# Modified Cauchy Distribution Model of High-Order Passive Intermodulation

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***Abstract* - In this paper, considering the statistical properties of PIM mainly, the probability density function (PDF) model - modified Cauchy distribution model is proposed for the high-order PIM. In the satellite communication system with PIM interference and noise, the modified Cauchy and Gaussian(C-G) mixture distribution extended model is developed. The proposed distribution model is proved to be rational by comparison between simulation results and calculations of PIM from various orders.**

***Index Terms* —high-order passive intermodulation, probability density function, modified Cauchy distribution**

## 1. Introduction

Passive intermodulation (PIM) interference may lead to severe degradation for the performance of satellite communication system [1][2]. A growing body of research is focusing on the behavioral model of PIM, including power-series models, Volterra-series models, double exponential function models, neural networks models, non-analytic models and hybrid models [3][4]. However, there are few analyses on the statistical properties of PIM as random signals [5].

In this paper, a modified Cauchy distribution model is employed to characterize the surface morphology of PIM interference to improve the overweight tail of Cauchy distribution. For another, owing to the inescapable effect of noise in the system, we extend the PIM distribution utilizing convolution of modified Cauchy and Gaussian distribution [6].

## 2. Modified Cauchy Distribution Model

In digital satellite communication, the nonlinearity of passive component is the primary cause to PIM products. Power series model can be used to describe the nonlinear characteristics of the passive components.

$$Y(t) = a_1 X(t) + a_2 X^2(t) + a_3 X^3(t) + \cdots + a_n X^n(t) + \cdots \quad (1)$$

where $Y(t)$ is the PIM interference, $X(t)$ is a $m$-channel downlink carrier signal, and can be expressed as

$$X(t) = \sum_{i=1}^{m} R_i(t)\cos(2\pi f_i t) \quad (2)$$

where $R_i(t)$ and $f_i$ $(i = 1, 2, \cdots, m)$ are the respective envelope and frequency of the independent carriers. When only considering the intermodulation products in the first zone, that is, only considering the results of odd order of $X(t)$, for the case of two carriers, the $n^{th}$ ($n$=2$p$+1) order PIM products falling into the receiving frequency band can be expressed as

$$Y_n(t) = R_1^{p+1} R_2^{p} \cos\left(2\pi\left((p+1)f_2 - pf_1\right)t\right) \quad (3)$$

The simulation of the probability density of the 9th order PIM interference is shown in Fig.1, irrespective of additive white Gaussian noise (AWGN).

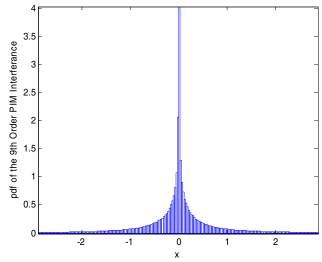


Fig. 1. the PDF of the 9th Order PIM signal

The distribution is non-Gaussian obviously. Cauchy distribution can match the curve in shape roughly. But for the heavy tail and the inexistence of expectation and variance of Cauchy distribution, Gaussian function which has light tail is used to reduce the tail weight of Cauchy distribution by multiplying the two together. The revised Cauchy distribution is expressed as follows

$$f(x) = \frac{1}{A} \cdot \frac{\lambda}{x^2 + \lambda^2} \cdot e^{\frac{-x^2}{K^2}} \quad (4)$$

where $\lambda$ is the scale parameter of Cauchy distribution, $K$ is correction parameter in Gaussian function, and $A = \pi[1 - erf(\frac{\lambda}{K})] \cdot e^{(\frac{\lambda}{K})^2}$.

Parameters $\lambda$ and $K$ are associated with the intermodulation order $N$ if normalizing PIM power. With the mean of extensive test values, the relationship between $\lambda$ and $N$, $K$ and $N$ are obtained by curve-fitting technique shown in Fig.2. They can be respectively expressed by exponential approximation as (5) and (6).

$$\lambda = 0.62 \times e^{-0.13N} \quad (5)$$

$$K = 2.6 \times e^{0.12N} \quad (6)$$

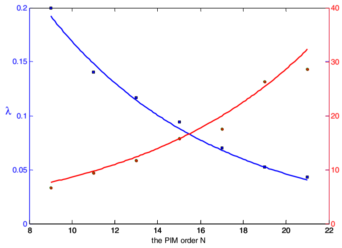


Fig. 2. The relationship between $\lambda$ and $N$, $K$ and $N$

The PDF result of the 9th order PIM interference by using (4) is shown in Fig.3 with logarithm scale for y-axis. It is observed that the outcome of model fitting is in accord with the simulation result with minor error particularly in the tail we concerned about.

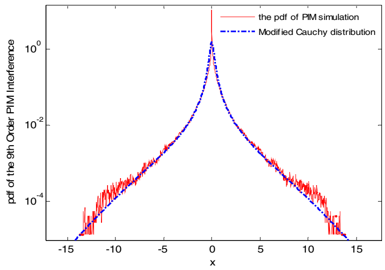


Fig. 3. Simulated PDF of the 9th order PIM vs. modified Cauchy distribution

## 3. Properties of Modified Cauchy Distribution

For the PDF $f_N(x)$ of the Nth order PIM, it has the same fundamental characteristics as other probability density functions

$$\begin{cases} f_N(x) \geq 0 \\ F_N(\infty) = \int_{-\infty}^{\infty} f_N(x)dx = \frac{1}{A}\int_{-\infty}^{\infty} e^{\frac{-x^2}{K}} \frac{\lambda}{x^2+\lambda^2}dx = 1 \end{cases} \quad (7)$$

Different from the Cauchy distribution, the expectation and variance exist in this modified Cauchy distribution model as follows

$$E_N(x) = \int_{-\infty}^{\infty} x \cdot f_N(x)dx = \frac{1}{A}\int_{-\infty}^{\infty} e^{\frac{-x^2}{K^2}} \frac{\lambda x}{x^2+\lambda^2}dx = 0 \quad (8)$$

$$D_N(X) = \frac{1}{A}\int_{-\infty}^{\infty} e^{\frac{-x^2}{K^2}} \frac{\lambda x^2}{x^2+\lambda^2}dx = \frac{\lambda K}{\sqrt{\pi}[1-erf(\frac{\lambda}{K})] \cdot e^{(\frac{\lambda}{K})^2}} - \lambda^2 \quad (9)$$

## 4. Extended Modified C-G Mixture Distribution

Due to noises existing in the communication system, we assume that the model is described as $Z = X + Y$, where $X$ is the PIM interference that obeys the modified Cauchy distribution, and $Y$ is AWGN that obeys Gaussian distribution. The modified C-G mixture distribution model is educed from the model. Assuming that $X$ and $Y$ are independent of each other, then the PDF of $Z$ is derived as

$$f_{C-G}(z) = \pi \cdot \frac{\mathrm{Re}\left[ e^{-(\frac{z+i\beta}{\alpha})}(1+\frac{2i}{\sqrt{\pi}}\int_0^{\frac{z+i\beta}{\alpha}} e^{t^2}dt)\right] e^{\frac{-z^2}{\gamma}}}{A \cdot \sqrt{2\pi\sigma}} \quad (10)$$

where $\alpha = \frac{\sqrt{2}\sigma\sqrt{2\sigma^2+K^2}}{K}$, $\beta = \lambda \cdot \frac{2\sigma^2+K^2}{K^2}$, $\gamma = 2\sigma^2+K^2$.

The PDF result of modified C-G mixture distribution by using (10) is shown in Fig.4, agreed with the simulated PDF of the 9th Order PIM with AWGN quite well.

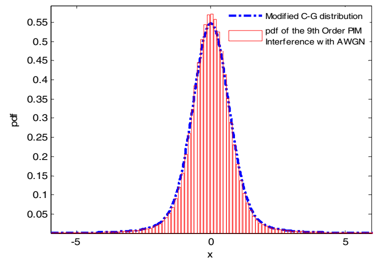


Fig. 4. Simulated PDF of the 9th Order PIM with AWGN vs. Modified C-G mixture distribution

## 5. Conclusion

In this paper, a modified Cauchy distribution model multiplied by a Gaussian function is put forward to describe the statistical distribution characteristics of PIM interference. The modified Cauchy distribution model can improve the overweight tail and the inexistence of expectation and variance of Cauchy distribution. Also the parameters in this distribution are acquired by exponential fitting. Numerical simulation results shows that the proposed model obtains a favorable effect for distribution fitting.

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