phi \frac{\partial \psi_d}{\partial n'} \right) dv' \tag{2}$$

$$\frac{1}{2}\phi = \phi^{inc} + \frac{j}{4}\int_c \left(\frac{\partial \phi}{\partial n'} \psi_f - \phi \frac{\partial \psi_f}{\partial n'} \right) dv' \qquad ; P \in C \tag{3}$$

where $$\psi_d = H_0^{(2)}(\kappa k_0 |\rho - \rho'|) \qquad \psi_f = H_0^{(2)}(k_0 |\rho - \rho'|) \tag{4}$$

$$\phi^{inc} = E_z^{inc} \tag{5}$$

and the time factor $\exp(j\omega t)$ is supposed. ρ and ρ' are radial vectors, P is the observation point, k_0 is the free-space wavenumber, $H_0^{(2)}(\cdot)$ is Hankel function of the second kind, zeroth order, and n is the coordinate along the outward normal to the cylinder periphery line C.

If the wavenumber k is equal to any one of eigen wavenumbers k_i of the set of integral equations obtained by setting $\phi^{inc} = 0$ in (2) and (3), the set of integral equations (2) and (3) has resonant solutions. These wavenumbers k_i are just the same as those for resonant modes in the conducting hollow cylinder, which can easily be checked up for the case of a circular cylinder. The method extending the boundary conditions is already known to be useful in eliminating the resonant solutions for the problem of conductors[5]. This idea was shown to be also effective to the problem of dielectric bodies[9]. However, we thus far felt inconvenience in the case of choosing the interior points, because more than one points are needed and there are no criterions for choosing them. Therefore, we wish to get away with only one interior point, e.g., the coordinate origin. This wish is fulfilled if we use some lower order equations of the "Extended Boundary Condition (E B C) method" as the additional equations. These EBC equations are given as[9],[10]

$$\frac{j}{4}\int_c \left[\frac{\partial \phi}{\partial n'} H_m^{(2)}(k_0\rho')e^{-jm\alpha'} - \phi \frac{\partial}{\partial n'}\{ H_m^{(2)}(k_0\rho')e^{-jm\alpha'} \} \right] dv'$$

$$= -j^{-m} \qquad ; P \in S_d , \quad m = 0, \pm 1, ---- \tag{6}$$

where ρ' and α' are the cylindrical coordinates of the integration point, and S_d denotes the interior dielectric region. The addition of only a few lower order equations works well for the purpose of obtaining the acceptable solutions, which is shown through numerical examples in the next section.

III. Numerical Computation

The numerical computation was carried out for a dielectric square cylinder illuminated by a wave normally incident to a side. The matrix condition number C_n and the scattering cross section σ_s were calculated; the former can be used to detect the possibility of appearance of ill-conditioning and the latter to check the degree of errors involved in the results. The scattering cross sections were calculated using the definition of Ref.[11]. As usual, the discretization of the integral equations was done by dividing the perimeter into equal length segments where currents are assumed to be uniform, and the integrations on each segment were performed by dividing them into five smaller segments. Some numerical results were checked to be in good agreement with those by the Müller-type equations[3].

The calculated results of σ_s and C_n (κ=2.0) for the TM case are shown in Figs. 1 and 2; the solid curves represent the direct solutions of the surface integral equations, the dashed curves represent the solutions of the overdetermined equations using the zeroth order (m=0) EBC equation as an additional equation, and the dotted curves represent the solutions of the overdetermined equations using zeroth and first order (m=0 and m=1) EBC equations as additional ones. The side width W of the square cylinder normalized by $1/k_o$ is chosen as the abscissa.

We see from these examples that the parameter ranges where the direct integral equation solutions give the erroneous results are rather narrow. This can be said also for the TE case although its figures are not shown here. Furthermore, we see that the addition of only the zeroth and first order equations works well for removing the erroneous solutions.

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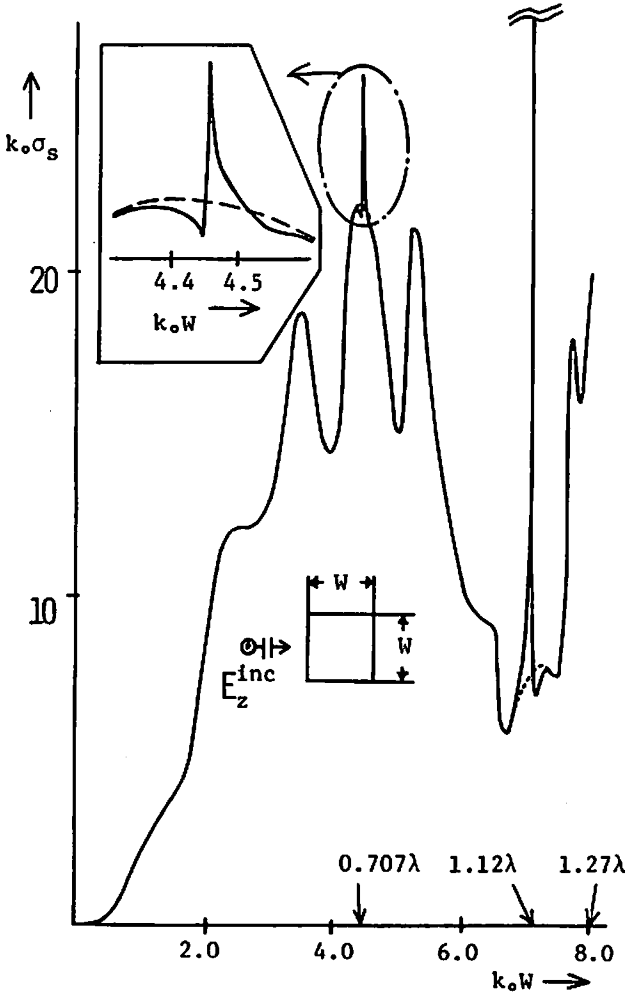

Fig.1. Scattering cross section σ_s (κ=2.0).

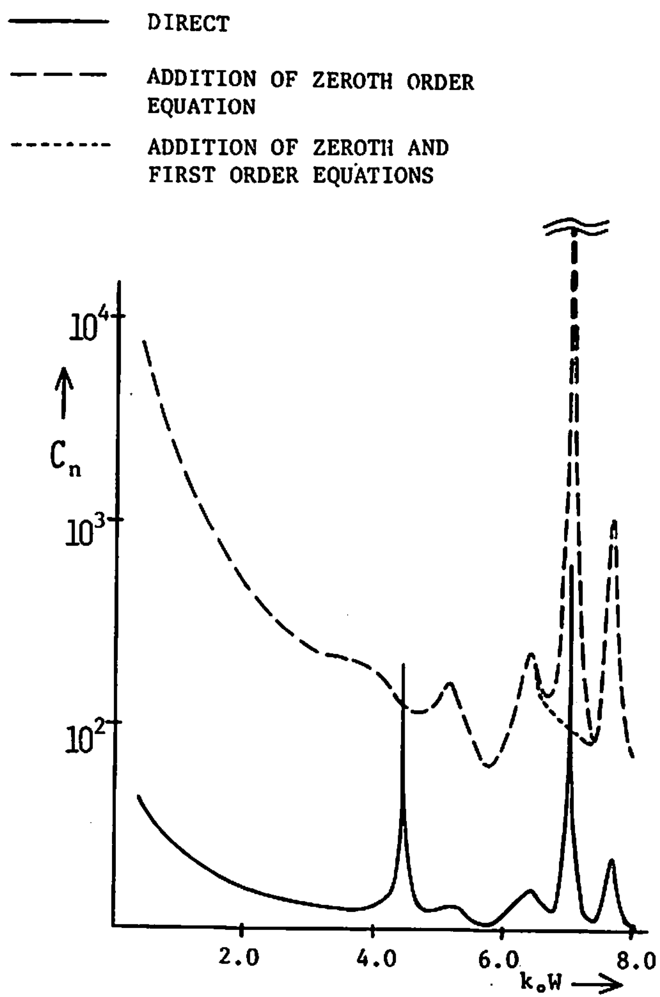

Fig.2. Matrix condition number C_n (κ=2.0).