THE MOMENT METHOD for ELECTROMAGNETIC COUPLING between ARBITRARILY BENT WIRE and SLOT STRUCTURES

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1. Introduction

A system composed of straight wire and slot structures has been analyzed as an antenna problem [1][2] or an electromagnetic coupling problem [3]-[5]. Some of these problems are solved on the basis of a Pocklington-type integral equation or a Hallen-type integral equation. It should be noted that unlike the Hallen-type integral equation the Pocklington-type integral equation does not have arbitrary constants with subsequent less confusion in the numerical procedure of the moment method.

The purpose of the present paper is to extend the Pocklington-type integral equation for the problem of electromagnetic coupling between arbitrarily bent wire and slot structures. The integral equations are arranged in simple algebraic form for the moment method. An example of analysis is also presented. The details of the obtained results will be discussed during the presentation.

2. Boundary value problem on a bent slot

Fig.1 illustrates a system to be considered here. The incident field ($\bar{E}^{in}$, $\bar{H}^{in}$) penetrates through a narrow slot in a planar conducting screen, and excites a wire behind the screen. The screen is assumed to be perfectly conducting and vanishingly thin, and of infinite extent. The wire is also assumed to be perfectly conducting and is thin relative to the wavelength λ of the incident wave as well as to its length. Both the slot and wire structures are arbitrarily bent.

The boundary value problem on the narrow slot is formulated by using the magnetic fields $\bar{H}_{z-}$ in the region $z<0$ and $\bar{H}_{z+}$ in the region $z>0$. They are given by Eqs.(39) and (40) in Reference[5], respectively. The continuity requirement on the tangential magnetic field at $z=0$ gives

$$-\frac{1}{2\mu}(\nabla\times\bar{A})\cdot\hat{s}+\frac{1}{j\omega\mu\epsilon}(\beta^2\bar{F}+\nabla\nabla\cdot\bar{F})\cdot\hat{s}=-\frac{1}{2}\bar{H}_{sc}\cdot\hat{s} \tag{1}$$

where $\bar{A}$ and $\bar{F}$ are magnetic and electric vector potentials, respectively; $\beta^2=\omega^2\mu\epsilon$; $\hat{s}$ is a tangential unit vector at an observation point on the slot; and $\bar{H}_{sc}$ is the short-circuit field.

The first term of the left side of Eq.(1) is the term related to a magnetic field radiated from the bent wire. Focusing our attention on a bent wire element composed of two straight wires n-1 and n shown in Fig.1, we use the local cylindrical coordinates $(\varphi_{n-1},\rho_{n-1},z_{n-1})$ and (φ_n,ρ_n,z_n), respectively. The unit vectors corresponding to these coordinates are $(\hat{\varphi}_{n-1},\hat{\rho}_{n-1},\hat{z}_{n-1})$ and $(\hat{\varphi}_n,\hat{\rho}_n,\hat{z}_n)$, respectively.

Let us assume that the current on the bent wire element is as follows:

$$I(z') = I_n \frac{\sin\beta(z'-z_{n-1})}{\sin\beta e_{n-1}} \quad (z_{n-1} \leq z' \leq z_n), \quad I(z') = I_n \frac{\sin\beta(z_{n+1}-z')}{\sin\beta e_n} \quad (z_n \leq z' \leq z_{n+1}) \tag{2}$$

Then, a tangential component of the resultant magnetic field from the two wires is calculated to be

$$H_{wire}(I_n) \equiv I_n \cdot h_{n:wire} = -I_n \frac{j}{8\pi}\left(\frac{\bar{\Phi}_{n-1}}{\rho_{n-1}\sin\beta e_{n-1}} + \frac{\bar{\Phi}_n}{\rho_n \sin\beta e_n}\right)\cdot\hat{s} \tag{3}$$

where

$$\bar{\Phi}_{n-1} = \{ (j\cos\vartheta_U^{n-1}\cdot\sin\beta e_{n-1} - \cos\beta e_{n-1})\cdot e^{-j\beta R_U^{n-1}} + e^{-j\beta R_L^{n-1}} \}\,\hat{\varphi}_{n-1} \tag{4}$$

$$\bar{\Phi}_n = \{ -(j\cos\vartheta_L^n\cdot\sin\beta e_n + \cos\beta e_n)\cdot e^{-j\beta R_L^n} + e^{-j\beta R_U^n} \}\,\hat{\varphi}_n \tag{5}$$

in which the distances (R_L^{n-1}, R_U^{n-1}) and (R_L^n, R_U^n), the angles $(\vartheta_L^{n-1}, \vartheta_U^{n-1})$ and $(\vartheta_L^n, \vartheta_U^n)$, and the wire lengths (e_{n-1}, e_n) are defined in Fig.1.

The second term of the left side of Eq.(1) is the term related to a magnetic field radiated from the bent slot. We focus our attention on a bent slot element composed of two straight slots n-1 and n, and change I_n in Eq.(2) to M_n for a piecewise sinusoidal magnetic current. The resultant magnetic field is calculated to be

$$H_{slot}(M_n) \equiv M_n \cdot h_{n:slot} = -M_n \frac{j}{2\pi Z_0}\left[\left(\frac{\bar{P}_{n-1}}{\rho_{n-1}\sin\beta e_{n-1}} + \frac{\bar{P}_n}{\rho_n \sin\beta e_n}\right) - \left(\frac{\bar{Z}_{n-1}}{\sin\beta e_{n-1}} + \frac{\bar{Z}_n}{\sin\beta e_n}\right)\right]\cdot\hat{s} \tag{6}$$

where $Z_0=120\pi$ and

$$\bar{P}_{n-1} = \{ \cos\beta e_{n-1}\cdot\cos\vartheta_U^{n-1}\cdot e^{-j\beta R_U^{n-1}} - \cos\vartheta_L^{n-1}\cdot e^{-j\beta R_L^{n-1}} - j\sin\beta e_{n-1}\cdot e^{-j\beta R_U^{n-1}} \}\,\hat{\rho}_{n-1} \tag{7}$$

$$\bar{P}_n = \{ \cos\beta e_n\cdot\cos\vartheta_L^n\cdot e^{-j\beta R_L^n} - \cos\vartheta_U^n\cdot e^{-j\beta R_U^n} + j\sin\beta e_n\cdot e^{-j\beta R_L^n} \}\,\hat{\rho}_n \tag{8}$$

$$\bar{Z}_{n-1} = \{ \cos\beta e_{n-1}\cdot\frac{e^{-j\beta R_U^{n-1}}}{R_U^{n-1}} - \frac{e^{-j\beta R_L^{n-1}}}{R_L^{n-1}} \}\,\hat{z}_{n-1} \tag{9}$$

$$\bar{Z}_n = \{ -\frac{e^{-j\beta R_U^n}}{R_U^n} + \cos\beta e_n\cdot\frac{e^{-j\beta R_L^n}}{R_L^n} \}\,\hat{z}_n \tag{10}$$

3. Boundary value problem on a bent wire

The boundary value problem on a bent wire is formulated in such a way that at the surface of the perfectly conducting wire the sum of the scattered field and the incident field is zero:

$$\frac{1}{\epsilon}(\nabla\times\bar{F})\cdot\hat{s} + \frac{1}{j\omega\mu\epsilon}(\beta^2\bar{A} + \nabla\nabla\cdot\bar{A})\cdot\hat{s} = 0 \tag{11}$$

The piecewise sinusoidal expansion function described in Section 2 leads the left side of Eq.(11) to algebraic expressions. From the first term of the left side of Eq.(11), we have

$$E_{slot}(M_n) \equiv M_n \cdot e_{n:slot} = M_n \frac{j}{2\pi}\left(\frac{\bar{\Phi}_{n-1}}{\rho_{n-1}\sin\beta e_{n-1}} + \frac{\bar{\Phi}_n}{\rho_n \sin\beta e_n}\right)\cdot\hat{s} \tag{12}$$

The second term of the left side of Eq.(11) gives

$$E_{wire}(I_n) \equiv I_n \cdot e_{n:wire} = - I_n \, j30[(\frac{\bar{P}_{n-1}}{\rho_{n-1}\sin\beta e_{n-1}} + \frac{\bar{P}_n}{\rho_n \sin\beta e_n}) - (\frac{\bar{Z}_{n-1}}{\sin\beta e_{n-1}} + \frac{\bar{Z}_n}{\sin\beta e_n})]\cdot\hat{s} \quad (13)$$

4. Application

The generalized impedance matrix in the moment method for Eqs.(1) and (11) is calculated by using the obtained results of ($h_{n:wire}$, $h_{n:slot}$) and ($e_{n:wire}$, $e_{n:slot}$) together with a weighting function. The term $e_{n:wire}$ of Eq.(13) is familiar to antenna engineers, and is often used for an analysis of an arbitrarily bent wire antenna or wire scatterer [6][7].

Fig.2 shows an example of analysis using a piecewise sinusoidal weighting function (Galerkin method). A circularly polarized wave coming from the -Z direction excites a square loop wire through a crossed-slot. It is found that the crossed-slot acts as a receiving element for the circularly polarized wave, and that a travelling current wave appears on the square loop wire.

5. Conclusion

The tangential components of electric and magnetic fields in a Pocklington-type integral equation have been transformed into algebraic expressions, using piecewise sinusoidal expansion functions. The obtained algebraic expressions are used to produce the generalized impedance matrix of the moment method together with a weighting function. The numerical result of the electromagnetic coupling between a crossed-slot illuminated by a circularly polarized wave and a square loop wire shows that there exists a travelling electric current on the wire.

References

[1] K.Itoh and T.Matsumoto:"Theoretical analysis of mutual coupling between slot and unipole antennas", Trans.IECE Jpn, J61-B, 6, pp.391-397(1978).

[2] T.Ishizone, et al.:"Unipole-slot array antennas", Trans.IECE Jpn, J66-B, 3, pp.281-288(1983).

[3] C.M.Butler, et al.:"Electromagnetic excitation of a wire through an aperture-perforated conducting screen", IEEE Trans., AP-24, 4, pp.456-462(1976).

[4] Y.Naiheng, and R.F.Harrington,:"Electromagnetic coupling to an infinite wire through a slot in a conducting plane", IEEE Trans., AP-31, 2, pp.310-316(1983).

[5] H.Nakano, and R.F.Harrington,:"Integral equations on electromagnetic coupling to a wire through an aperture", Trans.IECE Jpn, vol.E66, 6, pp.383-389(1983).

[6] K.Hirasawa, and K.Fujimoto,:"On wire-grid method for analysis of wire antennas near/on a rectangular conducting body", Trans.IECE Jpn, vol.J65-B, 4, pp.382-389(1982).

[7] T.Tsukiji, and M.Yamasaki,:"Analysis of broad band triangular double loop antenna", Trans.IECE Jpn, vol.J65-B, 6, pp.769-776(1982).

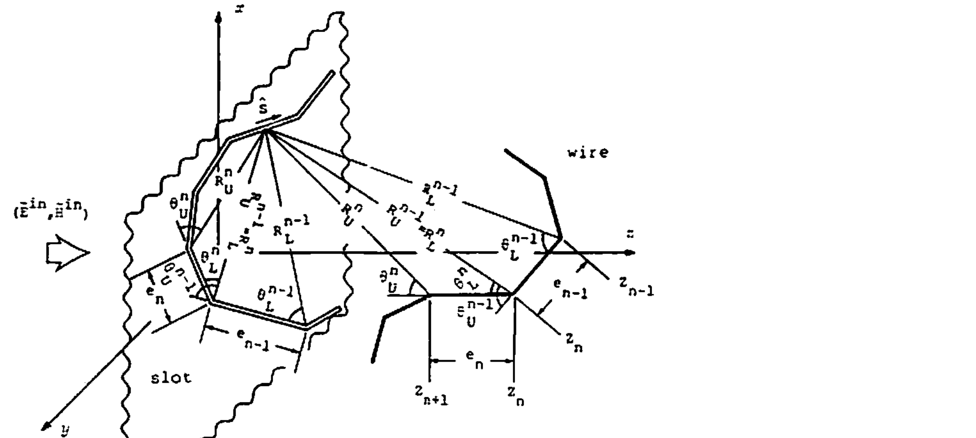
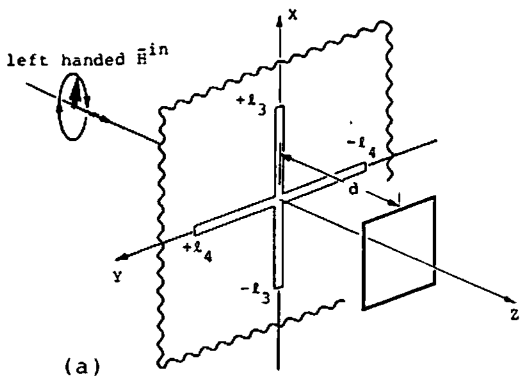
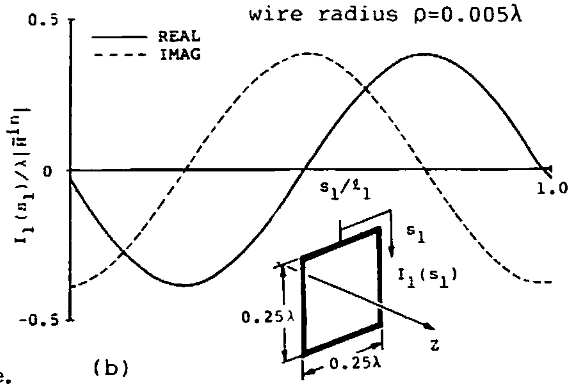


Fig.1 Problem of electromagnetic coupling between arbitrarily bent wire and slot structures.

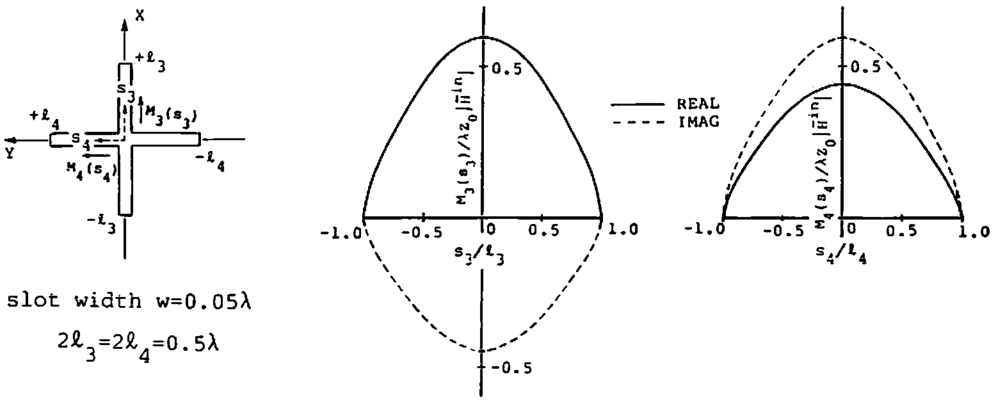


a crossed-slot and a square loop wire.



Electric current on the square loop wire.

loop length $\ell_1=1\lambda$, d=0.5λ



slot width w=0.05λ

$2\ell_3=2\ell_4=0.5\lambda$

(c) Magnetic current on the crossed-slot.

Fig.2 Electromagnetic coupling between a crossed-slot and a square loop wire.