On Extrapolated Absorbing Boundary Condition Based on CIP Method

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Abstract

In this paper we propose extrapolated absorbing boundary conditions (EABCs) based on the cubic-interpolated pseudo-particle (CIP) method for solving electromagnetic problems by using finite volume time domain (FVTD) method. Extrapolation of fields at the boundary of the computational region is performed on the basis of time-splitting of the wave equation into two equations; one is the advection equation and the other is an equation including non-advection terms. Numerical calculations are carried out for checking numerical accuracy of the present method.

1. INTRODUCTION.

With the rapid development of high speed and large memory computers, the finite difference time domain (FDTD) method have been widely used, since Yee first proposed the algorithm [1]. Because of the finite memory sizes of computers, we have to realize virtual computational spaces by introducing absorbing boundary conditions (ABCs) [2]. In the earlier times, Mur's ABCs [3] were used by many researchers, but it is inevitable for this type of ABCs that some amount of fictitious reflections are always observed. As is well known, PMLs were successfully introduced by Berenger [4] to overcome this difficult situation.

ABCs based on PMLs are excellent, but a lot of computer memories are needed to implement them on computers. We proposed the EABCs in a very compact form [5], where a boundary field is extrapolated in terms of the linear combination of the fields at the two inner points adjacent to the boundary. In case of 1D, it has been demonstrated that the accuracy of the EABCs is nearly the same order as PMLs and it requires much less memories than the PMLs. However, basic theory of EABCs has not been clarified, and its accuracy has not been checked compared with that of PMLs in case of 2D and 3D.

In this paper, we first discuss the basic theory of EABCs from the view point of the CIP method which has ingeniously utilized the essence of the advection type of wave equation [6], [7]. Next, we propose EABCs for FVTD method where the boundary fields are extrapolated by using the interpolated fields in terms of the three inner points including the boundary. Finally we show some numerical examples for checking the accuracy of the present method.

2. BASIC EQUATIONS.

The Maxwell's equations are written as follows:

$$\begin{aligned} \nabla \times \boldsymbol{E} &= -\frac{\partial}{c\partial t}\tilde{\boldsymbol{H}} \\ \nabla \times \tilde{\boldsymbol{H}} &= +\frac{\partial}{c\partial t}\boldsymbol{E} \end{aligned} \tag{1}$$

where we assume that the medium is lossless and $c = 1/\sqrt{\epsilon\mu}$ is the light velocity. The magnetic field is normalized by the intrinsic impedance of the medium as follows:

$$\tilde{\boldsymbol{H}} = \sqrt{\frac{\mu}{\epsilon}}\boldsymbol{H} \tag{2}$$

Splitting each field component into two terms as Berenger did [4], the Maxwell's equations are rewritten as follows:

$$\begin{aligned} \frac{\partial}{c\partial t}\tilde{H}_{xy} &= -\frac{\partial}{\partial y}(E_{zx}+E_{zy}) \\ \frac{\partial}{c\partial t}\tilde{H}_{xz} &= +\frac{\partial}{\partial z}(E_{yz}+E_{yx}) \\ \frac{\partial}{c\partial t}\tilde{H}_{yz} &= -\frac{\partial}{\partial z}(E_{xy}+E_{xz}) \\ \frac{\partial}{c\partial t}\tilde{H}_{yx} &= +\frac{\partial}{\partial x}(E_{zx}+E_{zy}) \\ \frac{\partial}{c\partial t}\tilde{H}_{zx} &= -\frac{\partial}{\partial x}(E_{yz}+E_{yx}) \\ \frac{\partial}{c\partial t}\tilde{H}_{zy} &= +\frac{\partial}{\partial y}(E_{xy}+E_{xz}) \end{aligned} \tag{3}$$

$$\begin{aligned} \frac{\partial}{c\partial t}E_{xy} &= +\frac{\partial}{\partial y}(\tilde{H}_{zx}+\tilde{H}_{zy}) \\ \frac{\partial}{c\partial t}E_{xz} &= -\frac{\partial}{\partial z}(\tilde{H}_{yz}+\tilde{H}_{yx}) \\ \frac{\partial}{c\partial t}E_{yz} &= +\frac{\partial}{\partial z}(\tilde{H}_{xy}+\tilde{H}_{xz}) \\ \frac{\partial}{c\partial t}E_{yx} &= -\frac{\partial}{\partial x}(\tilde{H}_{zx}+\tilde{H}_{zy}) \\ \frac{\partial}{c\partial t}E_{zx} &= +\frac{\partial}{\partial x}(\tilde{H}_{yz}+\tilde{H}_{yx}) \\ \frac{\partial}{c\partial t}E_{zy} &= -\frac{\partial}{\partial y}(\tilde{H}_{xy}+\tilde{H}_{xz}) \end{aligned} \tag{4}$$

Adding and subtracting the above two equations, we have the advection equations with non-advection terms in the right hand sides as follows:

$$\frac{\partial}{c\partial t}(E_{yx} \pm \tilde{H}_{zx}) \pm \frac{\partial}{\partial x}(E_{yx} \pm \tilde{H}_{zx}) = \mp \frac{\partial}{\partial x}(E_{yz} \pm \tilde{H}_{zy})$$
$$\frac{\partial}{c\partial t}(E_{zx} \pm \tilde{H}_{yx}) \mp \frac{\partial}{\partial x}(E_{zx} \pm \tilde{H}_{yx}) = \pm \frac{\partial}{\partial x}(E_{zy} \pm \tilde{H}_{yz}) \quad (5)$$

$$\frac{\partial}{c\partial t}(E_{zy} \pm \tilde{H}_{xy}) \pm \frac{\partial}{\partial y}(E_{zy} \pm \tilde{H}_{xy}) = \mp \frac{\partial}{\partial y}(E_{zx} \pm \tilde{H}_{xz})$$
$$\frac{\partial}{c\partial t}(E_{xy} \pm \tilde{H}_{zy}) \mp \frac{\partial}{\partial y}(E_{xy} \pm \tilde{H}_{zy}) = \pm \frac{\partial}{\partial y}(E_{xz} \pm \tilde{H}_{zx}) \quad (6)$$

$$\frac{\partial}{c\partial t}(E_{xz} \pm \tilde{H}_{yz}) \pm \frac{\partial}{\partial z}(E_{xz} \pm \tilde{H}_{yz}) = \mp \frac{\partial}{\partial z}(E_{xy} \pm \tilde{H}_{yx})$$
$$\frac{\partial}{c\partial t}(E_{yz} \pm \tilde{H}_{xz}) \mp \frac{\partial}{\partial z}(E_{yz} \pm \tilde{H}_{xz}) = \pm \frac{\partial}{\partial z}(E_{yx} \pm \tilde{H}_{xy}) \quad (7)$$

3. CIP METHOD FOR SOLVING ADVECTION EQUATIONS.

All equations in Eqs.(5), (6) and (7) are reduced to one of the following two advection types of equations with non-advection terms in the right hand sides:

$$\frac{\partial}{c\partial t}\Phi_p + \frac{\partial}{\partial \xi}\Phi_p = \frac{\partial}{\partial \xi}\Psi_1$$
$$\frac{\partial}{c\partial t}\Phi_m - \frac{\partial}{\partial \xi}\Phi_m = \frac{\partial}{\partial \xi}\Psi_2 \quad (8)$$

The first equation shows advection in the plus ξ-direction (p), and the second shows advection in the minus ξ-direction (m). According to the CIP method, we can solve these two types of advection equations approximately by the method of time-splitting [7] described in the subsequent discussion.

At the first stage, we neglect the non-advection terms in Eq.(8), and then we have the following homogeneous advection equations:

$$\frac{\partial}{c\partial t}\Phi_p + \frac{\partial}{\partial \xi}\Phi_p = 0$$
$$\frac{\partial}{c\partial t}\Phi_m - \frac{\partial}{\partial \xi}\Phi_m = 0 \quad (9)$$

As is well-known, the above advection equations have solutions expressed as $\Phi_p = f(ct-\xi)$ and $\Phi_m = g(ct+\xi)$ where $f(\xi)$ and $g(\xi)$ are arbitrary continuous differentiable functions with respect to ξ. As a result, the value of Φ_p at $t = \tau+\Delta t$ and $\xi = a$ agrees with that at $t = \tau$ and $\xi = a-c\Delta t$. Similarly, the value of Φ_m at $t = \tau+\Delta t$ and $\xi = a$ agrees with that at $t = \tau$ and $\xi = a + c\Delta t$. If $\Phi_{p,m}$ are given in a discrete form, we can estimate their values in terms of interpolation. It should be noted that two points and two values, that is, field intensity and its derivative, at each point are used for interpolation in the CIP formulation. This is the basic idea of CIP [7]. Contrary to CIP, however, in case of FVTD it is difficult to use the derivatives of field components for field interpolation. This is why only the field components are computed in this FVTD method. In this paper, we use the Lagrange's method to interpolate the inner fields at one time-step in order to extrapolate the boundary fields at the next time-step.

Now we discretize the functions in Eq.(8) both in time and space as follows:

$$\Phi^n_{p,m}(i) = \Phi_{p,m}(n\Delta t, i\Delta\xi)$$
$$n = 0, 1, 2, \cdots \quad i = 0, \pm1, \pm2, \cdots \quad (10)$$

Then, as is stated before, interpolation in the advection equations leads to the following solutions:

$$\Phi^{n*}_p(i) = \Phi^n_p(i) + \Delta^{(+\xi)}_{adv}\Phi^n_p(i)$$
$$\Phi^{n*}_m(i) = \Phi^n_m(i) + \Delta^{(-\xi)}_{adv}\Phi^n_m(i) \quad (11)$$

where $n*$ is an appropriate time-step parameter within $n+1 > n* > n$ and the increments $\Delta^{(\pm\xi)}_{adv}\Phi^n_{p,m}(i)$ are evaluated in terms of Lagrange's interpolation as follows:

$$\Delta^{(+\xi)}_{adv}\Phi^n_p(i) = \sum_{k=0}^{M-1} w_k(x_0 - c\Delta t)[\Phi^n_p(i-k) - \Phi^n_p(i)]$$
$$\Delta^{(-\xi)}_{adv}\Phi^n_m(i) = \sum_{k=0}^{M-1} w_k(x_0 + c\Delta t)[\Phi^n_m(i+k) - \Phi^n_m(i)] \quad (12)$$

where M is the number of sampled points for interpolation. Moreover, the Lagrange's weights are given by

$$w_k(x) = \frac{\prod'^{M-1}_{j=0}(x - x_j)}{\prod'^{M-1}_{j=0}(x_k - x_j)} \qquad (k = 0, 1, 2, \cdots, M-1) \quad (13)$$

where $\prod'$ means that the product at $j = k$ is excluded. It should be noted that we have chosen as $M = 3$ for numerical computation of the Lagrange's interpolation.

At the next stage, based on the time-splitting approximation, we include the non-convection terms in Eq.(8) by solving the following equations [7]:

$$\frac{\partial}{c\partial t}\Phi_p = \frac{\partial}{\partial \xi}\Psi_1$$
$$\frac{\partial}{c\partial t}\Phi_m = \frac{\partial}{\partial \xi}\Psi_2 \quad (14)$$

In the FVTD formulation, no derivatives of field components are computed, although the right hand sides of Eq.(14) include them. So we follow the procedure that we first interpolate the field components and then we take their derivatives. Thus, the differences in time domain lead to the approximate solutions to Eq.(14) as follows:

$$\Phi^{n+1}_p(i) = \Phi^{n*}_p(i) + \Delta^{(+\xi)}_{non}\Psi^n_1(i)$$
$$\Phi^{n+1}_m(i) = \Phi^{n*}_m(i) + \Delta^{(-\xi)}_{non}\Psi^n_2(i) \quad (15)$$

where

$$\Delta^{(+\xi)}_{non}\Psi^n_1(i) = \frac{c\Delta t}{\Delta\xi}\sum_{k=0}^{M-1} w'_k(x_0)\Psi^n_1(i-k)$$
$$\Delta^{(-\xi)}_{non}\Psi^n_2(i) = \frac{c\Delta t}{\Delta\xi}\sum_{k=0}^{M-1} w'_k(x_0)\Psi^n_2(i+k) \quad (16)$$

where $w'_k(x)$ is the derivative of $w_k(x)$ in Eq.(13) with respect to x. As a result, approximate solutions to the advection equations in Eq.(8) are summarized in the following forms:

$$\begin{aligned}\Phi_p^{n+1}(i) &= \Phi_p^n(i) + \Delta_{adv}^{(+\xi)}\Phi_p^n(i) + \Delta_{non}^{(+\xi)}\Psi_1^n(i)\\ \Phi_m^{n+1}(i) &= \Phi_m^n(i) + \Delta_{adv}^{(-\xi)}\Phi_m^n(i) + \Delta_{non}^{(-\xi)}\Psi_2^n(i)\end{aligned} \tag{17}$$

4. Advection equations.

A. 3D advection equations.

Applying Eq.(17) to Eqs.(5), (6) and (7), we have the discretized CIP solutions to the Maxwell's equations as follows:

$$\begin{aligned}E_{yx}^{n+1}(i,j,k) \pm \tilde{H}_{zx}^{n+1}(i,j,k) &= E_{yx}^n(i,j,k) \pm \tilde{H}_{zx}^n(i,j,k)\\ &+ \Delta_{adv}^{(\pm x)}[E_{yx}^n(i,j,k) \pm \tilde{H}_{zx}^n(i,j,k)]\\ &\mp \Delta_{non}^{(\pm x)}[E_{yz}^n(i,j,k) \pm \tilde{H}_{zy}^n(i,j,k)]\\ E_{zx}^{n+1}(i,j,k) \pm \tilde{H}_{yx}^{n+1}(i,j,k) &= E_{zx}^n(i,j,k) \pm \tilde{H}_{yx}^n(i,j,k)\\ &+ \Delta_{adv}^{(\mp x)}[E_{zx}^n(i,j,k) \pm \tilde{H}_{yx}^n(i,j,k)]\\ &\pm \Delta_{non}^{(\mp x)}[E_{zy}^n(i,j,k) \pm \tilde{H}_{yz}^n(i,j,k)]\end{aligned} \tag{18}$$

$$\begin{aligned}E_{zy}^{n+1}(i,j,k) \pm \tilde{H}_{xy}^{n+1}(i,j,k) &= E_{zy}^n(i,j,k) \pm \tilde{H}_{xy}^n(i,j,k)\\ &+ \Delta_{adv}^{(\pm y)}[E_{zy}^n(i,j,k) \pm \tilde{H}_{xy}^n(i,j,k)]\\ &\mp \Delta_{non}^{(\pm y)}[E_{zx}^n(i,j,k) \pm \tilde{H}_{xz}^n(i,j,k)]\\ E_{xy}^{n+1}(i,j,k) \pm \tilde{H}_{zy}^{n+1}(i,j,k) &= E_{xy}^n(i,j,k) \pm \tilde{H}_{zy}^n(i,j,k)\\ &+ \Delta_{adv}^{(\mp y)}[E_{xy}^n(i,j,k) \pm \tilde{H}_{zy}^n(i,j,k)]\\ &\pm \Delta_{non}^{(\mp y)}[E_{xz}^n(i,j,k) \pm \tilde{H}_{zx}^n(i,j,k)]\end{aligned} \tag{19}$$

$$\begin{aligned}E_{xz}^{n+1}(i,j,k) \pm \tilde{H}_{yz}^{n+1}(i,j,k) &= E_{xz}^n(i,j,k) \pm \tilde{H}_{yz}^n(i,j,k)\\ &+ \Delta_{adv}^{(\pm z)}[E_{xz}^n(i,j,k) \pm \tilde{H}_{yz}^n(i,j,k)]\\ &\mp \Delta_{non}^{(\pm z)}[E_{xy}^n(i,j,k) \pm \tilde{H}_{yx}^n(i,j,k)]\\ E_{yz}^{n+1}(i,j,k) \pm \tilde{H}_{xz}^{n+1}(i,j,k) &= E_{yz}^n(i,j,k) \pm \tilde{H}_{xz}^n(i,j,k)\\ &+ \Delta_{adv}^{(\mp z)}[E_{yz}^n(i,j,k) \pm \tilde{H}_{xz}^n(i,j,k)]\\ &\pm \Delta_{non}^{(\mp z)}[E_{yx}^n(i,j,k) \pm \tilde{H}_{xy}^n(i,j,k)]\end{aligned} \tag{20}$$

B. 2D Advection equations.

The 2D advection equations can be derived from 3D equations by assuming that the 2D fields are uniform in z-direction, that is, $\partial/\partial z = 0$. As a result, Eqs.(5), (6) and (7) can be rewritten as follows:

$$\begin{aligned}\frac{\partial}{c\partial t}(E_{yx} \pm \tilde{H}_{zx}) \pm \frac{\partial}{\partial x}(E_{yx} \pm \tilde{H}_{zx}) &= -\frac{\partial}{\partial x}\tilde{H}_{zy}\\ \frac{\partial}{c\partial t}(E_{zx} \pm \tilde{H}_{yx}) \mp \frac{\partial}{\partial x}(E_{zx} \pm \tilde{H}_{yx}) &= \pm\frac{\partial}{\partial x}E_{zy}\end{aligned} \tag{21}$$

$$\begin{aligned}\frac{\partial}{c\partial t}(E_{zy} \pm \tilde{H}_{xy}) \pm \frac{\partial}{\partial y}(E_{zy} \pm \tilde{H}_{xy}) &= \mp\frac{\partial}{\partial y}E_{zx}\\ \frac{\partial}{c\partial t}(E_{xy} \pm \tilde{H}_{zy}) \mp \frac{\partial}{\partial y}(E_{xy} \pm \tilde{H}_{zy}) &= +\frac{\partial}{\partial y}\tilde{H}_{zx}\end{aligned} \tag{22}$$

The above equations can be divided into two independent fields E-wave (TM-wave) and H-wave (TE-wave) as follows:

$$\begin{aligned}\frac{\partial}{c\partial t}(E_{zx} \pm \tilde{H}_{y}) \mp \frac{\partial}{\partial x}(E_{zx} \pm \tilde{H}_{y}) &= \pm\frac{\partial}{\partial x}E_{zy}\\ \frac{\partial}{c\partial t}(E_{zy} \pm \tilde{H}_{x}) \pm \frac{\partial}{\partial y}(E_{zy} \pm \tilde{H}_{x}) &= \mp\frac{\partial}{\partial y}E_{zx}\end{aligned} \tag{23}$$

$$\begin{aligned}\frac{\partial}{c\partial t}(\tilde{H}_{zx} \pm E_{y}) \pm \frac{\partial}{\partial x}(\tilde{H}_{zx} \pm E_{y}) &= \mp\frac{\partial}{\partial x}\tilde{H}_{zy}\\ \frac{\partial}{c\partial t}(\tilde{H}_{zy} \pm E_{x}) \mp \frac{\partial}{\partial y}(\tilde{H}_{zy} \pm E_{x}) &= \pm\frac{\partial}{\partial y}\tilde{H}_{zx}\end{aligned} \tag{24}$$

It should be noted that $\tilde{H}_{yx} = \tilde{H}_y$ and $\tilde{H}_{xy} = \tilde{H}_x$ for E-wave, and $E_{yx} = E_y$ and $E_{xy} = E_x$ for H-wave, respectively.

The CIP algorithm discussed in the preceding section provides the following discretized solutions for E-wave and H-wave:

$$\begin{aligned}E_{zx}^{n+1}(i,j) \pm \tilde{H}_{y}^{n+1}(i,j) &= E_{zx}^n(i,j) \pm \tilde{H}_{y}^n(i,j)\\ &+ \Delta_{adv}^{(\mp x)}[E_{zx}^n(i,j) \pm \tilde{H}_{y}^n(i,j)] \pm \Delta_{non}^{(\mp x)}E_{zy}^n(i,j)\\ E_{zy}^{n+1}(i,j) \pm \tilde{H}_{x}^{n+1}(i,j) &= E_{zy}^n(i,j) \pm \tilde{H}_{x}^n(i,j)\\ &+ \Delta_{adv}^{(\pm y)}[E_{zy}^n(i,j) \pm \tilde{H}_{x}^n(i,j)] \mp \Delta_{non}^{(\pm y)}E_{zx}^n(i,j)\end{aligned} \tag{25}$$

$$\begin{aligned}\tilde{H}_{zx}^{n+1}(i,j) \pm E_{y}^{n+1}(i,j) &= \tilde{H}_{zx}^n(i,j) \pm E_{y}^n(i,j)\\ &+ \Delta_{adv}^{(\pm x)}[\tilde{H}_{zx}^n(i,j) \pm E_{y}^n(i,j)] \mp \Delta_{non}^{(\pm x)}\tilde{H}_{zy}^n(i,j)\\ \tilde{H}_{zy}^{n+1}(i,j) \pm E_{x}^{n+1}(i,j) &= \tilde{H}_{zy}^n(i,j) \pm E_{x}^n(i,j)\\ &+ \Delta_{adv}^{(\mp y)}[\tilde{H}_{zy}^n(i,j) \pm E_{x}^n(i,j)] \pm \Delta_{non}^{(\mp y)}\tilde{H}_{zx}^n(i,j)\end{aligned} \tag{26}$$

C. 1D Advection equations.

The 1D advection equations can be derived from 2D ones by assuming that the 1D fields are uniform both in z and y-direction, that is, $\partial/\partial z = 0$ and $\partial/\partial y = 0$. As a result, the second equation in Eq.(21) can be rewritten as follows:

$$\frac{\partial}{c\partial t}(E_z \pm \tilde{H}_y) \mp \frac{\partial}{\partial x}(E_z \pm \tilde{H}_y) = 0 \tag{27}$$

where $E_{zx} = E_z$ and $\tilde{H}_{yx} = \tilde{H}_y$ are introduced. It should be noted that the above relations include no non-advection terms, and therefore we have accurate solutions to the advection equations as follows:

$$\begin{aligned}E_z^{n+1}(i) \pm \tilde{H}_y^{n+1}(i) &= E_z^n(i) \pm \tilde{H}_y^n(i)\\ &+ \Delta_{adv}^{(\mp x)}[E_z^n(i) \pm \tilde{H}_y^n(i)]\end{aligned} \tag{28}$$

5. Application to EABCs for FVTD.

A. 1D EABCs.

In this section we study the ABCs for FVTD computations. First we consider the most simple case, that is, 1D problem. Assume that there exist two boundaries at $i = -N_x$ and $i = N_x$ with a source between the two boundaries. Then the radiated waves are always traveling outward at the boundaries. In other words, it is enough to consider only the advection terms for difference equations, since the non-advection terms are always zero in this 1D case. Consequently, the EABCs at $i = N_x$ are expressed as follows:

$$\begin{aligned} E_z^{n+1}(N_x) + \tilde{H}_y^{n+1}(N_x) &= E_z^n(N_x) + \tilde{H}_y^n(N_x) \\ E_z^{n+1}(N_x) - \tilde{H}_y^{n+1}(N_x) &= E_z^n(N_x) - \tilde{H}_y^n(N_x) \\ &+ \Delta_{adv}^{(+x)}[E_z^n(N_x) - \tilde{H}_y^n(N_x)] \end{aligned} \tag{29}$$

The above relations can be rearranged in a more simplified form as follows:

$$E_z^{n+1}(N_x) = E_z^n(N_x) + \Delta_{adv}^{(+x)} E_z^n(N_x) \tag{30}$$

It should be noted that electric field always equals to magnetic field, that is, $E_z^n(N_x) = -\tilde{H}_y^n(N_x)$. Similarly, we have the following relation at $i = -N_x$

$$E_z^{n+1}(-N_x) = E_z^n(-N_x) + \Delta_{adv}^{(-x)} E_z^n(-N_x) \tag{31}$$

together with $E_z^n(-N_x) = \tilde{H}_y^n(-N_x)$.

Present EABCs are somewhat different from the former EABCs [5]. In the former case, the boundary fields were evaluated by the linear extrapolation using the inner fields at the two points adjacent to the boundary. In the present method, on the other hand, interpolated fields at one time step are used to extrapolted the boundary fields at next time step, where the interpolation is made in terms of the fields at three points including the boundary. It is worth noting that present method shows a better improvement over the former one [5].

Fig. 1 shows numerical examples to check the accuracy of the present method in comparison with PML with 21 absorbing layers. Cell size for 1D EABC is chosen as 1000 and the large cell size for reference FVTD is chosen as 20000. As for the time step, n=100 corresponds to one cycle, and the error of PML at n=2000 is -84.6 [dB] and that of EABC is -82.2 [dB]. It is demonstrated that the accuracy EABC is almost the same order as that of PML.

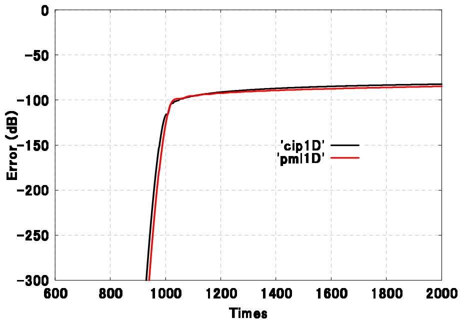

Fig. 1: Error of 1D EABC in Comparson with 1D PML

B. 2D EABCS

In this subsection, we consider 2D EABCs based on CIP at the boundaries $x = \pm N_x$ and $y = \pm N_y$. As for E-wave, Eq.(25) leads to the following relations:

$$\begin{aligned} &E_{zx}^{n+1}(+N_x, j) - \tilde{H}_y^{n+1}(+N_x, j) = \\ &\quad + E_{zx}^n(+N_x, j) - \tilde{H}_y^n(+N_x, j) \\ &\quad + \Delta_{adv}^{(+x)}[E_{zx}^n(+N_x, j) - \tilde{H}_y^n(+N_x, j)] \\ &\quad - \Delta_{non}^{(+x)} E_{zy}^n(+N_x, j) \\ &E_{zx}^{n+1}(+N_x, j) + \tilde{H}_y^{n+1}(+N_x, j) = \\ &\quad + E_{zx}^n(+N_x, j) + \tilde{H}_y^n(+N_x, j) \\ &\quad + \Delta_{non}^{(+x)} E_{zy}^n(+N_x, j) \end{aligned} \tag{32}$$

$$\begin{aligned} &E_{zx}^{n+1}(-N_x, j) + \tilde{H}_y^{n+1}(-N_x, j) = \\ &\quad + E_{zx}^n(-N_x, j) + \tilde{H}_y^n(-N_x, j) \\ &\quad + \Delta_{adv}^{(-x)}[E_{zx}^n(-N_x, j) + \tilde{H}_y^n(-N_x, j)] \\ &\quad + \Delta_{non}^{(-x)} E_{zy}^n(-N_x, j) \\ &E_{zx}^{n+1}(-N_x, j) - \tilde{H}_y^{n+1}(-N_x, j) = \\ &\quad + E_{zx}^n(-N_x, j) - \tilde{H}_y^n(-N_x, j) \\ &\quad - \Delta_{non}^{(-x)} E_{zy}^n(-N_x, j) \end{aligned} \tag{33}$$

$$\begin{aligned} &E_{zy}^{n+1}(i, +N_y) + \tilde{H}_x^{n+1}(i, +N_y) = \\ &\quad + E_{zy}^n(i, +N_y) + \tilde{H}_x^n(i, +N_y) \\ &\quad + \Delta_{adv}^{(+y)}[E_{zy}^n(i, +N_y) + \tilde{H}_x^n(i, +N_y)] \\ &\quad - \Delta_{non}^{(+y)} E_{zx}^n(i, +N_y) \\ &E_{zy}^{n+1}(i, +N_y) - \tilde{H}_x^{n+1}(i, +N_y) = \\ &\quad E_{zy}^n(i, +N_y) - \tilde{H}_x^n(i, +N_y) \\ &\quad + \Delta_{non}^{(+y)} E_{zx}^n(i, +N_y) \end{aligned} \tag{34}$$

$$\begin{aligned} &E_{zy}^{n+1}(i, -N_y) - \tilde{H}_x^{n+1}(i, -N_y) = \\ &\quad + E_{zy}^n(i, -N_y) - \tilde{H}_x^n(i, -N_y) \\ &\quad + \Delta_{adv}^{(-y)}[E_{zy}^n(i, -N_y) - \tilde{H}_x^n(i, -N_y)] \\ &\quad + \Delta_{non}^{(-y)} E_{zx}^n(i, -N_y) \\ &E_{zy}^{n+1}(i, -N_y) + \tilde{H}_x^{n+1}(i, -N_y) = \\ &\quad + E_{zy}^n(i, -N_y) + \tilde{H}_x^n(i, -N_y) \\ &\quad - \Delta_{non}^{(-y)} E_{zx}^n(i, -N_y) \end{aligned} \tag{35}$$

As for H-wave, Eq.(26) leads to the following relations:

$$\begin{aligned} &\tilde{H}_{zx}^{n+1}(+N_x, j) + E_y^{n+1}(+N_x, j) = \\ &\quad + \tilde{H}_{zx}^n(+N_x, j) + E_y^n(+N_x, j) \\ &\quad + \Delta_{adv}^{(+x)}[\tilde{H}_{zx}^n(+N_x, j) + E_y^n(+N_x, j)] \\ &\quad - \Delta_{non}^{(+x)} \tilde{H}_{zy}^n(+N_x, j) \\ &\tilde{H}_{zx}^{n+1}(+N_x, j) - E_y^{n+1}(+N_x, j) = \\ &\quad + \tilde{H}_{zx}^n(+N_x, j) - E_y^n(+N_x, j) \\ &\quad + \Delta_{non}^{(+x)} \tilde{H}_{zy}^n(+N_x, j) \end{aligned} \tag{36}$$

$$
\begin{aligned}
\tilde{H}_{zx}^{n+1}(-N_x,j) - E_y^{n+1}(-N_x,j) = \\
&+ \tilde{H}_{zx}^{n}(-N_x,j) - E_y^{n}(-N_x,j) \\
&+ \Delta_{adv}^{(-x)}[\tilde{H}_{zx}^{n}(-N_x,j) - E_y^{n}(-N_x,j)] \\
&+ \Delta_{non}^{(-x)}\tilde{H}_{zy}^{n}(-N_x,j) \\
\tilde{H}_{zx}^{n+1}(-N_x,j) + E_y^{n+1}(-N_x,j) = \\
&+ \tilde{H}_{zx}^{n}(-N_x,j) + E_y^{n}(-N_x,j) \\
&- \Delta_{non}^{(-x)}\tilde{H}_{zy}^{n}(-N_x,j)
\end{aligned} \tag{37}
$$

$$
\begin{aligned}
\tilde{H}_{zy}^{n+1}(i,+N_y) - E_x^{n+1}(i,+N_y) = \\
&+ \tilde{H}_{zy}^{n}(i,+N_y) - E_x^{n}(i,+N_y) \\
&+ \Delta_{adv}^{(+y)}[\tilde{H}_{zy}^{n}(i,+N_y) - E_x^{n}(i,+N_y)] \\
&- \Delta_{non}^{(+y)}\tilde{H}_{zx}^{n}(i,+N_y) \\
\tilde{H}_{zy}^{n+1}(i,+N_y) + E_x^{n+1}(i,+N_y) = \\
&+ \tilde{H}_{zy}^{n}(i,+N_y) + E_x^{n}(i,+N_y) \\
&+ \Delta_{non}^{(+y)}\tilde{H}_{zx}^{n}(i,+N_y)
\end{aligned} \tag{38}
$$

$$
\begin{aligned}
\tilde{H}_{zy}^{n+1}(i,-N_y) + E_x^{n+1}(i,-N_y) = \\
&+ \tilde{H}_{zy}^{n}(i,-N_y) + E_x^{n}(i,-N_y) \\
&+ \Delta_{adv}^{(-y)}[\tilde{H}_{zy}^{n}(i,-N_y) + E_x^{n}(i,-N_y)] \\
&+ \Delta_{non}^{(-y)}\tilde{H}_{zx}^{n}(i,-N_y) \\
\tilde{H}_{zy}^{n+1}(i,-N_y) - E_x^{n+1}(i,-N_y) = \\
&+ \tilde{H}_{zy}^{n}(i,-N_y) - E_x^{n}(i,-N_y) \\
&- \Delta_{non}^{(-y)}\tilde{H}_{zx}^{n}(i,-N_y)
\end{aligned} \tag{39}
$$

Fig. 2 shows errors [dB] in case of E-wave when a continuous-wave is excited at the center of the FVTD cells. Cell size for EABC is chosen as (200,200) and the large cell size for reference FVTD is chosen as (1200,1200). As for the time step, n=20 corresponds to one cycle, and the error of the present method at n=1000 is -35.1 [dB] and that of PML with 21 cells is -60.1 [dB]. It is demonstrated that although the accuracy of the present method is not so good as that of PML, it shows much better accuracy than that of Mur's ABC [2].

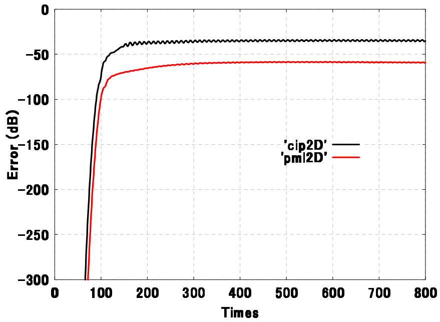

Fig. 2: Error of 2D EABC in Comparison with 2D PML

C. 3D EABCs.

In case of 3D, we assume that the boundaries are at $x = \pm N_x\Delta x$, $y = \pm N_y\Delta y$ and $z = \pm N_z\Delta z$. Then the EABCs for E_{yx} and E_{zx} are summarized as follows:

$$
\begin{aligned}
E_{yx}^{n+1}(+N_x,j,k) + \tilde{H}_{zx}^{n+1}(+N_x,j,k) = \\
&+ E_{yx}^{n}(+N_x,j,k) + \tilde{H}_{zx}^{n}(+N_x,j,k) \\
&+ \Delta_{adv}^{(+x)}[E_{yx}^{n}(+N_x,j,k) + \tilde{H}_{zx}^{n}(+N_x,j,k)] \\
&- \Delta_{non}^{(+x)}[E_{yz}^{n}(+N_x,j,k) + \tilde{H}_{zy}^{n}(+N_x,j,k)] \\
E_{yx}^{n+1}(+N_x,j,k) - \tilde{H}_{zx}^{n+1}(+N_x,j,k) = \\
&+ E_{yx}^{n}(+N_x,j,k) - \tilde{H}_{zx}^{n}(+N_x,j,k) \\
&+ \Delta_{non}^{(+x)}[E_{yz}^{n}(+N_x,j,k) - \tilde{H}_{zy}^{n}(+N_x,j,k)]
\end{aligned}
$$

$$
\begin{aligned}
E_{zx}^{n+1}(+N_x,j,k) - \tilde{H}_{yx}^{n+1}(+N_x,j,k) = \\
&+ E_{zx}^{n}(+N_x,j,k) - \tilde{H}_{yx}^{n}(+N_x,j,k) \\
&+ \Delta_{adv}^{(+x)}[E_{zx}^{n}(+N_x,j,k) - \tilde{H}_{yx}^{n}(+N_x,j,k)] \\
&- \Delta_{non}^{(+x)}[E_{zy}^{n}(+N_x,j,k) - \tilde{H}_{yz}^{n}(+N_x,j,k)] \\
E_{zx}^{n+1}(+N_x,j,k) + \tilde{H}_{yx}^{n+1}(+N_x,j,k) = \\
&+ E_{zx}^{n}(+N_x,j,k) + \tilde{H}_{yx}^{n}(+N_x,j,k) \\
&+ \Delta_{non}^{(+x)}[E_{zy}^{n}(+N_x,j,k) + \tilde{H}_{yz}^{n}(+N_x,j,k)]
\end{aligned} \tag{40}
$$

$$
\begin{aligned}
E_{yx}^{n+1}(-N_x,j,k) - \tilde{H}_{zx}^{n+1}(-N_x,j,k) = \\
&E_{yx}^{n}(-N_x,j,k) - \tilde{H}_{zx}^{n}(-N_x,j,k) \\
&+ \Delta_{adv}^{(-x)}[E_{yx}^{n}(-N_x,j,k) - \tilde{H}_{zx}^{n}(-N_x,j,k)] \\
&+ \Delta_{non}^{(-x)}[E_{yz}^{n}(-N_x,j,k) - \tilde{H}_{zy}^{n}(-N_x,j,k)] \\
E_{yx}^{n+1}(-N_x,j,k) + \tilde{H}_{zx}^{n+1}(-N_x,j,k) = \\
&E_{yx}^{n}(-N_x,j,k) + \tilde{H}_{zx}^{n}(-N_x,j,k) \\
&- \Delta_{non}^{(-x)}[E_{yz}^{n}(-N_x,j,k) + \tilde{H}_{zy}^{n}(-N_x,j,k)]
\end{aligned}
$$

$$
\begin{aligned}
E_{zx}^{n+1}(-N_x,j,k) + \tilde{H}_{yx}^{n+1}(-N_x,j,k) = \\
&E_{zx}^{n}(-N_x,j,k) + \tilde{H}_{yx}^{n}(-N_x,j,k) \\
&+ \Delta_{adv}^{(-x)}[E_{zx}^{n}(-N_x,j,k) + \tilde{H}_{yx}^{n}(-N_x,j,k)] \\
&+ \Delta_{non}^{(-x)}[E_{zy}^{n}(-N_x,j,k) + \tilde{H}_{yz}^{n}(-N_x,j,k)] \\
E_{zx}^{n+1}(-N_x,j,k) - \tilde{H}_{yx}^{n+1}(-N_x,j,k) = \\
&E_{zx}^{n}(-N_x,j,k) - \tilde{H}_{yx}^{n}(-N_x,j,k) \\
&- \Delta_{non}^{(-x)}[E_{zy}^{n}(-N_x,j,k) - \tilde{H}_{yz}^{n}(-N_x,j,k)]
\end{aligned} \tag{41}
$$

Similar expressions are obtained for E_{zy}, E_{xy}, E_{xz} and E_{yz}, but they are omitted here for brevity of the paper.

Fig. 3 shows errors [dB] in case of 3D when a continuous sine-wave is excited at the center of the FVTD cells. Cell size for EABC is chosen as (50,50,50) and the large cell size for reference FVTD is chosen as (130,130,130). As for the time step, n=20 corresponds to one cycle, and the error of the present method at n=150 is -20.1 [dB]. Numerical results based on the 3D PMLs are not shown here, but it might be concluded that the accuracy of the present 3D EABCs is not satisfactory for practical applications.

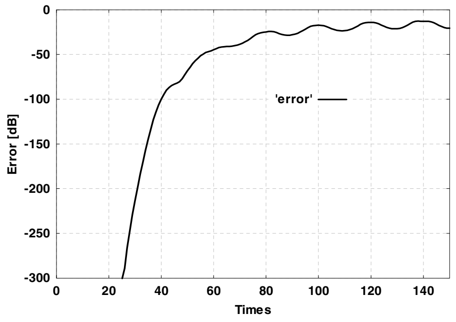

Fig. 3: Error of 3D EABC

7. Conclusion.

In this paper, we have proposed the EABCs for FVTD method by extrapolating the electromagnetic fields at the computational boundaries by employing the theory of CIP. In the present FVTD formulations, we have utilized Lagrange's interpolation based on the curves of second order for evaluating both advection and non-advection terms. Numerical calculations were carried out for checking the accuracy of the present method in comparison with PML in case of 1D and 2D problems. In case of 1D the proposed method improves the absorbing efficiency compared with the former EABC based on the linear extrapolation. It is concluded that although numerical accuracy of the present method is not so good as PML in case of 2D, it saves much more computer memories than PML and the accuracy might be acceptable for some practical problems.

It deserves as a future problem to develop a new algorithm which can cope with the 3D non-advection terms more accurately than the present one.

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