Radiation and Scattering of a Waveguide Antenna with Aperture Array

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Abstract

Radiation and scattering properties from a rectangular waveguide with aperture array are considered. Eigenfunction expansion methods are utilized for field representations. The Fourier transform and mode matching technique are used and the boundary conditions are enforced to obtain simultaneous equations for the modal coefficients. The equations are solved to represent the radiation pattern and scattering expressions in convergent series. Our formulation is useful for analyzing radiation and scattering from a rectangular waveguide antenna with aperture array.

1. Introduction

A waveguide antenna with aperture array has received considerable attention as a candidate for practical microwave antennas. For instance, the propagation constant for infinitely slotted waveguide is evaluated using Floquet theorem in [1]. Mutual coupling effects between slots on waveguide are calculated in [2] using transmission line theory. The resonant slot length is calculated using method of moments (MoM) for longitudinal slots on the broad wall of waveguide in [3] and [4] and for transverse slots in [5]. The present paper shows a theoretical model for a waveguide antenna with aperture array, which has multiple rectangular apertures in the broad wall of a rectangular waveguide. We assume aperiodic and multiple apertures to make the formulation flexible for antenna design applications. In this paper, the Fourier transform and mode matching technique are used to study the radiation and scattering property of a waveguide with aperture array, which is excited by TE_{10} mode. Using the stationary phase approximation, the radiation patterns are evaluated. The formulation presented in this paper is a fast convergent series form, which is numerically efficient. The analysis method and expressions in this paper are similar to those in [6].

2. Field Expressions

Electromagnetic wave radiation and scattering from a rectangular waveguide with aperture array are analyzed. The problem geometry is shown in Fig. 1. The waveguide has multiple apertures on a broad wall as shown Fig. 1, which has only two apertures ($l = 1, 2$) for drawing purpose. We assume an infinitely large conducting flange at $z = 0$. On the infinitely large conducting flange, an aperture array is present.

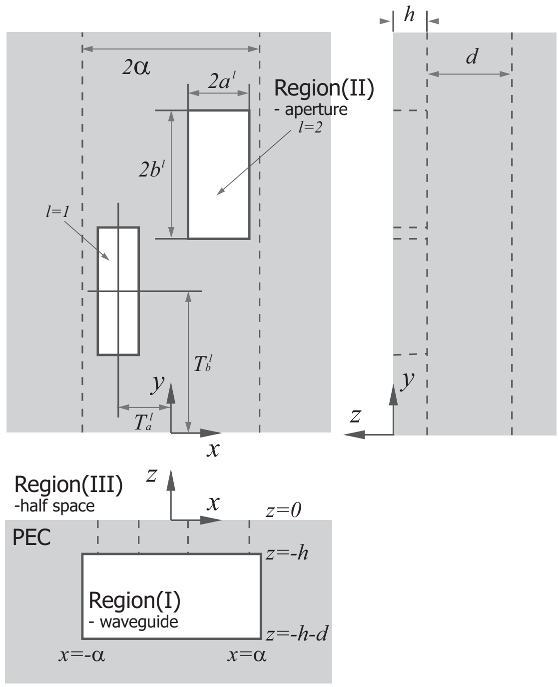

Fig. 1: Problem Geometry

We can separate the geometry into three parts, regions (I), (II), and (III). Regions (I), (II), and (III) denote an interior of rectangular waveguide, a thick rectangular aperture array, and the upper half space ($z > 0$), respectively. The sides of apertures assumed to be parallel with the x and y axes. The apertures are located arbitrarily over the broad wall of waveguide. The waveguide is excited by TE_{10} mode. The time convention $e^{-i\omega t}$ is suppressed. The total field in region (I) consists of the incident and scattered components. The incident electric vector potential F_y^i for TE_{10} mode is

$$F_y^i = \cos\frac{\pi}{2\alpha}(x+\alpha)e^{i\beta_y y} \tag{1}$$

where $\beta_y = \sqrt{k_w^2 - \left(\frac{\pi}{2\alpha}\right)^2}$ and $k_w = \sqrt{\mu_w \epsilon_w}$ is the wave number in the waveguide region.

The scattered electric and magnetic vector potentials in region (I) are given as

$$F_y^I(x,y,z) \tag{2}$$
$$= \frac{1}{2\pi}\sum_{g=0}^{\infty}\cos\alpha_g(x+\alpha)\int_{-\infty}^{\infty} e_g(\eta)\cos\gamma_g(z+h+d)e^{-i\eta y}d\eta$$

$$A_y^I(x,y,z) \tag{3}$$
$$= \frac{1}{2\pi}\sum_{g=1}^{\infty}\sin\alpha_g(x+\alpha)\int_{-\infty}^{\infty} \tilde{e}_g(\eta)\sin\gamma_g(z+h+d)e^{-i\eta y}d\eta$$

where $\alpha_g = \frac{g\pi}{2\alpha}$ and $\gamma_g = \sqrt{k_w^2 - \alpha_g^2 - \eta^2}$.

The scattered magnetic and electric vector potentials in regions (II) and (III) have the same forms as in [6].

3. Boundary Conditions and Analysis

We enforce the boundary conditions of the tangential electric and magnetic field continuities in order to determine the unknown modal coefficients C_{mn}, D_{mn}, $\tilde{C}_{mn}$, and $\tilde{D}_{mn}$. First of all, the tangential $E_{x,y}$ components continuities at $z=-h$ require

$$E_{x,y}^I(x,y,-h) \tag{4}$$
$$= \begin{cases} E_{x,y}^{II}(x,y,-h) & \text{for } |x-T_a^l|<a^l,\ |y-T_b^l|<b^l \\ 0 & \text{otherwise.}\end{cases}$$

We multiply the E_x and E_y continuities of Eq. (4) by $\cos\alpha_{g'}(x+\alpha)$ and $\sin\alpha_{g'}(x+\alpha)$, respectively. We then integrate from $-\alpha$ to α with respect to x and apply the Fourier transform to get $e_g(\eta)$ and $\tilde{e}_g(\eta)$ in terms of C_{mn}^l, D_{mn}^l, $\tilde{C}_{mn}^l$, and $\tilde{D}_{mn}^l$.

$$e_g(\eta) = \frac{\epsilon_w}{\epsilon_a}\frac{\alpha_g}{\alpha\gamma_g(k_w^2-\eta^2)\sin(\gamma_g d)}$$
$$\times\sum_{l=1}^{N}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\left(C_{mn}^l e^{-i\xi_{mn}^l h} + D_{mn}^l e^{i\xi_{mn}^l h}\right) \tag{5}$$
$$\times\left\{(a_m^l)^2\eta^2 - (b_n^l)^2(k_w^2-\eta^2)\right\}E_{lmg}(\alpha,a^l)E_{ln}(\eta,b^l)$$
$$+\frac{k_w^2\epsilon_w}{\omega\mu_a\epsilon_a}\frac{\alpha_g}{\alpha\gamma_g(k_w^2-\eta^2)\sin(\gamma_g d)}$$
$$\times\sum_{l=1}^{N}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\tilde{C}_{mn}^l e^{-i\xi_{mn}^l h} - \tilde{D}_{mn}^l e^{i\xi_{mn}^l h}\right)$$
$$\times a_m^l b_n^l \xi_{mn}^l E_{lmg}(\alpha,a^l)E_{ln}(\eta,b^l)$$

$$\tilde{e}_g(\eta) = \frac{\omega\mu_w\epsilon_w}{\epsilon_a}\frac{1}{\alpha(k_w^2-\eta^2)\sin(\gamma_g d)}$$
$$\times\sum_{l=1}^{N}\sum_{m=1}^{\infty}\sum_{n=0}^{\infty}\left(C_{mn}e^{-i\xi_{mn}h} + D_{mn}e^{i\xi_{mn}h}\right) \tag{6}$$
$$\times(a_m^l)^2\eta E_{lmg}(\alpha,a^l)E_{ln}(\eta,b^l)$$
$$+\frac{\mu_w\epsilon_w}{\mu_a\epsilon_a}\frac{1}{\alpha(k_w^2-\eta^2)\sin(\gamma_g d)}$$
$$\times\sum_{l=1}^{N}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\tilde{C}_{mn}^l e^{-i\xi_{mn}^l h} - \tilde{D}_{mn}^l e^{i\xi_{mn}^l h}\right)$$
$$\times a_m^l b_n^l \xi_{mn}^l \eta E_{lmg}(\alpha,a^l)E_{ln}(\eta,b^l).$$

where

$$E_{lmg}(u,v) = \frac{1}{\left(\frac{m\pi}{2v}\right)^2 - \left(\frac{g\pi}{2u}\right)^2}$$
$$\times\left\{\sin\frac{g\pi}{2u}(T_a^l - v + u) - (-1)^m\sin\frac{g\pi}{2u}(T_a^l + v + u)\right\}$$
$$E_{ln}(u,v) = \frac{e^{iuT_b^l}}{\left(\frac{n\pi}{2v}\right)^2 - u^2}\left\{e^{-iuv} - (-1)^n e^{iuv}\right\}.$$

Note that Eq. (5) can not account for the case of $g=0$. When $g=0$ and $m=0$, $e_g(\eta)$ has a form different from Eq. (5) as follows

$$e_0(\eta) = -\frac{\epsilon_w}{\epsilon_a}\frac{\delta_{g0}\delta_{m0}}{\alpha\gamma_g\sin(\gamma_g d)} \tag{7}$$
$$\times\sum_{l=1}^{N}\sum_{n=1}^{\infty}\left(C_{mn}e^{-i\xi_{mn}h} + D_{mn}e^{i\xi_{mn}h}\right)a^l(b_n^l)^2E_{ln}(\eta,b^l)$$

where $\delta_{gg'}$ is Kronecker delta.

The tangential $H_{x,y}$ continuities at $z=-h$ within the slot require

$$H_{x,y}^I(x,y,-h) = H_{x,y}^{II}(x,y,-h) \tag{8}$$
$$\text{for } |x-T_a^l|<a^l,\ |y-T_b^l|<b^l.$$

Substituting $e_g(\eta)$ and $\tilde{e}_g(\eta)$ into the H_x continuity of (8), multiplying $\sin a_p(x-T_a^r+a^r)\cos b_q(y-T_b^r+b^r)$, and integrating over the aperture ($|x-T_a^r|<a^r$, $|y-T_b^r|<b^r$) gives

$$-\frac{i}{\omega\mu_w\epsilon_w}\alpha_1 a_p^r(\beta_y)^2E_{rp1}(\alpha,a^r)E_{rq}(\beta_y,b^r)$$
$$-\frac{i}{\omega\mu_w\epsilon_a}\frac{1}{2\pi\alpha}\sum_{g=1}^{\infty}\sum_{l=1}^{N}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}a_p^r \tag{9}$$
$$\times\left(C_{mn}^l e^{-i\xi_{mn}h} + D_{mn}^l e^{i\xi_{mn}h}\right)E_{lmg}(\alpha,a^l)E_{rpg}(\alpha,a^r)$$
$$\times\left[k_w^2(a_m^l)^2 - \alpha_g^2\left\{(a_m^l)^2+(b_n^l)^2\right\}\right]I_1$$
$$-\frac{i\epsilon_w}{\mu_a\epsilon_a}\frac{1}{2\pi\alpha}\sum_{g=1}^{\infty}\sum_{l=1}^{N}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}a_m^l b_n^l a_p^r\xi_{mn}^l$$
$$\times\left(\tilde{C}_{mn}^l e^{-i\xi_{mn}^l h} - \tilde{D}_{mn}^l e^{i\xi_{mn}^l h}\right)E_{lmg}(\alpha,a^l)E_{rpg}(\alpha,a^r)I_1$$
$$=\frac{a^r b^r}{\omega\mu_a\epsilon_a}\left(C_{pq}^r e^{-i\xi_{pq}^r h} - D_{pq}^r e^{i\xi_{pq}^r h}\right)a_p^r\xi_{pq}^r\varepsilon_q\delta_{lr}\delta_{mp}\delta_{nq}$$
$$+\frac{a^r b^r}{\mu_a}\left(\tilde{C}_{pq}^r e^{-i\xi_{pq}^r h} + \tilde{D}_{pq}^r e^{i\xi_{pq}^r h}\right)b_q^r\delta_{lr}\delta_{mp}\delta_{nq}$$

where

$$I_1 = \int_{-\infty}^{\infty}\frac{\eta^2}{\gamma_g\tan(\gamma_g d)}E_{ln}(\eta,b^l)E_{rq}(-\eta,b^r)d\eta$$
$$\varepsilon_n = \begin{cases} 2 & n=0 \\ 1 & \text{otherwise.}\end{cases}$$

Similarly, the tangential H_y continuity at $z=-h$ gives

$$
\begin{aligned}
&\frac{i}{\omega\mu_w\epsilon_w}(\alpha_1)^3 b_q^r E_{rp1}(\alpha,a^r)E_{rq}(\beta_y,b^r)\\
&-\frac{i}{\omega\mu_w\epsilon_a}\frac{\delta_{g0}\delta_{m0}\delta_{p0}}{\pi\alpha}\sum_{l=1}^{N}\sum_{n=1}^{\infty}a^l a^r (b_n^l)^2 b_q^r\\
&\times\left(C_{mn}^l e^{-i\xi_{mn}h}+D_{mn}^l e^{i\xi_{mn}h}\right)\left(k_w^2 I_2 - I_1\right) \qquad (10)\\
&+\frac{i}{\omega\mu_w\epsilon_a}\frac{1}{2\pi\alpha}\sum_{g=1}^{\infty}\sum_{l=1}^{N}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}(\alpha_g)^2 b_q^r\\
&\times\left(C_{mn}^l e^{-i\xi_{mn}h}+D_{mn}^l e^{i\xi_{mn}h}\right)E_{lmg}(\alpha,a^l)E_{rpg}(\alpha,a^r)\\
&\times\left[\left\{(a_m^l)^2-(b_n^l)^2\right\}I_1 - k_w^2 (b_n^l)^2 I_2\right]\\
&+\frac{i\epsilon_w}{\mu_a\epsilon_a}\frac{1}{2\pi\alpha}\sum_{g=1}^{\infty}\sum_{l=1}^{N}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}(\alpha_g)^2 a_m^l b_n^l b_q^r \xi_{mn}^l\xi_{mn}\\
&\times\left(\tilde{C}_{mn}^l e^{-i\xi_{mn}^l h}-\tilde{D}_{mn}^l e^{i\xi_{mn}^l h}\right)E_{lmg}(\alpha,a^l)E_{rpg}(\alpha,a^r)I_2\\
&=\frac{a^r b^r}{\omega\mu_a\epsilon_a}\left(C_{pq}^r e^{-i\xi_{pq}^r h}-D_{pq}^r e^{i\xi_{pq}^r h}\right)b_q^r\xi_{pq}^r\varepsilon_p\delta_{lr}\delta_{mp}\delta_{nq}\\
&\quad-\frac{a^r b^r}{\mu_a}\left(\tilde{C}_{pq}^r e^{-i\xi_{pq}^r h}+\tilde{D}_{pq}^r e^{i\xi_{pq}^r h}\right)a_p^r\delta_{lr}\delta_{mp}\delta_{nq}
\end{aligned}
$$

where

$$I_2=\int_{-\infty}^{\infty}\frac{1}{\gamma_g\tan(\gamma_g d)}E_{ln}(\eta,b^l)E_{rq}(-\eta,b^r)d\eta.$$

Equations (9) and (10) represent a set of simultaneous equations for the unknown coefficient C_{mn}^l, D_{mn}^l, $\tilde{C}_{mn}^l$, and $\tilde{D}_{mn}^l$. It is necessary to obtain another set of the simultaneous equation for C_{mn}^l, D_{mn}^l, $\tilde{C}_{mn}^l$, and $\tilde{D}_{mn}^l$, by using the boundary conditions at $z=0$. Another set of the simultaneous equations is given by Eq. (10) and (11), respectively, in [6].

The far-zone electric fields in region (III) can be evaluated as

$$E_\theta^{III}(\rho,\theta,\phi)=-\frac{k_o^3\cos\theta\sin\theta e^{i\rho k_o}}{2\pi\rho\omega\mu_o\epsilon_o}\tilde{A}_z^{III}(\zeta_o,\eta_o) \qquad (11)$$

$$E_\phi^{III}(\rho,\theta,\phi)=\frac{k_o^2\cos\theta\sin\theta e^{i\rho k_o}}{2\pi\rho\epsilon_o}\tilde{F}_z^{III}(\zeta_o,\eta_o) \qquad (12)$$

where the stationary points are $\zeta_o=-k_o\sin\theta\cos\phi$ and $\eta_o=-k_o\sin\theta\sin\phi$. $\tilde{A}_z^{III}(\zeta,\eta)$ and $\tilde{F}_z^{III}(\zeta,\eta)$ can be written in terms of C_{mn}^l, D_{mn}^l, $\tilde{C}_{mn}^l$, and $\tilde{D}_{mn}^l$.

The time-averaged incident power of TE_{10} mode is given by

$$
\begin{aligned}
P_{inc}&=\frac{1}{2}\int_{-h-d}^{-h}\int_{-\alpha}^{\alpha}\mathrm{Re}\left[E_z^i\left(H_x^i\right)^*\right]dxdz\\
&=\frac{\alpha d(\alpha_1)^2\beta_y}{2\omega\mu_w(\epsilon_w)^2} \qquad (13)
\end{aligned}
$$

where $()^*$ is complex conjugate.

To calculate reflected and transmitted fields in region (I), we arrange the scattered electric and magnetic field as follows

$$
\begin{aligned}
E_z^{I\pm}&=\frac{1}{\epsilon_a}\frac{1}{2\pi\alpha}\sum_{g=1}^{\infty}\sum_{l=1}^{N}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\sin\alpha_g(x+\alpha) \qquad (14)\\
&\times\left(C_{mn}^l e^{-i\xi_{mn}^l h}+D_{mn}^l e^{i\xi_{mn}^l h}\right)E_{lmg}(\alpha,a^l)I_{s1}^{lmng\pm}\\
&+\frac{1}{\omega\mu_a\epsilon_a}\frac{1}{2\pi\alpha}\sum_{g=1}^{\infty}\sum_{l=1}^{N}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}a_m^l b_n^l\xi_{mn}^l\sin\alpha_g(x+\alpha)\\
&\times\left(\tilde{C}_{mn}^l e^{-i\xi_{mn}^l h}-\tilde{D}_{mn}^l e^{i\xi_{mn}^l h}\right)E_{lmg}(\alpha,a^l)I_{s2}^{lmng\pm}
\end{aligned}
$$

$$
\begin{aligned}
H_x^{I\pm}&=-\frac{1}{\omega\mu_w\epsilon_a}\frac{1}{2\pi\alpha}\sum_{g=1}^{\infty}\sum_{l=1}^{N}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\sin\alpha_g(x+\alpha)\\
&\times\left[k_w^2(a_m^l)^2-\alpha_g^2\left\{(a_m^l)^2+(b_n^l)^2\right\}\right] \qquad (15)\\
&\times\left(C_{mn}^l e^{-i\xi_{mn}^l h}+D_{mn}^l e^{i\xi_{mn}^l h}\right)E_{lmg}(\alpha,a^l)I_{s3}^{lmng\pm}\\
&-\frac{\epsilon_w}{\mu_a\epsilon_a}\frac{1}{2\pi\alpha}\sum_{g=1}^{\infty}\sum_{l=1}^{N}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}a_m^l b_n^l\xi_{mn}^l\sin\alpha_g(x+\alpha)\\
&\times\left(\tilde{C}_{mn}^l e^{-i\xi_{mn}^l h}-\tilde{D}_{mn}^l e^{i\xi_{mn}^l h}\right)E_{lmg}(\alpha,a^l)I_{s3}^{lmng\pm}
\end{aligned}
$$

where

$$
\begin{aligned}
&I_{s1}^{lmng\pm}\\
&=\int_{-\infty}^{\infty}\frac{(a_m^l)^2\eta^2-(\alpha_g)^2(b_n^l)^2}{\gamma_g\sin(\gamma_g d)}\cos\gamma_g(z+h+d)\\
&\qquad\times E_{ln}(\eta,b^l)e^{-i\eta y}d\eta\\
&=\pm\frac{2\pi i}{d}\sum_{s=0}^{\infty}\left[\frac{(-1)^s}{\varepsilon_s}\frac{(a_m^l)^2\eta^2-\alpha_g^2(b_n^l)^2}{\eta}\cos\gamma_g(z+h+d)\right.\\
&\qquad\left.\times E_{ln}(\eta,b^l)e^{-i\eta y}\right]_{\eta=\mp\eta_s}
\end{aligned}
$$

$$
\begin{aligned}
&I_{s2}^{lmng\pm}\\
&=\int_{-\infty}^{\infty}\frac{\alpha_g^2+\eta^2}{\gamma_g\sin(\gamma_g d)}\cos\gamma_g(z+h+d)E_{ln}(\eta,b^l)e^{-i\eta y}d\eta\\
&=\pm\frac{2\pi i}{d}\sum_{s=0}^{\infty}\left[\frac{(-1)^s}{\varepsilon_s}\frac{\alpha_g^2+\eta^2}{\eta}\cos\gamma_g(z+h+d)\right.\\
&\qquad\left.\times E_{ln}(\eta,b^l)e^{-i\eta y}\right]_{\eta=\mp\eta_s}
\end{aligned}
$$

$$
\begin{aligned}
&I_{s3}^{lmng\pm}\\
&=\int_{-\infty}^{\infty}\frac{\eta}{\gamma_g\sin(\gamma_g d)}\cos\gamma_g(z+h+d)E_{ln}(\eta,b^l)e^{-i\eta y}d\eta\\
&=\pm\frac{2\pi i}{d}\sum_{s=0}^{\infty}\left[\frac{(-1)^s}{\varepsilon_s}\cos\gamma_g(z+h+d)\right.\\
&\qquad\left.\times E_{ln}(\eta,b^l)e^{-i\eta y}\right]_{\eta=\mp\eta_s}
\end{aligned}
$$

$$\eta_s=\sqrt{k_w^2-\alpha_g^2-\left(\frac{s\pi}{d}\right)}.$$

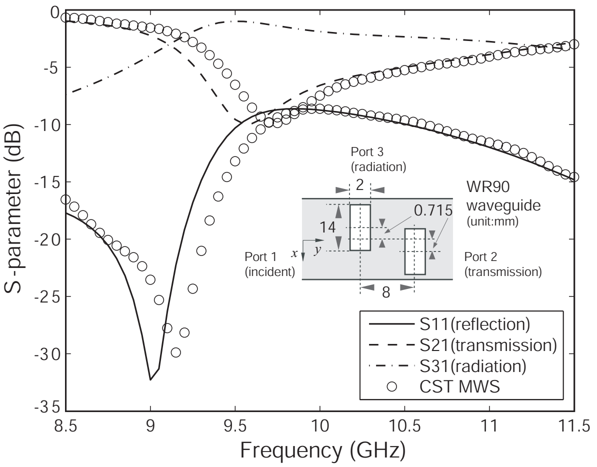

Fig. 2: S-parameter (slot thickness $h = 0.508mm$).

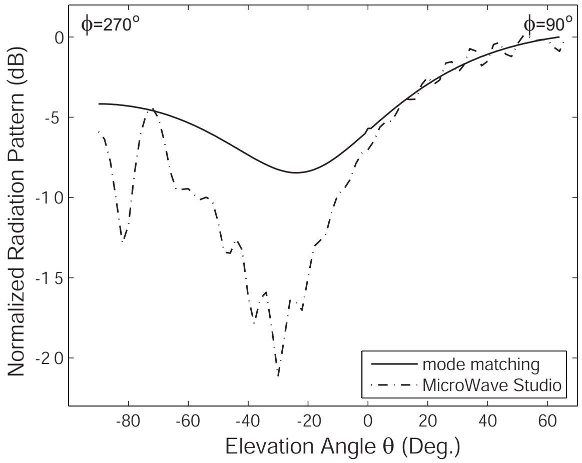

Fig. 3: Normalized E-plane radiation pattern.

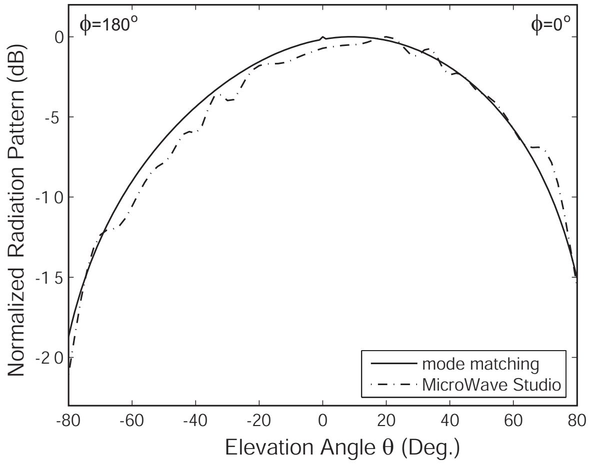

Fig. 4: Normalized H-plane radiation pattern.

The symbols $\pm$ in the above equations represent travelling waves along $\pm\hat{y}$ axes. The integrals $I_{s1}^{lmng\pm}$, $I_{s2}^{lmng\pm}$, and $I_{s3}^{lmng\pm}$ can be expressed in rapidly convergent series by utilizing residue calculus. We can calculate time-averaged reflected and transmitted coefficients using Eq. (14), Eq. (15), and the incident fields.

The time-averaged power radiated into region (III) from the apertures at $z = 0$ is

$$\begin{aligned} P_{rad} &= \frac{1}{2}\sum_{l=1}^{N}\int_{T_b^l-b^l}^{T_b^l+b^l}\int_{T_a^l-a^l}^{T_a^l+a^l}\mathrm{Re}\left[E_x^{II}\left(H_y^{II}\right)^* \right. \\ &\qquad \left. -E_y^{II}\left(H_x^{II}\right)^*\right]dxdy \\ &= \frac{1}{2\omega\mu_a\epsilon_a}\sum_{l=1}^{N}\mathrm{Re} \qquad (16) \\ &\times\left[\frac{1}{\epsilon_a}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}CD_{mn}^l\{(a_m^l)^2\varepsilon_n+(b_n^l)^2\varepsilon_m\}\left(\xi_{mn}^l\right)^* a^l b^l \right. \\ &\qquad \left. +\frac{1}{\mu_a}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\widetilde{CD}_{mn}^l\{(a_m^l)^2+(b_n^l)^2\}\xi_{mn}^l a^l b^l\right] \end{aligned}$$

where

$$\begin{aligned} CD_{mn}^l &= \left(C_{mn}^l+D_{mn}^l\right)\left(C_{mn}^l-D_{mn}^l\right)^* \\ \widetilde{CD}_{mn}^l &= \left(\tilde{C}_{mn}^l+\tilde{D}_{mn}^l\right)^*\left(\tilde{C}_{mn}^l-\tilde{D}_{mn}^l\right). \end{aligned}$$

4. COMPUTATIONS

Our numerical results of S-parameters are compared with other results to check the validity of our computation. Figure 2 shows the S-parameters for WR90 typical rectangular waveguide that has two identical transverse slots on a broad wall. The thickness of slots h is $0.508mm$. Our results agree well with CST MicroWave Studio(MWS) results while resonant frequency is slightly shifted. Figures 3 and 4 show normalized E- and H-plane radiation patterns at 9.5GHz, respectively. Our H-plane radiation pattern agrees well with that of CST MWS while reasonable agreement is seen between the E-plane radiation patterns. We assumed a finite-sized conducting flange ($50cm\times 50cm$) in CST MWS simulation. The reason for discrepancies in E-plane is due to diffraction from edges of the flange. Our computation uses the number of modes ($m = 2$, $n = 0$, $g = 3$, and $s = 1$). These numerical results show our formulation is efficient for evaluation.

5. CONCLUSION

We developed a rigorous formulation to investigate radiation and scattering properties of waveguide with slot array By using the Fourier transform and mode matching technique, we obtained the expressions for radiation and scattering in fast convergent series. Our numerical results are seen to agree with others. Our formulation is suitable for analyzing and designing a rectangular waveguide antenna with aperture array.

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