SCATTERING OF ELECTROMAGNETIC PLANE WAVES BY A COLUMN OF MAGNETIZED HOT PLASMA

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INTRODUCTION

Scatterings of electromagnetic waves from bounded plasma [1],[2] are of continuing interest in connection with the studies of plasma diagnostic. In this paper, the scattering of electromagnetic plane waves by a column of hot plasma magnetized in the axial direction is investigated based on the kinetic theory. The numerical results of the back scattering cross section are obtained for both of TM and TE plane waves. The present analysis is valid when the transverse wavelength of electromagnetic fields inside the plasma is much larger than the Larmor radius of electrons.

FORMULATION OF THE PROBLEM

A column of plasma of radius a is immersed in an axial magnetic field B_0 (Fig. 1). The plasma is composed of hot electrons with cold massive ions neutralizing the electrons on the average. The unperturbed state of electrons is assumed to be subjected to a bimaxwellian velocity distribution with different thermal velocities $v_{\parallel}$ and $v_{\perp}$ in the axial and transverse directions, respectively. The linearized Boltzmann equation with a Gross-Krook collision term is solved by using a method of orbit integral [3], to have the dielectric tensor of the plasma. The resulting expressions are expanded in the powers of the ratio of the Larmor radius of electrons to the gradient scale length of the electromagnetic fields, and only the first-order terms of the expansion are retained. Thus for the perturbation of the form $\exp[-i(\omega t - k_z z)]$, after extensive manipulations, the dielectric displacement of this hot plasma can be deduced as follows:

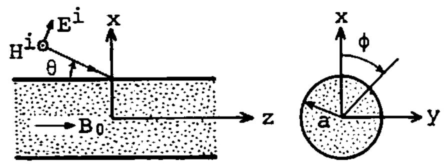


Fig. 1. Geometry of the problem.

$$\mathbf{D} = \varepsilon_0 \begin{bmatrix} \varepsilon_1 & i\varepsilon_2 & 0 \\ -i\varepsilon_2 & \varepsilon_1 & 0 \\ 0 & 0 & \varepsilon_3 \end{bmatrix} \cdot \mathbf{E} + i\varepsilon_0 \lambda_1 (\nabla_t \hat{z} + \hat{z}\nabla_t)\cdot \mathbf{E} + \varepsilon_0 \lambda_2 \nabla_t \times \mathbf{E}, \tag{1}$$

where

$$\varepsilon_1 = 1 + \frac{\omega_p^2 P_+}{2\omega k_z V} - \frac{\omega_p^2(\delta - 1)Q_+}{4\omega^2}, \quad \varepsilon_2 = \frac{\omega_p^2 P_-}{2\omega k_z V} - \frac{\omega_p^2(\delta - 1)Q_-}{4\omega^2},$$
$$\varepsilon_3 = 1 - \frac{\omega' \omega_p^2 Z'(s_0)}{\omega k_z^2 V^2}, \quad \lambda_1 = -\frac{\omega_p^2}{4\omega^2 k_z}\left[\frac{\delta\omega'}{\omega_c} Q_- + (\delta - 1)Q_+\right], \tag{2}$$
$$\lambda_2 = -\frac{\delta\omega' \omega_p^2 Z'(s_0)}{2\omega^2 \omega_c k_z} + \frac{\omega_p^2}{4\omega^2 k_z}\left[\frac{\delta\omega'}{\omega_c} Q_+ + (\delta - 1)Q_-\right],$$

$$P_{\pm} = Z(s_1) \pm Z(s_{-1}), \quad Q_{\pm} = Z'(s_1) \pm Z'(s_{-1}),$$
$$s_m = (\omega' + m\omega_c)/(k_z V) \quad (m = 0, \pm 1), \tag{3}$$
$$\omega' = \omega + i\nu, \quad \delta = (V_\perp/V_{||})^2.$$

In the above equations, ∇_t denotes the vector derivative transverse to the z direction, ω_p and ω_c are the electron plasma and cyclotron frequencies, ν is an effective collision frequency, $Z(s_m)$ is the usual plasma dispersion function [4], and the prime signifies the derivative of the function. Using (1) in Maxwell's equations, the coupled-wave equations for E_z and H_z are derived. Their solutions can be expressed as follows:

$$E_z = E_0 \sum_{\ell=1}^{2} \sum_{n=-\infty}^{\infty} A_{\ell n}(-i)^n J_n(\chi_\ell \rho)\, e^{in\phi} e^{ik_z z} \tag{4}$$

$$H_z = -i(E_0/\zeta_0) \sum_{\ell=1}^{2} \sum_{n=-\infty}^{\infty} A_{\ell n}(-i)^n \xi_\ell J_n(\chi_\ell \rho)\, e^{in\phi} e^{ik_z z} \tag{5}$$

where $\zeta_0 = \sqrt{\mu_0/\varepsilon_0}$, E_0 is a constant amplitude, $J_n(\chi_\ell \rho)$ is the Bessel function of the nth order, and $A_{\ell n}$ are unknown constants which can be determined by applying the boundary conditions at $\rho = a$. χ_ℓ are the two roots of the quadratic equation

$$\alpha\chi^4 - k_0^2[\beta(\alpha + \varepsilon_3 + k_0^2\lambda_2^2) - \varepsilon_2(\varepsilon_2 - 2\gamma k_0 \lambda_2)]\chi^2 + k_0^4 \eta \varepsilon_3 = 0, \tag{6}$$

and ξ_ℓ $(\ell = 1, 2)$ are given by

$$\xi_\ell = (\alpha\chi_\ell^2/k_0^2 - \beta\varepsilon_3)/(\beta k_0 \lambda_2 + \gamma\varepsilon_2) \tag{7}$$

where

$$\alpha = \varepsilon_1 + k_0\lambda_1(\gamma - k_z/k_0), \quad \beta = \varepsilon_1 - k_z^2/k_0^2,$$
$$\gamma = k_0\lambda_1 - k_z/k_0, \quad \eta = \beta^2 - \varepsilon_2^2, \quad k_0 = \omega\sqrt{\varepsilon_0\mu_0}. \tag{8}$$

Other field components can be obtained in terms of E_z and H_z, though their expressions have been omitted for the sake of brevity.

SCATTERING OF PLANE WAVES

We consider first the case of incidence of TM plane wave. A plane wave with magnetic vector polarized in the y direction is assumed to impinge on the plasma column obliquely at an angle θ (Fig. 1). Then the axial components of the incident wave take the form

$$E_z^i = E_0 \sin\theta \sum_{n=-\infty}^{\infty} (-i)^n J_n(\kappa\rho)\, e^{in\phi} e^{ik_z z}, \quad H_z^i = 0, \tag{9}$$

where $k_z = k_0\cos\theta$ and $\kappa = k_0\sin\theta$. The scattered waves outside the column are composed of both of TM wave and TE wave because of the anisotropy of the plasma. The axial components of scattered waves may be expressed as follows:

$$E_z^s = E_0 \sin\theta \sum_{n=-\infty}^{\infty} (-i)^n H_n^{(1)}(\kappa\rho) R_n^E\, e^{in\phi} e^{ik_z z},$$
$$H_z^s = (E_0/\zeta_0) \sin\theta \sum_{n=-\infty}^{\infty} (-i)^n H_n^{(1)}(\kappa\rho) R_n^H\, e^{in\phi} e^{ik_z z}, \tag{10}$$

where $H_n^{(1)}(\kappa\rho)$ is the Hankel function of the nth order of the first kind, and R_n^E and R_n^H are the unknown constants which characterize the strength of scattering for the TM and TE wave components, respectively. Matching the tangential electric and magnetic fields at $\rho = a$, simultaneous equations to determine R_n^E and R_n^H can be derived in the following expressions:

$$\begin{bmatrix} -h_n\sin\theta & 0 & j_{n1} & j_{n2} \\ 0 & h_n\sin\theta & i\xi_1 j_{n1} & i\xi_2 j_{n2} \\ (\frac{n}{\kappa a})h_n\cos\theta & ih_n' & p_{n1} & p_{n2} \\ ih_n' & -(\frac{n}{\kappa a})h_n\cos\theta & iq_{n1} & iq_{n2} \end{bmatrix} \begin{bmatrix} R_n^E \\ R_n^H \\ A_{1n} \\ A_{2n} \end{bmatrix} = \begin{bmatrix} J_n(\kappa a)\sin\theta \\ 0 \\ -(\frac{n}{\kappa a})J_n(\kappa a)\cos\theta \\ -iJ_n'(\kappa a) \end{bmatrix} \quad (11)$$

where

$$p_{n\ell} = [(n/a)e_\ell j_{n\ell} + f_\ell \chi_\ell j_{n\ell}']/(k_0\eta) \quad (\ell = 1,\ 2),$$
$$q_{n\ell} = [(n/a)f_\ell j_{n\ell}\cos\theta - g_\ell \chi_\ell j_{n\ell}']/(k_0\eta), \quad (12)$$
$$h_n = H_n^{(1)}(\kappa a),\ \ h_n' = H_n^{(1)\prime}(\kappa a),\ \ j_{n\ell} = J_n(\chi_\ell a),\ \ j_{n\ell}' = J_n'(\chi_\ell a),$$

$$e_\ell = \beta\gamma + \varepsilon_2 w_\ell,\ \ f_\ell = \gamma\varepsilon_2 + \beta w_\ell,$$
$$g_\ell = \beta(\varepsilon_1 - \lambda_1 k_z) - \varepsilon_2^2 - \varepsilon_2 w_\ell k_z/k_0,\ \ w_\ell = k_0\lambda_2 - \xi_\ell. \quad (13)$$

The bistatic scattering cross section per unit length of the column is obtained as

$$\sigma(\phi) = \frac{4}{k_0\sin\theta}[\,|\Gamma^E(\phi)|^2 + |\Gamma^H(\phi)|^2\,], \quad (14)$$

where

$$|\Gamma^E(\phi)| = |\sum_{n=-\infty}^{\infty}(-1)^n R_n^E e^{in\phi}|,\ \ |\Gamma^H(\phi)| = |\sum_{n=-\infty}^{\infty}(-1)^n R_n^H e^{in\phi}|. \quad (15)$$

The far-field patterns of the scattered TM and TE waves are proportional to $|\Gamma^E(\phi)|$ and $|\Gamma^H(\phi)|$, respectively.

The case of incidence of TE wave with the electric vector polarized in the -y direction can be treated in an analogous manner. In this case, the axial components of the incident wave may be written as

$$H_z^i = -i(E_0/\zeta_0)\sum_{n=-\infty}^{\infty}(-i)^n J_n(\kappa\rho)\, e^{in\phi} e^{ik_z z},\ \ E_z^i = 0. \quad (16)$$

The expressions for the axial components of the fields inside the plasma and the scattered fields are obtained by writing $-iE_0$ to replace the constant amplitude E_0 in (4), (5), and (10). Simultaneous equations to determine the scattering coefficients are given by replacing the right hand side of (11) by the following column vector:

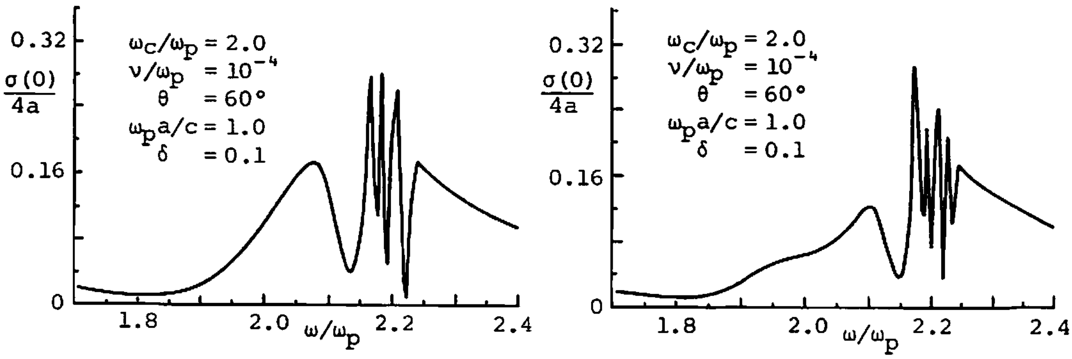
$$F_n = [0,\ \ -J_n(\kappa a)\sin\theta,\ \ -iJ_n'(\kappa a),\ \ (n/\kappa a)J_n(\kappa a)\cos\theta]^T. \quad (17)$$

The bistatic scattering cross section and the far-field patterns can be calculated from the same equations as (14) and (15).

NUMERICAL RESULTS

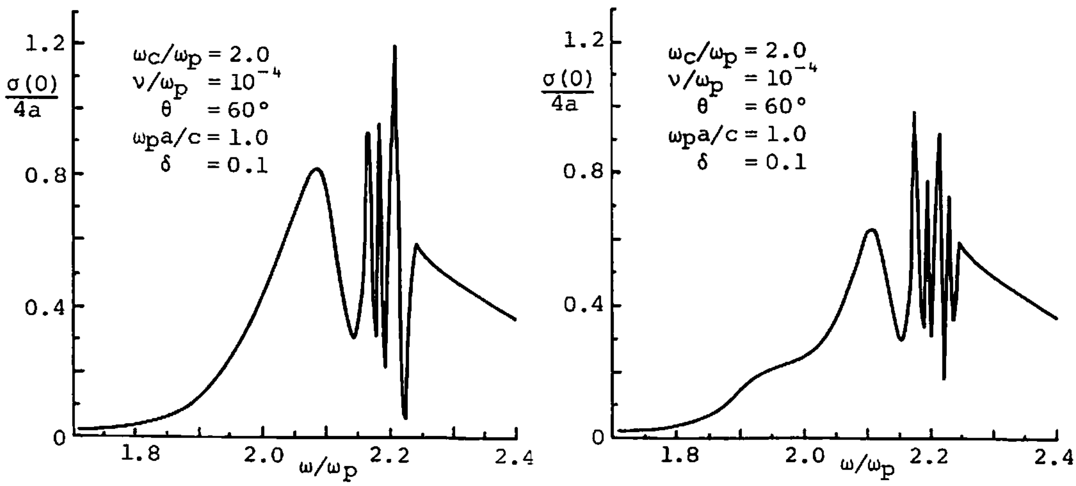
In order to have numerical appreciations of the scattering cross section, we have carried out computations on (11) together with (14), (15), and (17). The results of the normalized back-scattering cross section $\sigma(0)/4a$ are plotted in Fig. 2 for the case of TM-wave incidence and in Fig. 3 for the case of TE-wave incidence, as a function of ω/ω_p with the following parameters fixed: $\omega_c/\omega_p = 2.0$, $\nu/\omega_p = 10^{-4}$, $\theta = 60°$, $\omega_p a/c = 1.0$, and $\delta = 0.1$. Note that, for these choise of parameters, the cyclotron frequency and the upper-hybrid frequency are located at $\omega/\omega_p = 2.0$ and $\omega/\omega_p = 2.236$, respectively. The thermal velocity of the plasma is taken to be

$V_{\parallel}/c = 10^{-5}$ in Figs. 2(a) and 3(a), whereas that is $V_{\parallel}/c = 5 \times 10^{-2}$ in Figs. 2(b) and 3(b). The effect of the finite temperature of plasma on the scattering cross section can be readily understood from these figures.



(a) $V_{\parallel}/c = 10^{-5}$ (b) $V_{\parallel}/c = 5 \times 10^{-2}$

Fig. 2. Back-scattering cross section for the incidence of TM plane wave.



(a) $V_{\parallel}/c = 10^{-5}$ (b) $V_{\parallel}/c = 5 \times 10^{-2}$

Fig. 3. Back-scattering cross section for the incidence of TE plane wave.

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