LOD-FDTD Analysis of Optical Waveguides with Nondispersive and Dispersive Media

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Abstract

We formulate an efficient implicit FDTD method based on the locally one-dimensional (LOD) scheme and introduce the recursive convolution (RC), piecewise linear recursive convolution (PLRC) and auxiliary differential equation (ADE) methods into the LOD-FDTD for the analysis of frequency-dependent materials. As an application, an optical waveguide with an endfacet is analyzed, demonstrating the validity and the efficiency of the LOD-FDTD. Furthermore, an optical waveguide with a metal cladding is analyzed. It is shown that the results obtained from the PLRC- and ADE-based LOD-FDTDs agree well with the result from the explicit FDTD, even with a time step being ten times as large as that used in the explicit FDTD.

1. INTRODUCTION

The finite-difference time-domain (FDTD) method has widely been used to obtain characteristics of various optical waveguides. Recall, however, that a time increment (Δt) of the FDTD is limited by the Courant-Friedrich-Levy (CFL) condition. To remove this restriction, the FDTD based on the alternating-direction implicit (ADI) scheme [1], [2] has been developed. On the other hand, we have developed an unconditionally stable FDTD based on the locally one-dimensional (LOD) scheme [3], [4]. The LOD-FDTD was also independently formulated in [5], and the split-field perfectly matched layer was investigated [6]. The main advantage of the LOD-FDTD is that the algorithm is quite simple with a subsequent reduction in the computational time, while maintaining the accuracy comparable to the ADI-FDTD. Note, however, that no detailed formulation of the LOD-FDTD has been given for the TM mode. In addition, the LOD-FDTD has not been extended to the problem with material dispersions.

In this paper, we present the TM mode formulation of the LOD-FDTD and introduce the recursive convolution (RC), piecewise linear recursive convolution (PLRC) and auxiliary differential equation (ADE) methods into the LOD-FDTD for the analysis of dispersive media. As an application, we analyze optical waveguides with an endfacet and with a metal cladding.

2. FORMULATION OF THE LOD-FDTD

We first consider a TM case for nondispersive media. Maxwell's equations are expressed as

$$\frac{\partial \phi}{\partial t} = ([A]+[B])\phi \tag{1}$$

where $\phi = [H_y, E_x, E_z]^T$ and

$$[A] = \begin{bmatrix} 0 & 0 & -\frac{\partial}{\varepsilon \partial z} \\ 0 & 0 & 0 \\ -\frac{\partial}{\mu \partial z} & 0 & 0 \end{bmatrix}, \quad [B] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & \frac{\partial}{\varepsilon \partial x} \\ 0 & \frac{\partial}{\mu \partial x} & 0 \end{bmatrix}$$

in which ε and μ represent permittivity and permeability, respectively. After applying the Crank-Nicolson scheme to (1) and factoring the resultant equation, we obtain

$$\phi^{n+1} = \frac{\left([I]+\frac{\Delta t}{2}[A]\right)\left([I]+\frac{\Delta t}{2}[B]\right)}{\left([I]-\frac{\Delta t}{2}[A]\right)\left([I]-\frac{\Delta t}{2}[B]\right)}\phi^n . \tag{2}$$

(2) can be solved in two steps using not only the ADI scheme but also the LOD scheme. Application of the LOD scheme to (2) results in

$$\phi^{n+1/2} = \frac{\left([I]+\frac{\Delta t}{2}[B]\right)}{\left([I]-\frac{\Delta t}{2}[B]\right)}\phi^n \tag{3a}$$

for the first step and

$$\phi^{n+1} = \frac{\left([I]+\frac{\Delta t}{2}[A]\right)}{\left([I]-\frac{\Delta t}{2}[A]\right)}\phi^{n+1/2} \tag{3b}$$

for the second step. Note that unlike the ADI-FDTD, in each half step of the LOD-FDTD, we move forward only in the x or z direction.

From (3a) and (3b), we derive

$$E_x^{n+1/2} = E_x^n \tag{4a}$$

$$\frac{E_z^{n+1/2} - E_z^n}{\Delta t/2} = \frac{1}{\varepsilon}\left(\frac{\partial H_y^{n+1/2}}{\partial x} + \frac{\partial H_y^n}{\partial x}\right) \tag{4b}$$

$$\frac{H_y^{n+1/2} - H_y^n}{\Delta t/2} = \frac{1}{\mu}\left(\frac{\partial E_z^{n+1/2}}{\partial x} + \frac{\partial E_z^n}{\partial x}\right) \tag{4c}$$

for the first step and

$$E_z^{n+1} = E_z^{n+1/2} \tag{5a}$$

$$\frac{E_x^{n+1} - E_x^{n+1/2}}{\Delta t/2} = -\frac{1}{\varepsilon}\left(\frac{\partial H_y^{n+1}}{\partial z} + \frac{\partial H_y^{n+1/2}}{\partial z}\right) \tag{5b}$$

$$\frac{H_y^{n+1} - H_y^{n+1/2}}{\Delta t/2} = -\frac{1}{\mu}\left(\frac{\partial E_x^{n+1}}{\partial z} + \frac{\partial E_x^{n+1/2}}{\partial z}\right) \tag{5c}$$

for the second step. In the first step, we substitute (4c) into (4b) and implicitly solve the resultant equation. Then, (4c) is explicitly solved. In the second step, the equations are calculated in the same way as in the first step. It should be noted that for the LOD-FDTD two implicit and two explicit equations are solved. As a result, the number of explicit equations to be solved is reduced, when compared with the ADI-FDTD in which two implicit and four explicit equations should be solved.

For dispersive media, the RC [8] and PLRC [9] methods are developed, in which the convolution can be efficiently performed using recursion. It is assumed that for the RC method the electric field is constant over Δt, while for the PLRC method the electric field has piecewise linear functional dependence over Δt. Therefore, the PLRC method may achieve better accuracy than the RC method. On the other hand, the ADE method [10], [11] is a technique that uses the differential equation involving the electric field and the electric flux density. Although the ADE method provides a simple formulation when compared with the RC and PLRC methods, the computational time and memory are increased due to the calculation of the electric flux density.

We here introduce the RC and PLRC methods into the LOD-FDTD for the analysis of dispersive media, expressed in a Drude model. Application of the PLRC method to the LOD-FDTD results in

$$E_z^{n+1} = \frac{\varepsilon_\infty - \xi^0}{\varepsilon_\infty + \chi^0 - \xi^0} E_z^n + \frac{1}{\varepsilon_\infty + \chi^0 - \xi^0}\phi_z^n + \frac{\Delta t}{2\varepsilon_0\left(\varepsilon_\infty + \chi^0 - \xi^0\right)}\left(\frac{\partial H_y^{n+1/2}}{\partial x} + \frac{\partial H_y^n}{\partial x}\right) \tag{6a}$$

$$\frac{H_y^{n+1/2} - H_y^n}{\Delta t/2} = \frac{1}{\mu}\left(\frac{\partial E_z^{n+1}}{\partial x} + \frac{\partial E_z^n}{\partial x}\right) \tag{6b}$$

for the first step and

$$E_x^{n+1} = \frac{\varepsilon_\infty - \xi^0}{\varepsilon_\infty + \chi^0 - \xi^0} E_x^n + \frac{1}{\varepsilon_\infty + \chi^0 - \xi^0}\phi_x^n - \frac{\Delta t}{2\varepsilon_0\left(\varepsilon_\infty + \chi^0 - \xi^0\right)}\left(\frac{\partial H_y^{n+1}}{\partial z} + \frac{\partial H_y^{n+1/2}}{\partial z}\right) \tag{7a}$$

$$\frac{H_y^{n+1} - H_y^{n+1/2}}{\Delta t/2} = -\frac{1}{\mu}\left(\frac{\partial E_x^{n+1}}{\partial z} + \frac{\partial E_x^n}{\partial z}\right) \tag{7b}$$

for the second step, where ε_0 is the permittivity of free space, and ε_∞ the dielectric constant of the materials at infinite frequency. ϕ^n is expressed as

$$\begin{aligned}\phi^n &= \sum_{m=0}^{n-1}\left[E^{n-m}\Delta\chi^m + \left(E^{n-m-1} - E^{n-m}\right)\Delta\xi^m\right] \\ &= \left(\Delta\chi^0 - \Delta\xi^0\right)E^n + \Delta\xi^0 E^{n-1} + e^{-\nu_c\Delta t}\phi^{n-1}\end{aligned} \tag{8}$$

where ν_c is the collision frequency. The expressions for χ^0, ξ^0, $\Delta\chi^m$ and $\Delta\xi^m$ are given in [9]. When ξ^m =0 and $\Delta\xi^m$ = 0 are adopted for all m, we obtain the equations for the RC method. In the first step, we substitute (6b) into (6a) and implicitly solve the resultant equation. Then, (6b) is explicitly solved. In the second step, the equations are calculated in the same way as in the first step. As a result, we solve two implicit and two explicit equations.

On the other hand, applying the ADE method to the LOD-FDTD, we obtain

$$\frac{D_z^{n+1} - D_z^n}{\Delta t/2} = \frac{\partial H_y^{n+1/2}}{\partial x} + \frac{\partial H_y^n}{\partial x} \tag{9a}$$

$$\frac{H_y^{n+1/2} - H_y^n}{\Delta t/2} = \frac{1}{\mu}\left(\frac{\partial E_z^{n+1}}{\partial x} + \frac{\partial E_z^n}{\partial x}\right) \tag{9b}$$

$$\begin{aligned}E_z{}^{n+1} = &\left\{(\nu_c\Delta t + 2)D_z{}^{n+1} - 4D_z{}^n + (-\nu_c\Delta t + 2)D_z{}^{n-1}\right. \\ &\left. + 4\varepsilon_0 E_z{}^n - \varepsilon_0\left(\omega_p^2\Delta t^2 - \nu_c\Delta t + 2\right)E_z{}^{n-1}\right\} \\ &/\,\varepsilon_0\left(\omega_p^2\Delta t^2 + \nu_c\Delta t + 2\right)\end{aligned} \tag{9c}$$

for the first step and

$$\frac{D_x^{n+1} - D_x^n}{\Delta t/2} = -\left(\frac{\partial H_y^{n+1}}{\partial z} + \frac{\partial H_y^{n+1/2}}{\partial z}\right) \tag{10a}$$

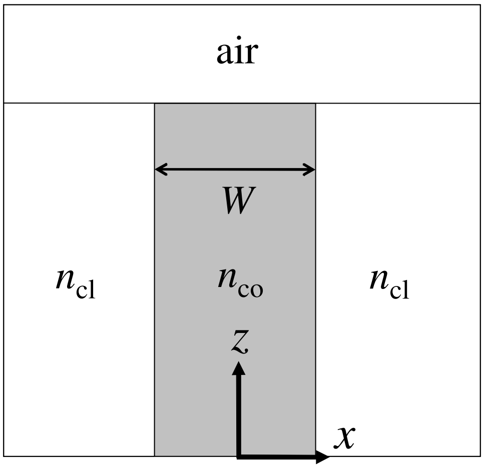


Fig. 1: Configuration

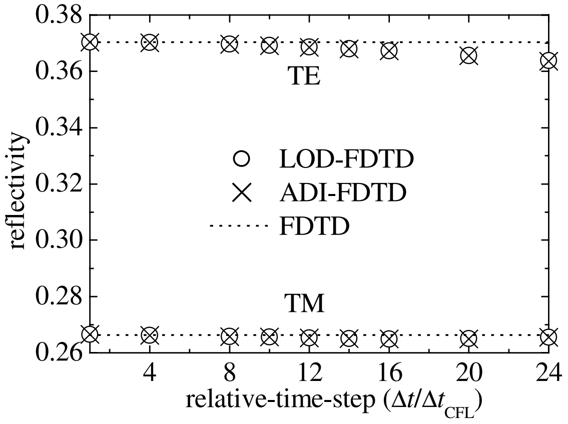


Fig. 2: Reflectivity

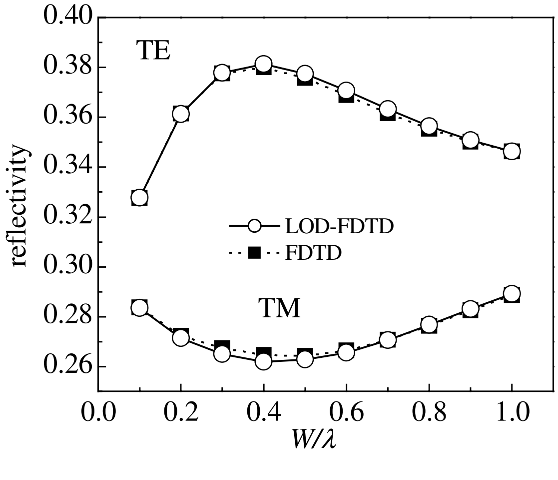


Fig. 3: Reflectivity

$$\frac{H_y^{n+1} - H_y^{n+1/2}}{\Delta t / 2} = -\frac{1}{\mu}\left(\frac{\partial E_x^{n+1}}{\partial z} + \frac{\partial E_x^{n}}{\partial z}\right) \tag{10b}$$

$$\begin{aligned} E_x^{\,n+1} = &\left\{(\nu_c \Delta t + 2) D_x^{\,n+1} - 4 D_x^{\,n} + (-\nu_c \Delta t + 2) D_x^{\,n-1} \right. \\ &\left. + 4\varepsilon_0 E_x^{\,n} - \varepsilon_0\left(\omega_p^2 \Delta t^2 - \nu_c \Delta t + 2\right) E_x^{\,n-1}\right\} \\ &/ \varepsilon_0\left(\omega_p^2 \Delta t^2 + \nu_c \Delta t + 2\right) \end{aligned} \tag{10c}$$

for the second step, where ω_p is the radian plasma frequency. In the first step, we substitute (9b) into (9a), and substitute the resultant equation into (9c). Then, the obtained equation is implicitly solved, and (9a) and (9b) are explicitly solved. In the second step, the equations are calculated in the same way as in the first step. As a result, we need to solve two implicit and four explicit equations.

For each LOD-FDTD implementation, the number of explicit equations to be solved is reduced by two, when compared with the ADI counterpart. This leads to simple implementation of the algorithm, with a subsequent reduction in the computational time.

3. Numerical Results

To show the validity of the LOD-FDTD, we calculate the facet reflectivity of an optical waveguide [7]. The configuration is shown in Fig. 1, in which the refractive indices of the core and cladding are n_{co} = 3.6 and n_{cl} = 3.42, respectively. A wavelength of λ = 0.86 μm is used. The sampling widths are $\Delta x = W/10$ and Δz = 0.01 μm.

Fig. 2 shows the reflectivity obtained from the LOD-FDTD, when $W = 0.6\lambda$ is used. The upper limit of the CFL condition of the FDTD is defined as Δt_{CFL}. For reference, included are the results obtained from the ADI-FDTD indicated as the cross and those from the FDTD with Δt_{CFL} as the straight broken line. It is found that the results obtained from the LOD-FDTD perfectly follow those from the ADI-FDTD for both TE and TM cases. In addition, the results obtained from the LOD-FDTD agree well with those from the FDTD up to 10 Δt_{CFL}.

Fig. 3 shows the reflectivity obtained from the LOD-FDTD with $\Delta t = 10\Delta t_{CFL}$ as a function of W/λ. For comparison, the results obtained from the FDTD with Δt_{CFL} are also shown. It is found that the results obtained from the LOD-FDTD agree well with those from the FDTD for both TE and TM cases. For $\Delta t = 10\Delta t_{CFL,}$ the computational time of the LOD-FDTD is reduced to about 80% and 50% of those of the ADI-FDTD and the FDTD, respectively.

Next, to evaluate the performance of the frequency-dependent LOD-FDTD, we analyze the optical waveguide with the metal cladding shown in Fig. 4. The refractive index of the metal (Au) is expressed in the following Drude model:

$$n_m^2 = 1 + \frac{\omega_p^2}{\omega(j\nu_c - \omega)} \tag{11}$$

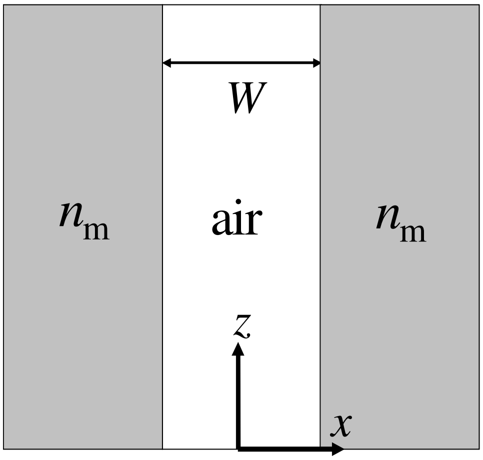
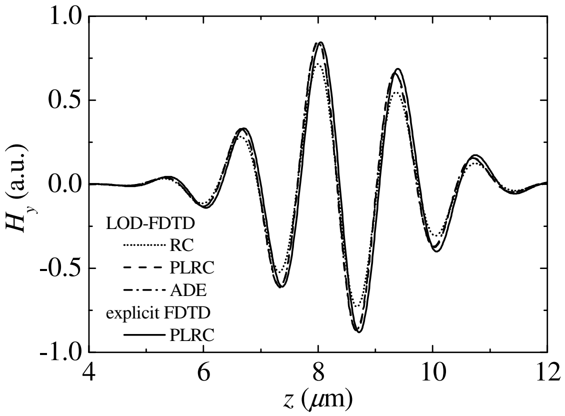
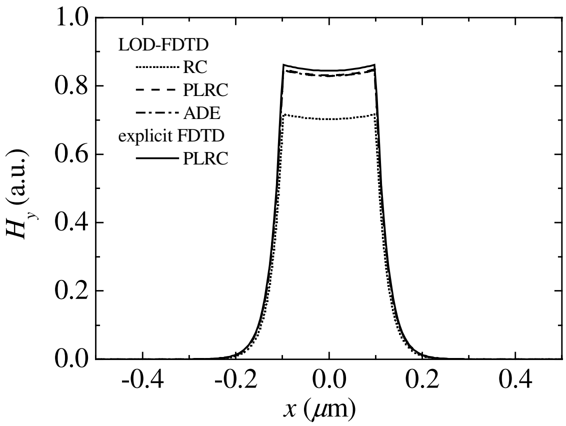


Fig. 4: Configuration



(a) $x = 0$ (μm)



(b) $z = 8.04$ (μm)

Fig. 5: Field distribution

in which ω_p and v_c are determined by (11) with $n_m = 0.18 - j10.2$ at a wavelength of $\lambda = 1.55$ μm. The core width is $W = 0.2$ μm. The sampling widths are $\Delta x = 0.005$ μm and $\Delta z = 0.01$ μm. The incident wave is launched using the pulse excitation for the TM wave.

Fig. 5(a) shows the field distribution at $x = 0$ μm observed at $t = 50$ fs for $\Delta t = 10\ \Delta t_{CFL}$. For comparison, the result of the FDTD using the PLRC method with Δt_{CFL} is included (although not shown, good agreement is found to exist among the results of the FDTDs using the RC, PLRC and ADE methods). Fig. 5(b) shows the field distribution at $z = 8.04$ μm, where the field amplitude of the FDTD becomes maximal in Fig. 5(a). It is found that the result obtained from the PLRC-LOD-FDTD agrees well with that from the ADE-LOD-FDTD, demonstrating almost the same accuracy for both FDTDs. In addition, their results are in good agreement with the result from the explicit PLRC-FDTD. In contrast, the field amplitude obtained from the RC-LOD-FDTD decreases. This is because the use of the large time step violates the assumption that the electric field is constant over Δt for the RC method. It is noteworthy, in the above analysis, that the computational times of the PLRC- and ADE-LOD-FDTDs with $\Delta t = 10\ \Delta t_{CFL}$ are reduced to about 23% and 29%, respectively, of the time of the explicit PLRC-FDTD.

4. CONCLUSION

We have formulated an FDTD based on the locally one-dimensional scheme and introduced the RC, PLRC and ADE methods into the LOD-FDTD. Several numerical results show the effectiveness of the LOD-FDTD for the analyses of both nondispersive and dispersive media.

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