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A METHOD OF SOLUTION FOR THE DIFFRACTED FIELD BY N PARALLEL SLIT ARRAY

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A method of deriving the solution for the diffracted field of electromagnetic plane wave by N parallel slit array is presented. Starting with the formulation which are derived by applying the Weber-Schafheitlin discontinuos integrals[4], the determinantal equations for the expansion coefficients of the diffracted field are derived in a matrix form. The matrix equation may be solved by the iterative method. When the spacing d between the slits is large compared with wavelength, the diffracted field could be expressed in inverse powers of d. In a special case of two slits, the present result is found to agree to order $(kd)^{-3/2}$ with that derived by the method of Zitron and Karp[1] (the ZK method). The method of solution is readily extended to cylindrical wave excitation. Numerical results of the diffracted patterns for two strips is also presented.

Consider N parallel infinite slits in a thin conducting sheet. The width of the I-th slit is $2a_I$ and the separation between the centers of the I-th and the J-th slits is d_{JI}. The expressions for the incident wave, the reflected wave and the diffracted wave in H-polarization are given by;

$$H_z^{(i)} = \exp[jk(x\cos\theta_0 + y\sin\theta_0)] \qquad (1a)$$

$$H_z^{(r)} = \exp[jk(x\cos\theta_0 - y\sin\theta_0)] \qquad (1b)$$

$$H_z^{(d)} = \sum_{I=1}^{N} H_z^{(d)}(I)$$

$$H_z^{(d)}(I) = \sum_{m=0}^{\infty} \int_0^{\infty} \exp[\mp\sqrt{\xi^2 - K_I^2}\, v_I] \cdot \left\{ A_m^{(I)} \frac{J_{2m}(\xi)}{\sqrt{\xi^2 - K_I^2}} \cos\xi u_I + B_m^{(I)} \frac{J_{2m+1}(\xi)}{\sqrt{\xi^2 - K_I^2}} \sin\xi u_I \right\} \qquad (1c)$$

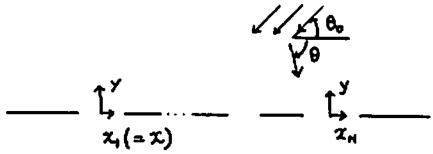

Fig. 1 N slit array

where (x,y) and (x_I, y) are the rectangular coordinates whose origins are located at the centers of the first and the I-th slits, respectively, θ_0 is the incident angle, $u_I = x_I/a_I$, $v_I = y/a_I$ are the normalized variables, $K_I = ka_I$ is the normalized width of the I-th slit. $A_m^{(I)}$ and $B_m^{(I)}$ are the expansion coefficients for the diffracted field and they are to be determined from the boundary conditions. It is worthwhile to note that the expressions for $\partial H_z^{(d)}(I)/\partial v$ ($\sim E_x^d(I)$) reduce to Weber-Schafheitlin discontinuous integrals (WSDI) on the plane v=0 and they vanish on the conducting parts of the N slit array. This is main advantage of using the method of the WSDI. The expression for $H_z^{(d)}$ may be changed to simpler form after some manipulation

$$H_z^{(d)}(I) = \frac{K_I}{4j} \sum_{m=0}^{\infty} \int_0^{2\pi} \{ A_m^{(I)} e^{j2m\theta} + jB_m^{(I)} e^{j(2m+1)\theta} \} H_0^{(2)}(K_I\sigma)\, d\theta \qquad (2)$$

where $\sigma = \sqrt{(u+\sin\theta)^2 + v^2}$ and $H_0^{(2)}(x)$ is Hankel function of zero order. The expression (2) is newly derived here and seems to be convenient for calculating the near field by applying the Fast Fourier Transform since the integrals with respect to θ represent the expansion coefficients of Fourier series of $H_0^{(2)}(K_I\sigma)$. The far field expressions for $H_z^{(d)}(I)$ is calculated readily from eq.(2) by applying the saddle point method of integration and the result is

$$H_z^{d}(I) = C(k\rho_I)\, F(K_I, \theta_0, \theta_I) \qquad (3a)$$

$$F(\kappa_I,\theta_0,\theta)=\sum_{m=0}^{\infty}\left\{A_m^{(I)}J_{2m}(\kappa_I\cos\theta_I)+jB_m^{(I)}J_{2m+1}(\kappa_I\cos\theta_I)\right\} \tag{3b}$$

where $C(k\rho_I)=\sqrt{(2/\pi k\rho_I)}\,e^{-j(k\rho_I+\pi/4)}$ represents a cylindrical wave, $F(\kappa_I,\theta_0,\theta)$ is the pattern function for the diffracted wave, and (ρ_I,θ_I) are the cylindrical coordinates of observation point with respect to the center of the I-th slit. The expansion coefficients $A_m^{(I)}$ and $B_m^{(I)}$ are determined from the following matrix equations;

$$\sum_{m=0}^{\infty}A_m^{(I)}G(2m,2n;\kappa_I)=J_{2n}(\kappa_I\cos\theta_0)e^{jkd_{I1}\cos\theta_0}$$
$$-\sum_{J=1}^{N}{}'\sum_{m=0}^{\infty}\left[A_m^{(J)}GC(2m,2n;\delta_{JI})+B_m^{(J)}GS(2m+1,2n;\delta_{JI})\right] \tag{4a}$$

$$\sum_{m=0}^{\infty}B_m^{(I)}G(2m+1,2n+1;\kappa_I)=jJ_{2n+1}(\kappa_I\cos\theta_0)e^{jkd_{I1}\cos\theta_0}$$
$$-\sum_{J=1}^{N}{}'\sum_{m=0}^{\infty}\left[-A_m^{(J)}GS(2m,2n+1;\delta_{JI})+B_m^{(J)}GC(2m+1,2n+1;\delta_{JI})\right] \tag{4b}$$

$I=1,2,3,\ldots N.$

where the prime in Σ' means $J\neq I$ and $\delta_{JI}=d_{JI}/a_J$, and d_{I1} is the separation between the first and the I-th slits. In the above equations the function G, GC and GS, which give the matrix elements, are given by the integrals;

$$G(m,n;\kappa_I)=\int_0^{\infty}\frac{1}{\sqrt{\xi^2-\kappa_I^2}}J_m(\xi)J_n(\xi)\,d\xi \tag{5a}$$

$$GC(m,n;\delta_{JI})=\int_0^{\infty}\frac{1}{\sqrt{\xi^2-\kappa_J^2}}J_m(\xi)J_n\left(\frac{a_I}{a_J}\xi\right)\cos(\delta_{JI}\xi)\,d\xi \tag{5b}$$

$$GS(m,n;\delta_{JI})=\int_0^{\infty}\frac{1}{\sqrt{\xi^2-\kappa_J^2}}J_m(\xi)J_n\left(\frac{a_I}{a_J}\xi\right)\sin(\delta_{JI}\xi)\,d\xi \tag{5c}$$

As seen from the above equations, GC and GS depend on the spacing between the slits. Hence the second terms of right hand side of eqs.(4) may be considered to represent the mutual interaction between the slits. Functions G, GC and GS may be expressed in terms of power series of κ_I or κ_J , and GC and GS may also be represented in the inverse power series of the spacing δ_{JI}. The asymptotic expansion for GC is given by[6]

$$GC(m,n;\delta_{JI})=\frac{1}{2}\int_C g(\kappa_J,\kappa_I,\alpha)e^{jkd_{JI}\cos\alpha}\,d\alpha$$
$$=\frac{1}{2}C(kd_{JI})\sum_{\ell=0}^{\infty}\frac{(1+4\partial^2)(9+4\partial^2)\cdots([2\ell-1]^2+4\partial^2)}{(-j8kd_{JI})^{\ell}\,\ell!}g(\kappa_J,\kappa_I,0) \tag{6}$$

where $g(\kappa_J,\kappa_I,\cos\alpha)=J_m(\kappa_J\cos\alpha)J_n(\kappa_I\cos\alpha)$, ∂ denotes the symbol for differentiation with respect to α and $\partial g(\kappa_J,\kappa_I,0)$ means $(\partial/\partial\alpha)g(\kappa_J,\kappa_I,\cos\alpha)|_{\alpha=0}$. Similar expression for GS may be derived. The first term of right hand side (RHS) of eq.(6) is obtained by transforming the variable ($\xi=\kappa_J\cos\alpha$) and the contour C extends from $-j\infty$ to $\pi+j\infty$. The second term of the same equation is derived in a symbolic manner using the properties that the integral representation for GC resemble to that of Hankel function of order n,[6] but it may also be derived by applying the standard saddle point method of integration. Once all the matrix elements are determined, an approximate solutions for $A_m^{(I)}$ and $B_m^{(I)}$ can be obtained as follows. The zero-th approximate solutions are determined from

$$\sum_{m=0}^{\infty}A_m^{(I)}G(2m,2n;\kappa_I)=J_{2n}(\kappa_I\cos\theta_0)e^{jkd_{I1}\cos\theta_0} \tag{7a}$$

$$\sum_{m=0}^{\infty}B_m^{(I)}G(2m+1,2n+1;\kappa_I)=jJ_{2n+1}(\kappa_I\cos\theta_0)e^{jkd_{I1}\cos\theta_0} \tag{7b}$$

by setting $A_m^{(J)}=B_m^{(J)}=0$ for $I\neq J$ in eqs.(4). Eqs.(7) are the determinantal equations for an isolated slit. The first approximate solutions are derived by substituting the zero-th order solutions into RHS of eqs.(4), and the higher order approximate solutions are derived successively. If we retain the order $(kd)^{-3/2}$ in the asymptotic solutions forGC and GS to derive the solutions for $A_m^{(I)}$ and $B_m^{(I)}$, the diffracted field for two slits is expressed as $H_z^{(d)}=H_z^{(d)}(1)+H_z^{(d)}(2)$, where

$$\begin{aligned} H_z^{(d)}(1) = C(k\rho)^{-1}\cdot\Big[& F(\kappa_2,\theta_0,\pi)F(\kappa_1,0,\theta)e^{jkd\cos\theta_0} - F(\kappa_1,\theta_0,\theta)\cdot P \\ & + \frac{C(kd)}{j2kd}\Big\{F(\kappa_1,0,\theta)(D_\theta^2+\tfrac{1}{4})F(\kappa_2,\theta_0,\pi) + 2D_\theta F(\kappa_2,\theta_0,\pi)D_{\theta_0}F(\kappa_1,0,\theta) \\ & \qquad + F(\kappa_2,\theta_0,\pi)D_{\theta_0}^2 F(\kappa_1,0,\theta)\Big\}e^{jkd\cos\theta_0} \\ & + C(kd)^2 F(\kappa_1,\theta_0,0)F(\kappa_2,\pi,\pi)F(\kappa_1,0,\theta)\cdot P \\ & - C(kd)^3 F(\kappa_2,\theta_0,\pi)F(\kappa_1,0,0)F(\kappa_2,\pi,\pi)F(\kappa_1,0,\theta)e^{jkd\cos\theta_0}\Big] \end{aligned} \quad (8)$$

and P=1. The expression for $H_z^{(d)}(2)$ are obtained if we make replacements $\kappa_1\to\kappa_2$, $\kappa_2\to\kappa_1$, $\pi\leftrightarrows 0$, $\theta_0\leftrightarrows\theta$, $P=\exp(jkd(\cos\theta_0+\cos\theta))$ in the expression for $H_z^{(d)}(1)$. In eq.(8), $D_\theta F(\kappa,\theta_0,0)$ means $(\partial/\partial\theta)F(\kappa,\theta_0,\theta)|_{\theta=0}$, and similarly for $D_{\theta_0}F(\kappa,0,\theta)$. The above expressions completely agree with those derived by applying the ZK method. It is noted that eq.(8) may be considered to take into account changes in the curvature of the wave front of the diffracted wave through the derivatives of the pattern functions.

The procedure of deriving the solutions for the diffracted fields in various situations of N slits may be summerized as follows;

(a) When the spacing between the adjacent slits is large compared with wavelength, the asymptotic solutions (6) for GC and GS may be used, but if the spacing is rather small, the series representations for GC and GS should be used. The series may be derived in a similar manner as in reference (5). The expansion coefficients $A_m^{(I)}$ and $BB_m^{(I)}$ are obtained from eqs.(4) by the iterative method.

(b) Once the values of $A_m^{(I)}$ and $B_m^{(I)}$ are determined, the far diffracted field patterns and the near field patterns are calculated from eq.(3) and eq.(2), respectively.

(c) When the spacing between the adjacent slits is large, the diffracted field for two slits may be calculated from simple expression (8) which is valid to order $(kd)^{-3/2}$. Similar expression for N slits could be derived, but it seems to be more convenient to use the procedure (a).

(d) The response for line magnetic source $M=M_0\,\delta(x-x_0)\cdot\delta(y-y_0)$, instead of plane wave incidence, can be obtained readily if eqs.(4) are replaced by

$$\begin{aligned} \sum_{m=0}^{\infty} A_m^{(I)} G(2m,2n;\kappa_I) = & \frac{j\omega\varepsilon_0 M_0}{2\pi a_I}\int_0^\infty \frac{J_{2n}(\xi)}{\sqrt{\xi^2-\kappa_I^2}}\cos\xi(\delta_{I1}+u_0)\exp[-\sqrt{\xi^2-\kappa_I^2}\,v_0]\,d\xi \\ & -\sum_{J=1}^{N}{}'\sum_{m=0}^{\infty}\left[A_m^{(J)}GC(2m,2n;\delta_{JI}) + B_m^{(J)}GS(2m+1,2n;\delta_{JI})\right] \end{aligned} \quad (9a)$$

$$\begin{aligned} \sum_{m=0}^{\infty} B_m^{(I)} G(2m+1,2n+1;\kappa_I) = & \frac{j\omega\varepsilon_0 M_0}{2\pi a_I}\int_0^\infty \frac{J_{2n+1}(\xi)}{\sqrt{\xi^2-\kappa_I^2}}\sin\xi(\delta_{I1}+u_0)\exp[-\sqrt{\xi^2-\kappa_I^2}\,v_0]\,d\xi \\ & -\sum_{J=1}^{N}{}'\sum_{m=0}^{\infty}\left[-A_m^{(J)}GS(2m,2n+1;\delta_{JI}) + B_m^{(J)}GC(2m+1,2n+1;\delta_{JI})\right] \end{aligned} \quad (9b)$$

where $u_0=x_0/a_I$, $\delta_{I1}=d_{I1}/a_I$ and the integral with respect to ξ can be obtained by numerical method.

As a numerical example we show in Fig.2 the diffracted patterns by two strips, which are in a complementary relation with two slits. The solid lines represent the results calculated from eq.(8),which is equivalent to ones by the ZK method. The broken lines denote the GTD (Geometrical Theory of Diffraction) solutions which neglect the mutual interaction between the edges of the strips. The dotted lines are experimental results given by Deacetis and Lager together with the GTD solutions.[3] The agreement among three methods is fairly good. The advantages of the asymptotic solutions given in eq.(8) to the GTD solutions are; (a) the solutions give the finite values in any observation points, whereas GTD solution sometimes become infinity at the particular observation points[3]: (b) the only restriction to this method is that the spacing between the strips is large compared with the operating wavelength, on the other hand GTD is required that both the spacing and width of the strip are large.

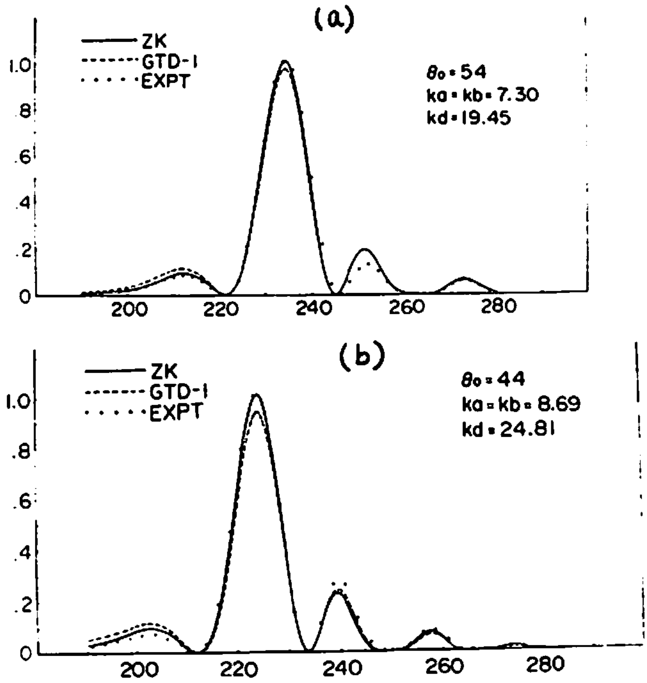

Fig.2 Diffracted patterns of plane wave by two parallel strips.

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