SCATTERING BY A FINITE SET OF STAGGERED PARALLEL HALF-PLANES

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Introduction

The scattering of electromagnetic field by a finite set of staggered parallel half-planes is analyzed by the use of a new formulation with the mutual fields scattered from each edge. The formulation is based on the exact representation of scattered field by one of half-planes, on the superposition of prospected fields in the scattering process point of view and the continuation of total fields on the fictitious boundary, or none-physical one. Using this formulation, the Wiener-Hopf technique can be applied for solving of the scattering problem of staggered parallel half-planes. Numerical results for the scattered far-field are presented and compared with the results by the G.T.D.method without multiple scattering.

Spectral properties of field concering a half-plane

In the first step, the spectral properties of the fields which are scattered by one of half-planes should be re-checked to employ to another incident field representation for the other half-planes.

Let us consider the n-th half-plane, $\#n$ (thin, perfect conducting) extended in the negative Z axis (Fig.1). We assume that field quantities are not y dependent. The time dependence $e^{-j\omega t}$ will be omitted. The regiorous representations of scattered fields by the W-H technique are found, if the incident field $E^i(X_n,Z_n)$, the one-sided Fourier Transformation $\Phi_-^i(X_n,Z_n)$ in the negative Z axis of the local coordinate $(-l_n \geq Z_n \geq -\infty)$ of E^i and the Fourier Transformation of the scattered field $E(X_n,Z_n)$ have the following properties[3].

$$|E^i(Xn,Zn)| \leq C_1 e^{-\tau_+ Zn} \quad , \quad Zn \to -\infty \qquad (1)$$

$$|\Phi_-^i(0,\alpha)| \leq C_2 |\alpha|^{-\eta} \quad , \quad |\sigma| \to \infty \qquad (2)$$

$$|\Phi(Xn,\alpha)| \leq C_3 |\alpha|^{-\eta} \quad , \quad |\sigma| \to \infty \qquad (3)$$

Where, the complex quantities α are defined as $\alpha = \sigma + j\tau$. The wave number k is given by the real quantities k_1 and k_2 as $k=k_1+jk_2$. The $C_n(n=1,2,\ )$ are constant, and $\eta \geq -1/2, \tau_+ \geq -k_2$

The total field E^t and the scattered field E for given incident field E^i are defined as,

$$E^t(Xn,Zn) = E^i(Xn,Zn) + E(Xn,Zn) \qquad (4)$$

It should be noticed that the scattered field by one half-plane will be another incident field to the other half-plane. In addition, each scattered field have to be solvable by the W-H technique. So, the equations (1) (2) (3) are essential conditions for the presented method.

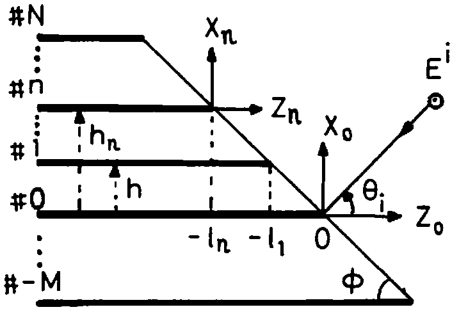

Fig. 1 A finite set of staggered parallel half-planes

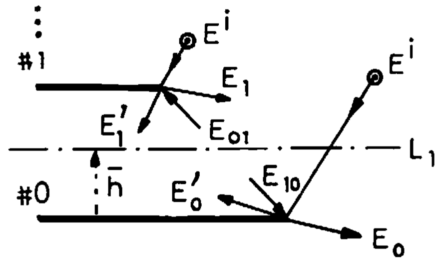

Fig. 2 Incident field E^i, scattered fields E_0, E_0', E_1, E_1', mutual fields E_{01}, E_{10}.

Scattered field by staggered half-planes

In this section, we consider the finite set of staggered parallel half-planes. In order to formulate for total fields, we set a fictitious boundary L_1 at an arbitrary position ($0 \leqq \bar{h} \leqq h_1$) between the two half-planes; #0 and #1. The L_1 is also parallel to the half-planes. Then the scatter field by two half-planes, #0 and #1 are denoted by the unknown fields; E_0, E_1 for the incident field E^i, and E_0', E_1' for the incident field E_{10}, E_{01} which are scattered by the half-planes #1 and #0, respectively. We call the fields E_{10}, E_{01} as the mutual fields. Now the scattered field E in the region between the two half-planes #1 and #0, is written as follow,

$$E = \begin{cases} E_1 + E_{01} + E_1' & X_1 \geqq \bar{h} \\ E_0 + E_{10} + E_0' & X_1 \leqq \bar{h} \end{cases}$$

The formulation to solve is set on the L_1 with all field representations by the continuious condition of the fields. The simular formulations are set on each L_n between the two half-planes #n and #n+1 sucessively. Using the solution of the W-H equation[1),2)] the scattered field representation is finally reduced to the following equations.

$$E = \sum_{n=-M}^{N} \frac{1}{\sqrt{2\pi}} \int_{i\tau'-\infty}^{i\tau'+\infty} \frac{Y_{n-}}{\sqrt{\alpha+k}} e^{-\gamma|Xn|-i\alpha Zn} \, d\alpha \quad , \quad -k_2 \leqq \tau' \leqq k_2\cos\theta_i$$

$$Y_{n-} + Y_{n+} = -\sqrt{\alpha+k}\,\Phi_{n-}^{i}(0,\alpha) - \sum_{\substack{n=-M \\ m=n}}^{N} Y_{m-} e^{-\gamma|h_n-h_m|+i\alpha(l_n-l_m)}$$

$$\Phi_{n-}(0,\alpha) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{0} E^i(0,Zn) e^{i\alpha Zn} \, dZn$$

The far-field solution of the W-H equation in this problem can be evaluated by the saddle point method. The formula for numerical calculation are not written here as they are rather lengthy formula.

However, it is noteworthy that they are expressed by each term corresponding to the geometrical optics representation. The Fig.3 show the far-field patterns $D(\theta)$ of the scattered fields when θ_i=30 , $h/\lambda=l/\lambda=0.75$, Φ=45 and the total numbers of half-planes are 3, 11, and 21. The dotted lines in Fig.3 indicate the results by the G.T.D method without the mutual fields.

Conclusion

The analysis by the use of the field formulation on the fictitious boundary gives us the rigorous equation as well as the W-H technique for solving this problem. The presented method is also useful to have the scattered field by many planes that are arbitarly arranged but not intersected each other.

References

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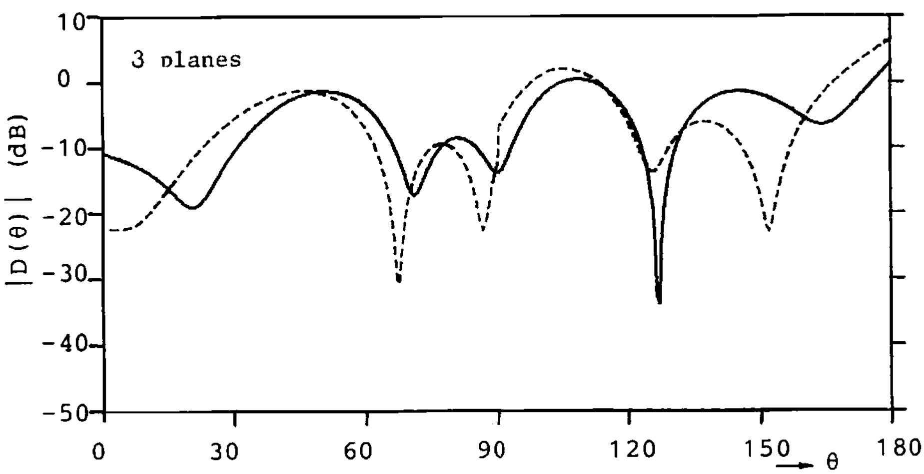
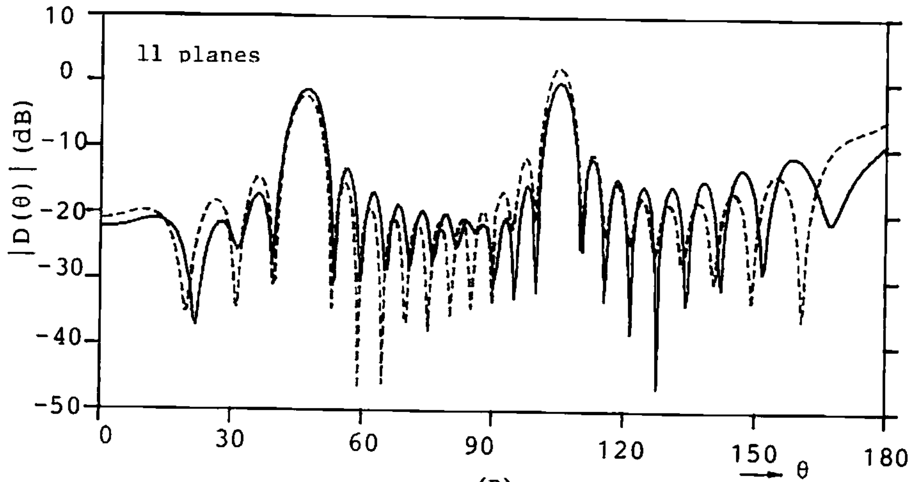
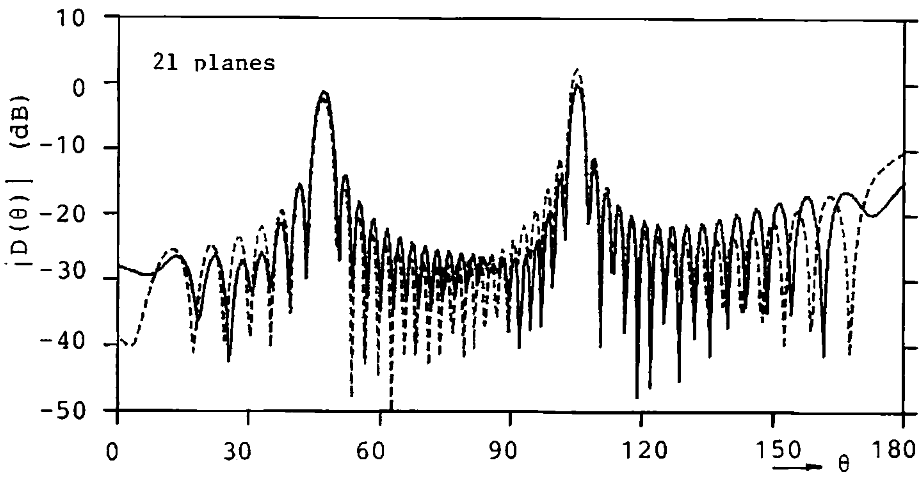


(A)



(B)



(C)

Fig. 3 Far-field patterns D(θ) when θ = 30°, h/λ = 1/λ = 0.75 φ = 45° and the total numbers of half-planes are (A) 3, (B) 11, and (C) 21. —— presented method, ---- G.T.D. method