Electromagnetic Scattering by Simplified Crack Models on Conducting Ground Plane

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***Abstract* – This paper analyzes electromagnetic scattering problems for simplified crack models, based on the formulation of the Kobayashi Potential (KP) method. Here, three types of simple crack models, a flanged waveguide, a rectangular trough, and a thick slit, are considered. The derived far zone field representations are utilized to obtain the approximate formulae of the scattered fields for the models.**

***Index Terms* — Scattering by cracks, The Kobayashi Potential, Weber-Schafheitlin discontinuous integrals.**

1. Introduction

A series of big earthquakes hit Kumamoto and Oita prefectures last April. To avoid the secondary disasters, one of the most important information is to promptly and exactly understand the hazard levels of the stricken residential areas. Hence, fast and accurate non-destructive testing (NDT), which can quickly detect fatal cracks on/in walls or pillars of the stricken residential houses, is strongly required.

In this paper, to realize easy and accurate non-destructive crack detections using electromagnetic waves [1], we shall try to derive simple formulae of the scattered fields by some crack models. H polarized plane wave incidence is here considered. In the formulation, the Kobayashi Potential (KP) method [2] – [4] first used to derive the rigorous representation of the scattered field. Then, the derived rigorous far zone field representation is used to obtain approximate formulae of the field when the crack aperture is electrically narrow. Due to limitation of space, the formulation for deriving the approximate formula is omitted in this manuscript.

2. Formulation

Let us now consider three types of crack models as (a) a narrow flanged waveguide, (b) a narrow rectangular trough, and (3) a narrow thick slit, on infinitely long perfectly electric conductor (PEC) plane, as shown in Fig.1. We consider scattering problem when H polarized plane wave $\phi^i(=H_z^i)=\exp\{-ik_0(x\cos\theta_0+y\sin\theta_0)\}$ impinges on each crack model, where $k_0(=\omega\sqrt{\varepsilon_0\mu_0})$ is free space wave number and θ_0 is incident angle. Width of the crack is $2a$. Depth of the models (a) and (b) is b. Here, the crack is filled by lossy material, whose relative permittivity is ε_r (the relative permeability $\mu_r=1$). For convenience, the entire region is divided into two or three regions: region I) semi-infinite upper half space ($y>0$), region II) waveguide space ($|x|<a, y<0$ for model (a), $|x|<a, -b<y<0$ for models (b) and (c)), and region III) semi-infinite lower half space ($y<-b$ for model (c)).

According to the Kobayashi Potential (KP) method, which is a kind of eigen function expansion method in terms of the discontinuous features of Weber-Schafheitlin type integrals, the scattered contribution ϕ_H^s for each model may be expressed by a same Fourier integral representation as

$$\phi_H^s=\sqrt{\frac{\pi u}{2}}\sum_{m=0}^{\infty}\int_0^{\infty}\frac{\sqrt{\xi}}{\sqrt{\xi^2-\kappa_0^2}}\{A_m J_{2m}(\xi)J_{-1/2}(\xi u) + B_m J_{2m+1}(\xi)J_{1/2}(\xi u)\}e^{-\sqrt{\xi^2-\kappa_0^2}v}d\xi \qquad (v>0) \tag{1}$$

where A_m and B_m are unknown expansion coefficients, and $u=x/a,\ v=y/a,\ b_0=b/a,\ \kappa_0=k_0a$. Taking into account the features of Weber-Schafheitlin's discontinuous integrals, the total fields in region I automatically satisfy the boundary condition on the PEC plane. In what follows, formulation for each crack model in region II is shown separately.

(1) Model (a): a flanged waveguide [3]

The field in region II of Model (a) may be expressed by a summation of the parallel plate waveguide modes excited at the aperture ($|x|<a,\ y=0$) as

$$\phi_{II}^{(tr)}=\sum_{n=0}^{\infty}F_n^{(wg)}\cos\frac{n\pi}{2}(1-u)\,e^{-ih_n av}, \tag{2}$$

where $F_n^{(wg)}$ is unknown excitation coefficients, and $h_n(=\{k^2-(n\pi/2a)^2\}^{1/2})$ is propagation constant with respect to y direction. By applying the remained boundary condition at the aperture ($|x|<a,\ y=0$), one may obtain the deterministic equations for the unknown coefficients as

$$\sum_{m=0}^{\infty}A_m\left\{G(2q,2m)+GN^{(wg)}(2q,2m)\right\} =-2J_{2q}(k_0a\cos\theta_0), \tag{3}$$

$$\sum_{m=0}^{\infty}B_m\left\{G(2q+1,2m+1)+GN^{(wg)}(2q+1,2m+1)\right\} =2iJ_{2q+1}(k_0a\cos\theta_0), \tag{4}$$

where

$$G(\alpha,\beta)=\int_0^{\infty}\frac{J_\alpha(\xi)J_\alpha(\xi)}{\sqrt{\xi^2-(k_0a)^2}}d\xi, \tag{5}$$

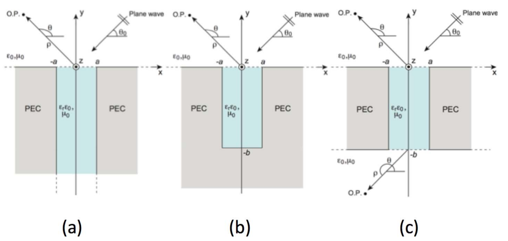

Fig. 1. Geometry of the problem.

$$GN^{(wg)}(2q,2m) = -\varepsilon_r \pi \sum_{m=0}^{\infty} \frac{J_{2q}(n\pi) J_{2m}(n\pi)}{(1+\delta_{0n})(ih_{2n}a)}, \quad (6)$$

$$GN^{(wg)}(2q+1,2m+1) = -\varepsilon_r \pi \sum_{m=0}^{\infty} \frac{J_{2q+1}(\frac{2n+1}{2}\pi) J_{2m+1}(\frac{2n+1}{2}\pi)}{ih_{2n+1}a}. \quad (7)$$

(2) Model (b): a rectangular trough [4]

For this model, by considering the +y propagation mode reflected at the bottom of the trough, in addition to the –y propagation mode Eq.(3) excited at the upper aperture, the field in region II may be represented as

$$\phi_{II}^{(tr)} = \sum_{n=0}^{\infty} F_n^{(tr)} \cos\frac{n\pi}{2}(1-u) \cdot \left\{ e^{-ih_n av} + e^{ih_n av} D_n^{(2b_0)} \right\}, \quad (8)$$

where $F_n^{(tr)}$ is the excitation coefficient, and $D_n^{(2b_0)} = \exp(ih_n a \cdot 2b_0)$. The deterministic equations for the unknown coefficietns for this model have same forms as Eqs.(3) and (4), except for $GN^{(wg)}(\cdot,\cdot)$. In Eqs. (3) and (4), $GN^{(wg)}(\cdot,\cdot)$is replaced by $GN^{(tr)}(\cdot,\cdot)$ as

$$GN^{(tr)}(2q,2m) = GN^{(wg)}(2q,2m) \cdot \frac{1+D_{2n}^{(2b_0)}}{1-D_{2n}^{(2b_0)}}, \quad (9)$$

$$GN^{(tr)}(2q+1,2m+1) = GN^{(wg)}(2q+1,2m+1) \cdot \frac{1+D_{2n+1}^{(2b_0)}}{1-D_{2n+1}^{(2b_0)}}. \quad (10)$$

(3) Model (c): a thick slit [3]

In region II of Model (c), there exist the propagation modes for both –y and +y directions. So the field in the region may be expressed as

$$\phi_{II}^{(sl)} = \sum_{n=0}^{\infty} \cos\frac{n\pi}{2}(1-u) \left\{ E_n^{(sl)} e^{ih_n av} + F_n^{(sl)} e^{-ih_n av} \right\}, \quad (11)$$

where $E_n^{(sl)}$and $F_n^{(sl)}$ are the excitation coefficients. For this model, the unknown expantion coefficients $C_m$and D_m in the scattered field ϕ_H^d in region III

$$\phi_H^d = \sqrt{\frac{\pi u}{2}} \sum_{m=0}^{\infty} \int_0^{\infty} \frac{\sqrt{\xi}}{\sqrt{\xi^2 - \kappa_0^2}} \{ C_m J_{2m}(\xi) J_{-1/2}(\xi u) + C_m J_{2m+1}(\xi) J_{1/2}(\xi u) \} e^{-\sqrt{\xi^2-\kappa_0^2}(v+b_0)} d\xi \quad (v < -b_0) \quad (12)$$

are also determined. So, by applying the boundary conditions at both upper and lower apertures, one may obtain

$$\sum_{m=0}^{\infty} P_m \left\{ G(2q,2m) + GN^{(sl)}(2q,2m) \right\} = -2J_{2q}(k_0 a \cos\theta_0), \quad (13)$$

$$\sum_{m=0}^{\infty} Q_m \left\{ G(2q,2m) + GP^{(sl)}(2q,2m) \right\} = -2J_{2q}(k_0 a \cos\theta_0), \quad (14)$$

$$\sum_{m=0}^{\infty} S_m \left\{ G(2q+1,2m+1) + GN^{(sl)}(2q+1,2m+1) \right\} = 2iJ_{2q+1}(k_0 a \cos\theta_0), \quad (15)$$

$$\sum_{m=0}^{\infty} T_m \left\{ G(2q+1,2m+1) + GP^{(sl)}(2q+1,2m+1) \right\} = 2iJ_{2q+1}(k_0 a \cos\theta_0), \quad (16)$$

where $P_m = A_m + C_m$, $Q_m = A_m - C_m$, $S_m = B_m + D_m$, $T_m = B_m - D_m$, and

$$GN^{(sl)}(2q,2m) = GN^{(tr)}(2q,2m)\Big|_{2b_0 \to b_0}, \quad (17)$$

$$GN^{(sl)}(2q+1,2m+1) = GN^{(tr)}(2q+1,2m+1)\Big|_{2b_0 \to b_0}, \quad (18)$$

$$GP^{(sl)}(2q,2m) = GN^{(sl)}(2q,2m)\Big|_{D_{2n}^{(b_0)} \to -D_{2n}^{(b_0)}}, \quad (19)$$

$$GP^{(sl)}(2q+1,2m+1) = GN^{(sl)}(2q+1,2m+1)\Big|_{D_{2n+1}^{(b_0)} \to -D_{2n+1}^{(b_0)}}. \quad (20)$$

3. Conclusions

In this paper, we derived the rigorous representations of the scattered fields for the three simple crack models, by using the KP method. In presentation, the approximation of the far zone fields will be carried out for the case that aperture width of the crack is electrically narrow.

Acknowledgment

This work was supported by JSPS KAKENHI Grant Numbers JP25350495, JP15K06083, JP16K01285, and Grant-In-Aid for test and research from The Uchida Energy Science Promotion, Japan.

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