SCATTERING OF ELECTROMAGNETIC FIELDS BY ARBITRARILY ORIENTED TWO PARALLEL STRIPS

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I. INTRODUCTION

Multiple scattering of electromagnetic fields by cylinders and spheres has been studied by numerous authors. The problem of the multiple scattering by two or many strips is also interesting and of importance for many applications. Walker W.A. and Butler C.M.[1] have developed a method for calculating the scattered fields from large arrays of narrow strips. More recently, Aoki K. et al.[2] and Iwashige J.[3] have studied the scattering by two staggered parallel strips by using the Wiener-Hopf technique and the method of the GTD respectively.

In this paper, the electromagnetic fields scattered by arbitrarily oriented two parallel conducting strips are analyzed by using the circular-harmonics expansion. Numerical results are given for the scattered far-field patterns.

II. ANALYSIS

Let us consider arbitrarily oriented two parallel conducting strips as shown in Fig.1. In the following formulation, a+b<d is assumed and time factor exp($i\omega t$) is suppressed throughout. The E-polarized incident plane wave is given by

$$E_z^{inc} = e^{ik\rho\cos(\phi-\phi_0)} \tag{1}$$

where $k=\sqrt{\varepsilon_0\mu_0} = 2\pi/\lambda$ (λ: wavelength). We define two circular cylindrical coordinate systems (ρ_1,ϕ_1,z) and (ρ_2,ϕ_2,z) as shown in Fig.1. Equation (1) can be expanded in these two coordinate systems as follows:

$$E_z^{inc} = \Phi_1 \sum_{n=-\infty}^{\infty} J_n(k\rho_1)\, e^{in(\phi_1-\phi_0)} = \Phi_2 \sum_{n=-\infty}^{\infty} J_n(k\rho_2)\, e^{in(\phi_2-\phi_0)} \tag{2}$$

where

$$\Phi_{\frac{1}{2}} = e^{\pm i(kd/2)\cos\phi_0} \tag{3}$$

and J_n is the Bessel function of the n-th order.

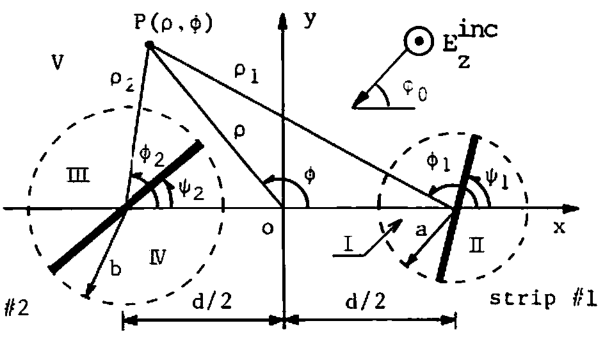

Fig.1. Geometry of the problem.

For convenience, let us define five regions I, II, III, IV and V as shown in Fig.1. The total field in each region is expressed as follows:

Region I $(\rho_1 \leqq a , \psi_1 \leqq \phi_1 \leqq \pi+\psi_1)$

$$E_z^{I} = \sum_{n=1}^{\infty} A_n^1 J_n(k\rho_1) \sin n(\phi_1 - \psi_1) \tag{4}$$

Region II $(\rho_1 \leqq a , -\pi+\psi_1 \leqq \phi_1 \leqq \psi_1)$

$$E_z^{II} = \sum_{n=1}^{\infty} B_n^1 J_n(k\rho_1) \sin n(\phi_1 - \psi_1) \tag{5}$$

Region III $(\rho_2 \leqq b , \psi_2 \leqq \phi_2 \leqq \pi \psi_2)$

$$E_z^{III} = \sum_{n=1}^{\infty} A_n^2 J_n(k\rho_2) \sin n(\phi_2 - \psi_2) \tag{6}$$

Region IV $(\rho_2 \leqq b , -\pi+\psi_2 \leqq \phi_2 \leqq \psi_2)$

$$E_z^{IV} = \sum_{n=1}^{\infty} B_n^2 J_n(k\rho_2) \sin n(\phi_2 - \psi_2) \tag{7}$$

Region V $(\rho_1 \geqq a , \rho_2 \geqq b)$

$$E_z^{V} = E_z^{s1} + E_z^{s2} + E_z^{inc} \tag{8}$$

where

$$E_z^{s1} = \sum_{n=-\infty}^{\infty} C_n^1 H_n^{(2)}(k\rho_1) e^{in\phi_1} , \quad E_z^{s2} = \sum_{n=-\infty}^{\infty} C_n^2 H_n^{(2)}(k\rho_2) e^{in\phi_2} \tag{9}$$

are the scattered fields from the strips #1 and #2, respectively, in which the multiple scattering effect is included, and A_n^1 , B_n^1 , A_n^2 , B_n^2 , C_n^1 and C_n^2 are unknown coefficients, and $H_n^{(2)}$ is the Hankel function of the second kind of n-th order. By means of the addition theorem of Bessel function, E_z^{s1} and E_z^{s2} can be expressed in another coordinate system as follows:

$$E_z^{s1} = \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} C_n^1 H_{m-n}^{(2)} (kd) J_m(k\rho_2) e^{im\phi_2} \tag{10}$$

$$E_z^{s2} = \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} C_n^2 H_{n-m}^{(2)} (kd) J_m(k\rho_1) e^{im\phi_1} . \tag{11}$$

Thus, in view of Eqs.(2),(8),(9),(10) and (11), we have two expressions for total field E_z^V in either coordinate system. The continuity conditions of electromagnetic fields on the boundaries ρ_1=a and ρ_2=b together with the orthogonality of the angular functions $\{\sin n(\phi_1-\psi_1)\}$ and $\{\sin n(\phi_2-\psi_2)\}$ lead to the infinite simultaneous equations to determine the unknown coefficients C_n^1 and C_n^2 . The matrix form of these equations is expressed as follows:

$$\begin{pmatrix} \mathbf{M}_{11} & \mathbf{M}_{12} \\ \mathbf{M}_{21} & \mathbf{M}_{22} \end{pmatrix} \begin{pmatrix} \mathbf{C}^1 \\ \mathbf{C}^2 \end{pmatrix} = \begin{pmatrix} \mathbf{D}_1 \\ \mathbf{D}_2 \end{pmatrix} \tag{12}$$

where $\mathbf{M}_{ij}$ (i,j=1,2) is the infinite known matrix, $\mathbf{D}_1$ and $\mathbf{D}_2$ are the infinite known column vectors and $\mathbf{C}^1$ and $\mathbf{C}^2$ are the infinite row vectors whose n-th elements are C_n^1 and C_n^2 respectively. Equation (12) can be solved numerically either by a direct matrix calculation or by an iterative process. In the latter case, multiple scattering can be evaluated iteratively. If the vectors $\mathbf{C}^1$ and $\mathbf{C}^2$ can be expressed as follows:

$$\mathbf{C}^1 = \sum_{\ell=1}^{\infty} {}_{\ell}\mathbf{C}^1 , \qquad \mathbf{C}^2 = \sum_{\ell=1}^{\infty} {}_{\ell}\mathbf{C}^2 \tag{13}$$

where ℓ is the iteration number, and ${}_{\ell}\mathbf{C}^1$ and ${}_{\ell}\mathbf{C}^2$ are the row vectors whose n-th elements are ${}_{\ell}C_n^1$ and ${}_{\ell}C_n^2$ respectively, then ${}_{\ell}\mathbf{C}^1$ and ${}_{\ell}\mathbf{C}^2$ can be determined iteratively by solving the following equations

$$\left.\begin{array}{lll} \mathbf{M}_{11}\,{}_{1}\mathbf{C}^1 = \mathbf{D}_1 & & \\ \mathbf{M}_{11}\,{}_{\ell}\mathbf{C}^1 = -\mathbf{M}_{12}\,{}_{\ell-1}\mathbf{C}^2 & & (\ell=2,3,4,\cdots) \\ \mathbf{M}_{22}\,{}_{1}\mathbf{C}^2 = \mathbf{D}_2 & & \\ \mathbf{M}_{22}\,{}_{\ell}\mathbf{C}^2 = -\mathbf{M}_{21}\,{}_{\ell-1}\mathbf{C}^1 & & (\ell=2,3,4,\cdots) \end{array}\right\} . \tag{14}$$

III. NUMERICAL EXAMPLES

Figure 2 shows the scattered far-field pattern. The broken line denotes the result obtained from the Wiener-Hopf method[2]. The agreement between them is good. Figure 3 shows the scattered far-field pattern when the multiple scattering effect is large. The broken line denotes the single scattering field (multiple scattering effect is neglected). this result shows the multiple scattering effect clearly.

IV. CONCLUSION

The electromagnetic fields scattered by arbitrarily oriented two parallel conducting strips are analyzed by using the circular-harmonics expansion and some numerical examples are given. The present method and results will be useful for the analysis of the multiple scattering by many strips such as strip grating of finite elements. The present method is also applicable to the problem of a cylindrical and beam wave incidence.

REFERENCES

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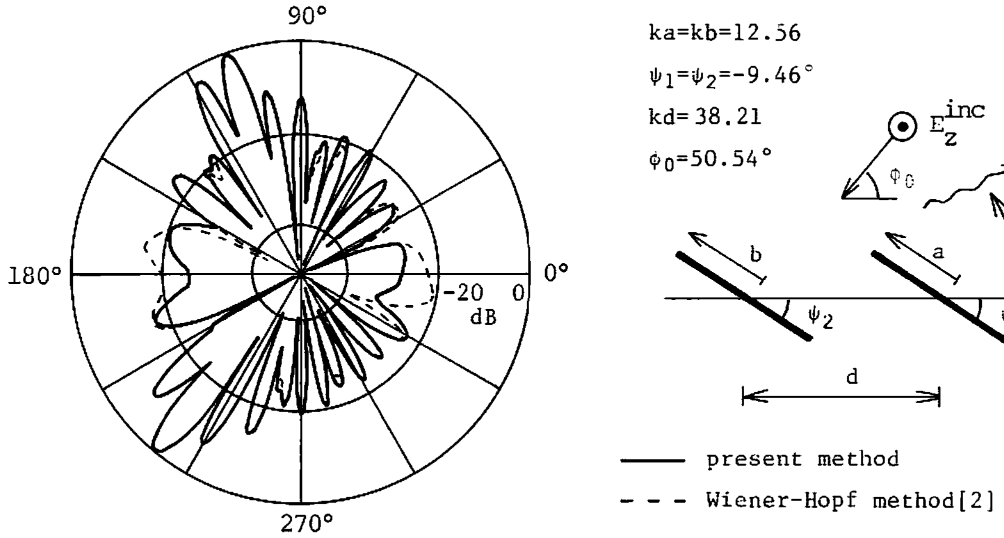

Fig.2. Scattered far-field pattern.

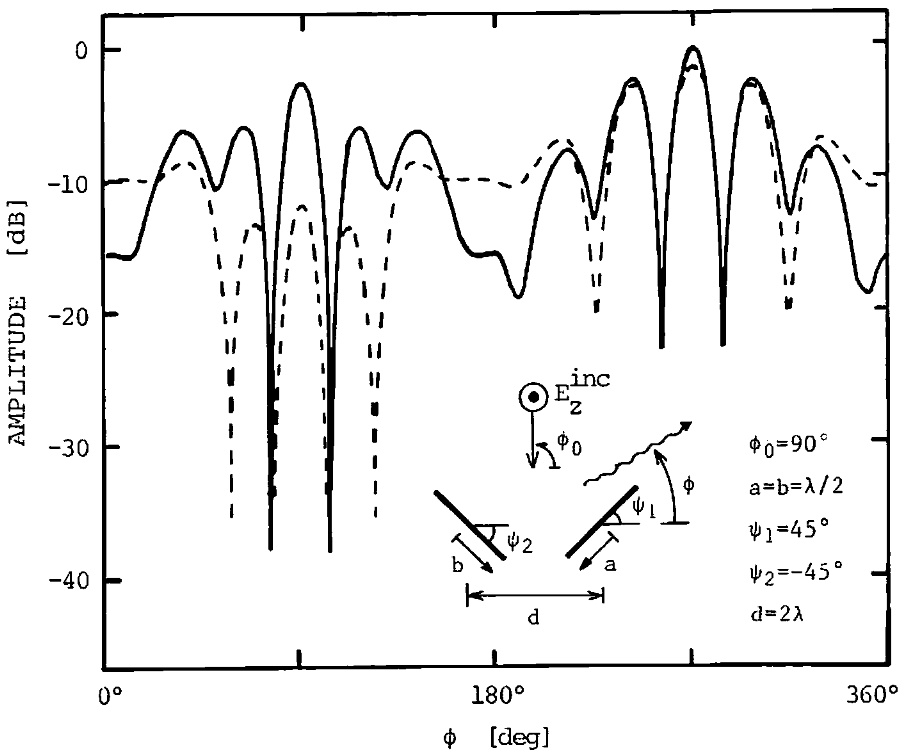

Fig.3. Scattered far-field pattern.
——— multiple scattering calculation
- - - - single scattering calculation