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SCATTERING OF A PLANE WAVE BY A SEMI-INFINITE DIELECTRIC SLAB

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The scattering problem of electromagnetic wave by a conducting rectangular cylinder has been investigated by many researchers with the aid of various computer aided numerical techniques. It seems also important to analyze the scattering problem by a lossy and dielectric material. C.M. Angulo [1] has discussed the diffraction of surface waves by a semi-infinite dielectric slab by means of the variational method. Recently, A.D. Rawlins[2] has analyzed, using the approximate boundary condition, the diffraction problem of a dielectric half plane by the Wiener-Hopf technique. In this paper, we have solved the two dimensional scattering problem of a plane electromagnetic wave by a semi-infinite thick dielectric slab with the aid of the Wiener-Hopf technique. Assume that the time factor is exp($i\omega t$).

Fig. 1 shows the geometry of this problem. Since the polarization is invariant for the diffraction from this structure, the electromagnetic field can be derived from the z-component of electric vector as follows:

$$\begin{cases} H_x = \dfrac{-i}{i\omega\mu_0}\dfrac{\partial E_z}{\partial y}, \\ H_y = \dfrac{1}{i\omega\mu_0}\dfrac{\partial E_z}{\partial x}, \\ E_x = E_y = H_z = 0 \end{cases} \quad (1)$$

where E_z is the solution of eq. (2)

$$\begin{cases} \left(\dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2} + \kappa_j^2\right) E_z(x,y) = 0, \\ \kappa_j = \begin{cases} \kappa_1 & x \le 0,\ |y| \le b \\ \kappa_0 & \text{otherwise} \end{cases} \\ \kappa_0^2 = \omega^2\varepsilon_0\mu_0 = (2\pi/\lambda)^2, \\ \kappa_1 = \kappa_0\sqrt{\varepsilon_1}, \end{cases} \quad (2)$$

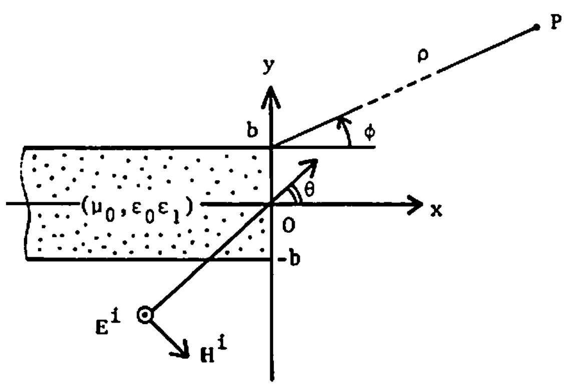

Fig. 1 semi-infinite dielectric slab and incident wave

Let the total field be ($\underline{E}^t$, $\underline{H}^t$) which is divided into two parts:

$$(\underline{E}^t, \underline{H}^t) = (\underline{E}^p, \underline{H}^p) + (\underline{E}^s, \underline{H}^s) \quad (3)$$

where the primary field ($\underline{E}^p$, $\underline{H}^p$) is the total field when the plane wave $\underline{E}^i$ = $\underline{i}_z E^i \exp\{-i\kappa_0(x\cos\theta + y\sin\theta)\}$ is incident upon the infinite dielectric slab of width 2b, which is expressed as follows for $|y| \le b$:

$$\begin{cases} E_z^p(x,y) = E^p(y)\ e^{-i\kappa_0 x\cos\theta},\ E^p(y) = E^p(e^{-i\kappa_1 y\sin\alpha} + R_1 e^{i\kappa_1 y\sin\alpha}) \\ H_x^p(x,y) = H^p(y)\ e^{-i\kappa_0 x\cos\theta},\ H^p(y) = -1/(i\omega\mu_0)\ dE^p(y)/dy, \end{cases} \tag{4}$$

($\kappa_1\cos\alpha = \kappa_0\cos\theta$, the constants E^p, R_1 are omitted here)

Accordingly, the secondary field ($\underline{E}^s$, $\underline{H}^s$) corresponds to the corrected field which is required as the result that the semi-infinite slab in $x > 0$ is removed.

Our present purpose is to find ($\underline{E}^s$, $\underline{H}^s$) satisfying the following boundary conditions:

(B1) ($\underline{E}^s$, $\underline{H}^s$) satisfies the radiation condition at $y = \pm\infty$.

(B2) the tangential components of electromagnetic field are continuous on the surface of slab.

It is convenient for the present analysis to divide ($\underline{E}^s$, $\underline{H}^s$) into two parts as follows:

$$(\underline{E}^s,\ \underline{H}^s) = (\underline{E}_1,\ \underline{H}_1) + (\underline{E}_2,\ \underline{H}_2) \tag{5}$$

where they are chosen to obey the following conditions

(i) $|y| > b$

$$\lim_{y \to \mp b \mp 0} \begin{cases} E_{1z}(x,\ y) = u_1(x,\ \mp b) + u_2(x,\ \mp b) \\ E_{2z}(x,\ y) = u_3(x,\ \mp b) \end{cases} \tag{6}$$

where

$$\begin{cases} u_1(x,\ \mp b) = \begin{cases} E_z^s(x,\ \mp b), & x < 0 \\ 0, & x > 0 \end{cases} \\ u_2(x,\ \mp b) = \begin{cases} 0, & x < 0 \\ -E_z^p(,\ \mp b), & x > 0 \end{cases} \\ u_3(x,\ \mp b) = \begin{cases} 0 & x < 0 \\ E_z^t(x,\ \mp b). & x > 0 \end{cases} \end{cases} \tag{7}$$

(ii) $|y| \leq b$

$$\begin{cases} (\underline{E}_1,\ \underline{H}_1) = \begin{cases} (\underline{E}^s,\ \underline{H}^s), & x < 0 \\ -(\underline{E}^p,\ \underline{H}^p), & x > 0 \end{cases} \\ (\underline{E}_2,\ \underline{H}_2) = \begin{cases} 0 & x < 0 \\ \underline{E}^t,\ \underline{H}^t, & x > 0 \end{cases} \end{cases} \tag{8}$$

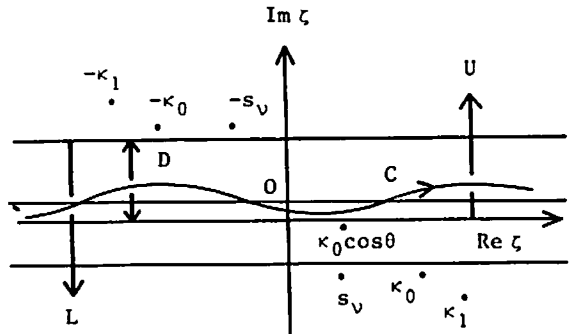

Fig. 2 regions U.L, D and contour C in the ζ-plane ($\mathrm{Im}\,\kappa_0 < 0$)

The Fourier transform and its inverse transform are defined by

$$\begin{cases} f(\zeta) = \mathscr{F}\{f(x)\} = \int_{\infty}^{\infty} f(x)e^{i\zeta x}\,dx, \\ f(x) = \mathscr{F}^{-1}\{f(\zeta)\} = \frac{1}{2\pi}\int_C f(\zeta)e^{-i\zeta x}\,d\zeta,\ (C \in D) \end{cases} \tag{9}$$

where C is the infinite contour in the region D as shown in Fig.2. (s_ν is the propagation constant of the ν-th surface wave supported by the infinite slab of width 2b)

The solution of eq. (2) can be written as

$$E_z^s(x,\ y) = \mathscr{F}^{-1}\{E(\zeta,\ y)\}$$

$$E(\zeta,\ y) = \begin{cases} \dfrac{U^-(\zeta,-b)+U^+(\zeta,-b)}{\zeta-\kappa_0\cos\theta}\, e^{ik_0(y+b)}, & y<-b \\ \dfrac{U^-(\zeta,b)+U^+(\zeta,b)}{\zeta-\kappa_0\cos\theta}\, e^{-ik_0(y-b)}, & y>b \\ E_1(\zeta,\ y)+E_2(\zeta,\ y) & |y|\le b \end{cases} \quad (10)$$

where the unknown functions $U^-(\zeta,\ \pm b)$, $U^+(\zeta,\ \pm b)$ are defined by

$$\begin{cases} U^-(\zeta,\ \mp b) = (\zeta-\kappa_0\cos\theta)u_1^-(\zeta,\ \mp b) - iE^p(\mp b) \\ U^+(\zeta,\ \mp b) = (\zeta-\kappa_0\cos\theta)\ u_3^+(\zeta,\ \mp b) \end{cases} \quad (11)$$

and

$$\begin{cases} u_1^-(\zeta,\ \mp b) = \mathscr{F}\{u_1(x,\ \mp b)\}, \\ u_3^+(\zeta,\ \mp b) = \mathscr{F}\{u_3(x,\ \mp b)\}, \end{cases} \quad (12)$$

$$\begin{cases} E_1(\zeta,\ y) = \dfrac{-U^-(\zeta,-b)\sin k_1(y-b) + U^-(\zeta,b)\sin k_1(y+b)}{(\zeta-\kappa_0\cos\theta)\sin 2k_1 b} + E_{1s}(\zeta,y) \\ E_2(\zeta,\ y) = \dfrac{-U^+(\zeta,-b)\sin k_0(y-b) + U^+(\zeta,b)\sin k_0(y+b)}{(\zeta-\kappa_0\cos\theta)\sin 2k_0 b} - E_{0s}(\zeta,y) \end{cases} \quad (13)$$

where

$$E_{js}(\zeta,y) = \sum_n \left\{\frac{\alpha_{1cn}-i\zeta\alpha_{2cn}}{\zeta^2-k_{jcn}^2}\cos b_{cn}y + \frac{\alpha_{1sn}-i\zeta\alpha_{2sn}}{\zeta^2-k_{jsn}^2}\sin b_{sn}y\right\} \quad (14)$$

and

$$\begin{cases} b_{cn} = (n-\frac{1}{2})\pi/b, & b_{sn} = n\pi/b \\ k_{jcn}^2 = \kappa_j^2 - b_{cn}^2, & k_{jsn}^2 = \kappa_j^2 - b_{sn}^2 \end{cases} \qquad j = 0,\ 1 \quad (15)$$

and

$$\begin{cases} \{\frac{\partial}{\partial x}(E_1+E_z^p)\}_{x=0_-} = \sum_n (\alpha_{1cn}\cos b_{cn}y + \alpha_{1sn}\sin b_{sn}y) \\ \{E_1+E_z^p\}_{x=0_-} = \sum_n (\alpha_{2cn}\cos b_{cn}y + \alpha_{2sn}\sin b_{sn}y) \end{cases} \quad |y| \le b \quad (16)$$

Employing the continuity conditions of $H_x(x,\ y)$ at $y = \pm b$, we have

$$\frac{G_{\ell 1}}{\zeta-\kappa_0\cos\theta}U_\ell^-(\zeta) + \frac{ik_0 b}{\zeta-\kappa_0\cos\theta}U_\ell^+(\zeta) - \sum_n (-1)^n\, 2bb_{\ell n}\frac{\alpha_{1\ell n}-i\zeta\alpha_{2\ell n}}{-k_n} = \phi_\ell^+(\zeta) \quad (17)$$

$$\frac{ik_0 bU_\ell^-(\zeta)}{\zeta-\kappa_0\cos\theta} + \frac{G_{\ell 0}U_\ell^+(\zeta)}{\zeta-\kappa_0\cos\theta} - \sum_n (-1)^n\, 2bb_{\ell n}\,\frac{\alpha_{1\ell n}-i\zeta\alpha_{2\ell n}}{\zeta^2-k_{0\ell}^2}$$

$$+ \frac{ib\{H^p(-b)+H^p(b)\}}{\zeta-\kappa_0\cos\theta} = \Phi_\ell^-(\zeta), \quad \ell = c,\ s \qquad (18)$$

where

$$\begin{cases} U_c^{\pm}(\zeta) = U^{\pm}(\zeta,\ -b) + U^{\pm}(\zeta,\ b) \\ U_s^{\pm}(\zeta) = U^{\pm}(\zeta,\ -b) - U^{\pm}(\zeta,\ b) \end{cases} \qquad (19)$$

$$\begin{cases} G_{sj} = ik_0 b + k_j b\cot k_j b \\ G_{cj} = ik_0 b - k_j b\tan k_j b \end{cases} \quad (j = 0,\ 1) \qquad (20)$$

Eqs.(17), (18) form the simultaneous Wiener-Hopf equations for $U_c^-(\zeta)$, $U_c^+(\zeta)$; $U_s^-(\zeta)$, $U_s^+(\zeta)$ which can be solved by the conventional factorization technique.

In order to obtain the far field pattern the expression eq.(10) is asymptotically evaluated for $\kappa_0\rho$ large, where $x = \rho\cos\phi$ and $y\mp b = \rho\sin\phi$.

Fig.3 shows the total field $|E_z^t(\rho,\phi)|$ which is obtained by an application of the saddle point method.

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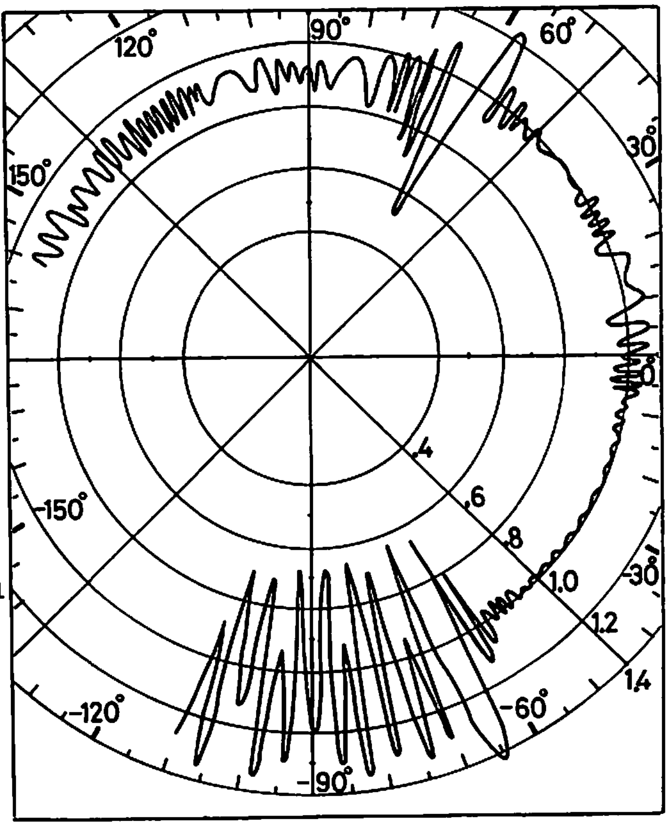

Fig.3 pattern of the total field $|E_z^t(\rho,\phi)|$.
$2b = 5\lambda/\pi$, $\varepsilon_1 = 4.0$,
$\kappa_0\rho = 1{,}000$, $E^i = 1.0$
$\theta = 60°$

References

[1] C.M. Angulo, "Diffraction of surface waves by a semi-infinite dielectric slab", Trans. IRE, AP-5, p.100(1957).

[2] A.D. Rawlins, "Diffraction by an acoustically penetrable or an electromagnetically dielectric half plane", Int. J. Engng. Sci., 15, p.569(1977).