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FORMULATION OF ELECTROMAGNETIC FIELDS IN SPATIAL NETWORK BY VECTOR POTENTIAL WITH LORENTZ GAUGE CONDITION

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1. Introduction

Recently, for many kinds of complex engineering problems, numerical analysis methods have become very useful with the remarkable development of the digital computer, especially the super-computer. The full-wave or vector analysis is essential in the three-dimensional problems especially in the time domain. The time-dependent analysis of electromagnetic fields has shown its utility not only in clarifying the variation of the fields at the transient state but also in gaining information on mechanisms by which the characteristics of an electromagnetic field at the stationary state are brought about. For this purpose, some finite difference methods such as the Finite-Difference Time-Domain method (FD-TD) and the Transmission Line Matrix method (TLM) have been proposed. I have recently proposed a new method for the vector analysis of the electromagnetic field, which is called as the Spatial Network Method (SNM).

For the analysis of some problems involving not only sources such as currents or charges and but also quantum effects such as superconductivity. It is known that the formulation by the vector potential is effective. So far, in the numerical methods such as the FEM or the BEM, the formulation by the magnetic vector potential has been generally proposed, and the introduction of the gauge conditions has been discussed, especially in the eddy current problems. But, for the FD-TD and the TLM methods, few studies about adaptation of the methods to the vector potential fields have appeared.

I have already shown that SNM can be expanded to the vector potential fields with Coulomb's gauge condition by using not only the magnetic vector potential but also the electric vector potential. Also in this formulation, both voltage and current variables are defined and the continuity law of currents occurs at each node in the network. These treatments have the capability of the method to more general problems involving complex medium and boundary conditions by equivalent lumped elements. The resultant iterative computation of the nodule equation at each node is fitted to the computation by the super computer.(1)-(4)

In this paper, it is shown that this treatment of the vector potential fields can be easily applied to the lossy fields. But, for this purpose the combined treatment of both the vector potential field and the scalar potential field with the Lorentz gauge condition, in that the divergence of the vector potential is not zero, is indispensable for the general application of the proposed method. By defining 'F' and 'F* ' function for the electric and magnetic scalar field, respectively, the relations of both currents and charges to both the scalar and the vector fields can be clarified. The equivalent networks for the scalar fields are presented.(5)

2. Equivalent circuit of vector potential

The magnetic vector potential **A** and electric vector potential **S** (=−**A*** by A. J. Stratton) are supposed to satisfy the following equations, respectively. [2]

$$\nabla \times A = \sigma^* S + \mu_0 \frac{\partial S}{\partial t} \qquad (=B) \qquad (1a)$$

$$\nabla \times S = -\sigma A - \varepsilon_0 \frac{\partial A}{\partial t} \qquad (=D) \qquad (1b)$$

where, **B** and **D** are the magnetic and electric flux densities, respectively. This pair of equations also satisfies the ordinary relation between the magnetic vector potential and the electromagnetic field and has the similar form to Maxwell's equation. **Fig. 1** shows the three-dimensional lattice network used in this formulation. Its structure is same as that in SNM for the electromagnetic fields, Therefor the lattice point is defined as the node where the continuity law of currents occurs and the line between nodes is supposed to be a one-dimensional transmission line in which the TEM wave propagates. **Table I** presents the correspondence between the vector potential quantities and the equivalent circuit variables at the six kind of nodes in the lattice network. These nodes are classified into two types. One is the electric node at which each component of the magnetic vector potential is defined as a voltage variable, and the other is the magnetic node at which each component of the electric vector potential is defined as a voltage variable. All circuit variables at the magnetic nodes are identified by the symbol "*" because of the duality of their physical meaning, as compared with their interpretation at the electric nodes. The resultant positions of the vector potential variable coincide to that of the electromagnetic fields expressed as conditions in (1a) and (1b).

In each fundamental cubic, for an example, plotted by the dashed and chained lines in Figure 1, the conditions about the divergence of the magnetic vector potential and electric vector potential are defined, respectively. For the former, the following Lorentz gauge condition can be defined.

$$\nabla \cdot A = -\varepsilon_0 \mu_0 \frac{\partial \phi}{\partial t} - \mu_0 \sigma \phi = -\mu_0 F. \qquad (2-1)$$

For the latter, the similar condition can be supposed.

$$\nabla \cdot S = -\varepsilon_0 \mu_0 \frac{\partial \phi_m}{\partial t} - \varepsilon_0 \sigma^* \phi_m = -\varepsilon_0 F^*. \qquad (2-2)$$

where, ϕ and ϕ_m are the scalar electric potential and the scalar magnetic potential, respectively. σ and σ^* are the conductivity for electric currents and magnetic currents, respectively. The variables 'F' and 'F*' and electric and magnetic field variables are defined as follows by using the analogy to the relation among the velocity potential and the pressure and the particle velocity in the sound field.

$$F = \varepsilon_0 \frac{\partial \phi}{\partial t} + \sigma \phi \qquad (3-1) \qquad F^* = \mu_0 \frac{\partial \phi_m}{\partial t} + \sigma^* \phi_m \qquad (4-1)$$

$$E_S = -\nabla \phi \qquad (3-2) \qquad H_S = -\nabla \phi_m \qquad (4-2)$$

As the difference interval becomes sufficiently small, the lattice network satisfies the next wave equation for the vector potential variables.

$$\nabla^2 \begin{bmatrix} A \\ - \\ S \end{bmatrix} - \varepsilon_0 \mu_0 \frac{\partial^2}{\partial t^2} \begin{bmatrix} A \\ - \\ S \end{bmatrix} - (\mu_0 \sigma + \varepsilon_0 \sigma^*) \frac{\partial}{\partial t} \begin{bmatrix} A \\ - \\ S \end{bmatrix} - \sigma \sigma^* \begin{bmatrix} A \\ - \\ S \end{bmatrix} = - \begin{bmatrix} \mu_0 J \\ --- \\ \varepsilon_0 J^* \end{bmatrix} \quad (4)$$

here J and J^* are the electric and magnetic current sources, respectively.

For the scalar fields, the following characteristic equation can be supposed by using the relation of (3) and (4).

$$-\nabla F = \varepsilon_0 \frac{\partial E_s}{\partial t} + \sigma E_s \quad (5-1) \qquad -\nabla F^* = \mu_0 \frac{\partial H_s}{\partial t} + \sigma^* H_s \quad (6-1)$$

$$-\nabla \cdot E_s = \mu_0 \frac{\partial F}{\partial t} - \frac{\rho}{\varepsilon_0} \quad (5-2) \qquad -\nabla \cdot H_s = \varepsilon_0 \frac{\partial F^*}{\partial t} - \frac{\rho_m}{\mu_0} \quad (6-2)$$

In above each equation, the suffix 's' indicates scalar fields. ρ and ρ_m are the electric and magnetic charges, respectively. The equivalent circuits for these equations are shown in **Figs. 2** and **3**, and **Table II** presents the correspondence between the scalar potential quantities and the equivalent circuit variables. The network satisfies the following wave equations when the spatial difference interval becomes sufficiently small;

$$\nabla^2 \begin{bmatrix} F \\ -- \\ F^* \end{bmatrix} - \varepsilon_0 \mu_0 \frac{\partial^2}{\partial t^2} \begin{bmatrix} F \\ -- \\ F^* \end{bmatrix} - \frac{\partial}{\partial t} \begin{bmatrix} \mu_0 \sigma F \\ ---- \\ \varepsilon_0 \sigma^* F^* \end{bmatrix} = \nabla \cdot \begin{bmatrix} J_s \\ -- \\ J_s^* \end{bmatrix} \quad (7)$$

$$\nabla^2 \begin{bmatrix} E_s \\ -- \\ H_s \end{bmatrix} - \varepsilon_0 \mu_0 \frac{\partial^2}{\partial t^2} \begin{bmatrix} E_s \\ -- \\ H_s \end{bmatrix} - \frac{\partial}{\partial t} \begin{bmatrix} \mu_0 \sigma E_s \\ ---- \\ \varepsilon_0 \sigma^* H_s \end{bmatrix} = \nabla \begin{pmatrix} \rho / \varepsilon_0 \\ ---- \\ \rho_m / \mu_0 \end{pmatrix} \quad (8)$$

3. Conclusion

The spatial networks for both the vector potential fields and scalar potential fields with the Lorentz gauge condition are presented by defining 'F' and 'F*' functions and by using the conductance elements and involving the current or charge sources. Application of this treatment to concrete problems is now being studied. those results will be reported in later papers.

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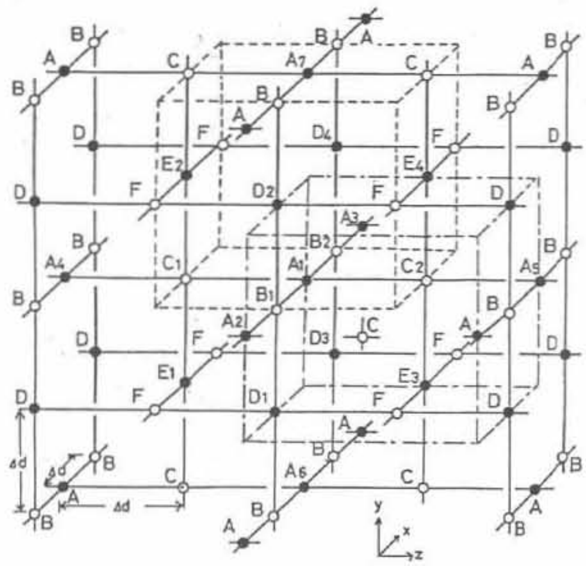


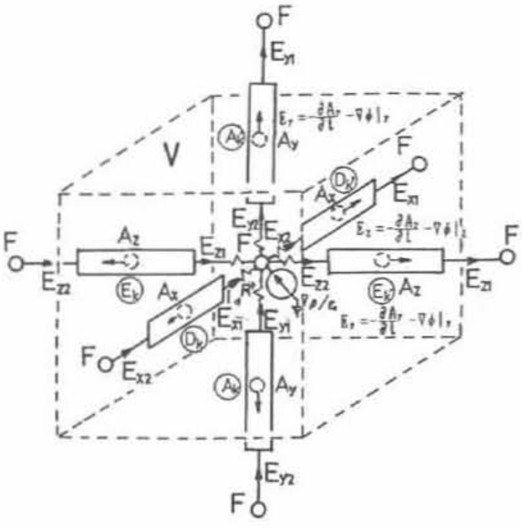
Electric Node			Magnetic Node		
Constituent Equation		Variables	Constituent Equation		Variables
A.	$\frac{\partial S_x}{\partial z}-\frac{\partial S_z}{\partial x}=-\varepsilon_0\frac{\partial A_y}{\partial t}$	$V_y = A_y$	F.	$\frac{\partial A_x}{\partial z}-\frac{\partial A_z}{\partial x}=\mu_0\frac{\partial S_y}{\partial t}$	$V^*_y = -S_y$
	$-\frac{\partial A_y}{\partial z}=\mu_0\frac{\partial S_x}{\partial t}$	$I_z = S_x$		$-\frac{\partial S_y}{\partial z}=-\varepsilon_0\frac{\partial A_x}{\partial t}$	$I^*_z = A_x$
	$\frac{\partial A_y}{\partial x}=\mu_0\frac{\partial S_z}{\partial t}$	$I_x = -S_z$		$\frac{\partial S_y}{\partial x}=-\varepsilon_0\frac{\partial A_z}{\partial t}$	$I^*_x = -A_z$
D.	$\frac{\partial S_z}{\partial y}-\frac{\partial S_y}{\partial z}=-\varepsilon_0\frac{\partial A_x}{\partial t}$	$V_x = A_x$	B.	$\frac{\partial A_y}{\partial x}-\frac{\partial A_x}{\partial y}=\mu_0\frac{\partial S_z}{\partial t}$	$V^*_z = -S_z$
	$\frac{\partial A_x}{\partial z}=\mu_0\frac{\partial S_y}{\partial t}$	$I_z = -S_y$		$\frac{\partial S_z}{\partial y}=-\varepsilon_0\frac{\partial A_x}{\partial t}$	$I^*_y = -A_x$
	$-\frac{\partial A_x}{\partial y}=\mu_0\frac{\partial S_z}{\partial t}$	$I_y = S_z$		$-\frac{\partial S_z}{\partial x}=-\varepsilon_0\frac{\partial A_y}{\partial t}$	$I^*_x = A_y$
E.	$\frac{\partial S_y}{\partial x}-\frac{\partial S_x}{\partial y}=-\varepsilon_0\frac{\partial A_z}{\partial t}$	$V_z = -A_z$	C.	$\frac{\partial A_z}{\partial y}-\frac{\partial A_y}{\partial z}=-\mu_0\frac{\partial S_x}{\partial t}$	$V^*_x = S_x$
	$\frac{\partial A_z}{\partial y}=\mu_0\frac{\partial S_x}{\partial t}$	$I_y = S_x$		$\frac{\partial S_x}{\partial z}=\varepsilon_0\frac{\partial A_y}{\partial t}$	$I^*_z = A_y$
	$-\frac{\partial A_z}{\partial x}=\mu_0\frac{\partial S_y}{\partial t}$	$I_x = -S_y$		$-\frac{\partial S_x}{\partial y}=\varepsilon_0\frac{\partial A_z}{\partial t}$	$I^*_y = -A_z$
Capacitance $C_0 = \varepsilon_0/2$ Inductance $L_0 = \mu_0/2$ Polarization $\Delta C = \varepsilon_0\chi_e/2\cdot\Delta d$ Conductivity $G = \sigma/2\cdot\Delta d$			Capacitance $C^*_0 = \mu_0/2$ Inductance $L^*_0 = \varepsilon_0/2$ Magnetization $\Delta C^* = \mu_0\chi_m/2\cdot\Delta d$ Conductivity $G^* = \sigma^*/2\cdot\Delta d$ (For magnetic Current)		

Fig. 1 3-dimensional lattice network.
● Electric Node,
○ Magnetic Node,
— 1-dimensional line.

Table I Correspondence between vector potential variables and equivalnet circuit ones.

Electric Field		Magnetic Field	
Characteristics Equations	Variable	Characteristics Equations	Variables
$\frac{\partial E_x}{\partial x}+\frac{\partial E_y}{\partial y}+\frac{\partial E_z}{\partial z}=-\mu_0\frac{\partial F}{\partial t}$	$V_s = F$	$\frac{\partial H_x}{\partial x}+\frac{\partial H_y}{\partial y}+\frac{\partial H_z}{\partial z}=-\varepsilon_0\frac{\partial F^*}{\partial t}$	$V_s^* = F^*$
$\frac{\partial F}{\partial x}=-\varepsilon_0\frac{\partial E_x}{\partial t}$	$I_{sx} = E_x$	$\frac{\partial F^*}{\partial x}=-\mu_0\frac{\partial H_x}{\partial t}$	$I^*_{sx} = H_x$
$\frac{\partial F}{\partial y}=-\varepsilon_0\frac{\partial E_y}{\partial t}$	$I_{sy} = E_y$	$\frac{\partial F^*}{\partial y}=-\mu_0\frac{\partial H_y}{\partial t}$	$I^*_{sy} = H_y$
$\frac{\partial F}{\partial z}=-\varepsilon_0\frac{\partial E_z}{\partial t}$	$I_{sz} = E_z$	$\frac{\partial F^*}{\partial z}=-\mu_0\frac{\partial H_z}{\partial t}$	$I^*_{sz} = H_z$
$C^*_{s0} = \mu_0\Delta d/3$ $\Delta C_s^* = \mu_0\chi_m\Delta d/3$ $L^*_{s0} = \varepsilon_0\Delta d$ $\Delta L_s^* = \varepsilon_0\chi_e\Delta d/3$ $R_s^* = \sigma\Delta d$		$C_{s0} = \varepsilon_0\Delta d/3$ $\Delta C_s = \varepsilon_0\chi_e\Delta d/3$ $L_{s0} = \mu_0\Delta d$ $\Delta L_s = \mu_0\chi_m\Delta d/3$ $R_s = \sigma^*\Delta d$	

Table II Correspondence between scalar field variables and equivalnet circuit ones.



Fog. 2 Equivalent circuit for electric scalar field.

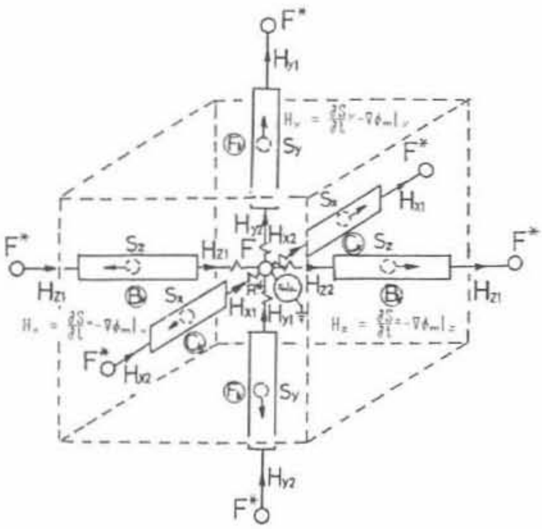


Fig. 3 Equivalent circuit for magnetic scalar field.