PROCEEDINGS OF ISAP2005, SEOUL, KOR

Calculation of Element Values of Antenna Equivalent Circuit

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Abstract: An arbitrary lossless antenna can be equivalent to a RLC circuit where the element values depend on the frequency. A general method is presented to determine these element values and some numerical results are given.

1. Formulation

In terms of the stored electric energy and magnetic energy and the radiated power from the antenna, a RLC equivalent circuit can be constructed for an ideal antenna, whose element values are defined by [1-2]

$$R_A^{rad} = 2P_{rad} / |I|^2 , L_A = 4\tilde{W}_m / |I|^2 , C_A = |I|^2 / 4\omega^2 \tilde{W}_e \quad (1)$$

where I is the terminal current; $\tilde{W}_e$ and $\tilde{W}_m$ are the stored electric energy and stored magnetic energy

$$\tilde{W}_e = \frac{1}{8}|I|^2 \left(\frac{\partial X_A}{\partial \omega} - \frac{X_A}{\omega} \right), \ \tilde{W}_m = \frac{1}{8}|I|^2 \left(\frac{\partial X_A}{\partial \omega} + \frac{X_A}{\omega} \right) \quad (2)$$

respectively. In the above expression, X_A is the antenna input reactance

$$X_A = 4\omega \left(\tilde{W}_m - \tilde{W}_e \right) / |I|^2 = \omega L_A - 1/\omega C_A \quad (3)$$

The antenna Q is then given by

$$Q = \begin{cases} 2\omega \tilde{W}_e / P_{rad} , \ \tilde{W}_e > \tilde{W}_m \\ 2\omega \tilde{W}_m / P_{rad} , \ \tilde{W}_m > \tilde{W}_e \end{cases} \quad (4)$$

Therefore once the antenna input impedance is known the element values of the equivalent RLC circuit are then determined. (2) is well known in circuit theory and can be easily derived for a bounded microwave system where the electromagnetic energy is confined in a finite region. In this case the stored electromagnetic energy is simply equal to the total electromagnetic energy for a lossless system. An antenna is an open system, and some of its energy radiates into free space. The total electromagnetic energy around the antenna and the radiated energy into free space are both infinite. But their difference is a finite quantity, which is defined as the stored electromagnetic energy. (2) and (3) indicate that to find the stored energies we only need to know the difference between the stored electric energy and magnetic energy. The difference $\tilde{W}_m - \tilde{W}_e$ can also be determined by making use of the Poynting theorem in the frequency domain (for definition of V_∞ and V_0 used below see Fig. 2 of ref. [2])

ISBN: 89-86522-79-9 94460©KEES

$$-\frac{1}{2}\int_{V_0}\mathbf{J}^*\cdot\mathbf{E}dv(\mathbf{r})=\frac{1}{2}\int_{\partial V_\infty}\mathbf{S}\cdot\hat{\mathbf{n}}ds(\mathbf{r})+j2\omega(W_m-W_e)=P_{rad}+j2\omega(\tilde{W}_m-\tilde{W}_e) \tag{5}$$

where W_m and W_e are the total average magnetic energy and electric energy stored in the region $V_\infty - V_0$ respectively; $P_{rad}=(1/2\int_{\partial V_\infty}\mathbf{S}\cdot\hat{\mathbf{n}}ds(\mathbf{r})$ is the radiated power. Note that in the above equation we have used the fact that $\tilde{W}_m-\tilde{W}_e=W_m-W_e$. The left-hand side of (5) can be expressed as

$$-(1/2)\int_{V_0}\mathbf{J}^*\cdot\mathbf{E}dv(\mathbf{r})=-(1/2)\int_{V_0}\mathbf{J}^*\cdot\left[-\nabla\phi-j\omega\mathbf{A}\right]dv(\mathbf{r}) \tag{6}$$

where ϕ and $\mathbf{A}$ are the scalar and vector potential functions given by

$$\phi(\mathbf{r})=\frac{\eta c}{4\pi}\int_{V_0}\frac{\rho(\mathbf{r}')e^{-jkR}}{R}dv(\mathbf{r}'),\quad \mathbf{A}(\mathbf{r})=\frac{\eta}{4\pi c}\int_{V_0}\frac{\mathbf{J}(\mathbf{r}')e^{-jkR}}{R}dv(\mathbf{r}')$$

where $R=|\mathbf{r}-\mathbf{r}'|$, $\eta=\sqrt{\mu_0/\varepsilon_0}$ and $c=1/\sqrt{\mu_0\varepsilon_0}$. Inserting the above equations into (6) we obtain

$$\begin{aligned}-(1/2)\int_{V_0}\mathbf{J}^*\cdot\mathbf{E}dv(\mathbf{r})&=\frac{\omega\eta c}{8\pi}\int_{V_0}\int_{V_0}R^{-1}\left[c^{-2}\mathbf{J}^*(\mathbf{r})\cdot\mathbf{J}(\mathbf{r}')-\rho^*(\mathbf{r})\rho(\mathbf{r}')\right]\sin(kR)dv(\mathbf{r})dv(\mathbf{r}')\\&+j\frac{\omega\eta c}{8\pi}\int_{V_0}\int_{V_0}R^{-1}\left[c^{-2}\mathbf{J}^*(\mathbf{r})\cdot\mathbf{J}(\mathbf{r}')-\rho^*(\mathbf{r})\rho(\mathbf{r}')\right]\cos(kR)dv(\mathbf{r})dv(\mathbf{r}')\end{aligned}$$

It follows from the above equation and (5) that

$$P_{rad}=\frac{\omega\eta c}{8\pi}\int_{V_0}\int_{V_0}R^{-1}\left[c^{-2}\mathbf{J}^*(\mathbf{r})\cdot\mathbf{J}(\mathbf{r}')-\rho^*(\mathbf{r})\rho(\mathbf{r}')\right]\sin(kR)dv(\mathbf{r})dv(\mathbf{r}') \tag{7}$$

$$\tilde{W}_m-\tilde{W}_e=\frac{\eta c}{16\pi}\int_{V_0}\int_{V_0}R^{-1}\left[c^{-2}\mathbf{J}^*(\mathbf{r})\cdot\mathbf{J}(\mathbf{r}')-\rho^*(\mathbf{r})\rho(\mathbf{r}')\right]\cos(kR)dv(\mathbf{r})dv(\mathbf{r}') \tag{8}$$

Thus once the current distribution is known the calculation of the energy difference is simply an integration.

2. An example

Let us consider a dipole antenna. A dipole antenna can be represented by various equivalent circuits [e.g., 3]. Assuming that the dipole antenna has radius a_0 and length $2a$ and current flows on the outer cylindrical surface of the dipole. The current distribution on the wire surface can be assumed to be

$$\mathbf{J}(\mathbf{r}) = \mathbf{z}\frac{I(z)}{2\pi a_0} = \mathbf{z}\frac{I_0}{2\pi a_0}\sin k\left(a - |z|\right); -a < z < a, r = a_0$$

The terminal current is given by $I = I_0 \sin ka$. The stored electric energy and stored magnetic energy are shown in Fig.1. A dipole has an infinite number of resonant frequencies, where the stored electric energy is equal to the stored magnetic energy. The element values of the RLC circuit are shown in Fig.2. The radiation Q is shown in Fig.3.

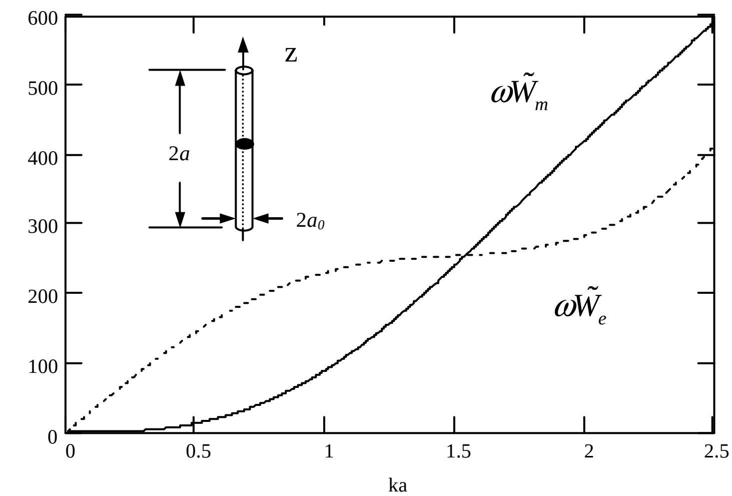

Fig.1 Stored energies of a dipole

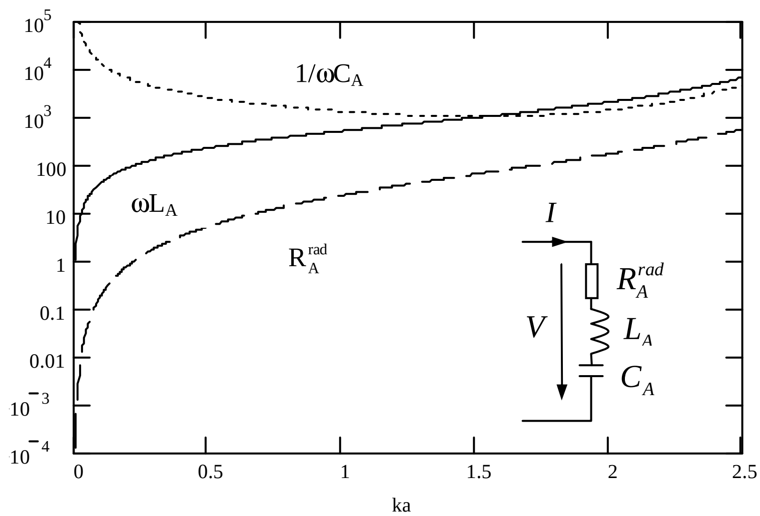

Fig.2 Element values of equivalent circuit for dipole antenna

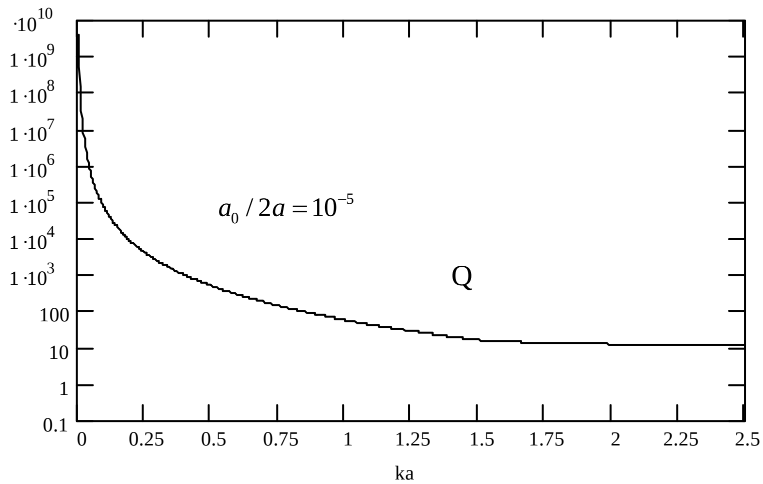

Fig.3 Radiation Q of a dipole antenna

3. Element values for small antennas

When the frequency is very low the calculation of the frequency derivative appearing in (2) is becoming a challenging task due to the numerical errors. Fortunately alternative expressions for the stored energies of small antennas have been derived in [1] to get rid of the frequency derivative, i.e.,

$$\tilde{W}_e = \frac{c\eta}{16\pi} \int_{V_0}\int_{V_0} \frac{1}{R}\left[\rho(\mathbf{r})\bar{\rho}(\mathbf{r}')\right] dv(\mathbf{r})dv(\mathbf{r}')$$

$$\tilde{W}_m = \frac{c\eta}{16\pi}\left[\frac{1}{c^2}\int_{V_0}\int_{V_0} \frac{\mathbf{J}(\mathbf{r})\cdot\bar{\mathbf{J}}(\mathbf{r}')}{R} dvdv' + \frac{k^2}{2}\int_{V_0}\int_{V_0} R\rho(\mathbf{r})\bar{\rho}(\mathbf{r}')dvdv'\right]$$

Note that the stored energies are always positive. The above equations can be used to evaluate the element values of small antennas.

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