B-9-4 DIFFRACTION OF A PLANE ELECTROMAGNETIC WAVE BY A PAIR OF PARALLEL, SEMI-INFINITE SCREENS WITH SURFACE IMPEDANCE

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1 Introduction

The diffraction of a plane electromagnetic wave by an imperfectly conducting sheet is an important problem for the electromagnetic wave propagation in the city and the antenna theory. This paper describes the diffraction by a pair of parallel, semi-infinite screens with surface impedance. To analyze such a problem, we use the angular spectrum method which is easier than others and is also useful for the diffraction problems.[1],[2],[3]

2 Analysis

Let us consider that the problem is two dimensional case. As shown in Fig.1, S1 and S2 are imperfectly conducting screens with equal surface impedance $\eta Z_0 (Z_0 = \sqrt{\mu_0/\varepsilon_0})$ which are occupying y=0, x > 0, and y=a, x > 0 in a free space region (ε_0, μ_0), respectively. It is assumed that their thickness τ satisfies that $\lambda_0 \gg \tau \gg d$, where λ_0 is the free space wavelength and d is the penetration depth.[1]

Consider an incident plane wave with E-polarization given by

$$E_z^i = e^{ik_0 r\cos(\theta-\alpha_0)} \quad (1)$$

where the time factor $\exp(-i\omega t)$ is suppressed throughout, $k_0 = \omega\sqrt{\varepsilon_0\mu_0} = 2\pi/\lambda_0$, and α_0 is real and $0<\alpha_0<\pi$. Then, using the angular spectra $P(k_0\cos\alpha)$'s, the scattered fields due to induced currents in S1 and S2 may be written as follows

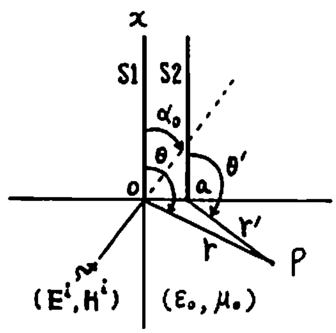

Fig.1 Geometry of the problem

$$E_z^{s1r} = \int_{C^r} P^{1r}(k_0\cos\alpha)\, e^{ik_0 r\cos(\theta+\alpha)}\, d\alpha, \quad (y<0), \quad (2)$$

$$E_z^{s1t} = \int_{C^t} P^{1t}(k_0\cos\alpha)\, e^{ik_0 r\cos(\theta-\alpha)}\, d\alpha, \quad (y>0), \quad (3)$$

$$E_z^{s2r} = \int_{C^r} P^{2r}(k_0\cos\alpha)\, e^{ik_0 r'\cos(\theta'+\alpha)}\, d\alpha, \quad (y<a), \quad (4)$$

$$E_z^{s2t} = \int_{C^t} P^{2t}(k_0\cos\alpha)\, e^{ik_0 r'\cos(\theta'-\alpha)}\, d\alpha, \quad (y>a), \quad (5)$$

where when we transform the above equations by $\lambda = k_0 \cos\alpha$, C^r and C^t are the integral paths in the complex α-plane coinciding with the real axis in the complex λ -plane.

The total fields $\mathbb{E}$ and $\mathbb{H}$ are expressed as follows

$$\mathbb{E} = \mathbb{E}^i + \mathbb{E}^{s1} + \mathbb{E}^{s2}, \quad \mathbb{H} = \mathbb{H}^i + \mathbb{H}^{s1} + \mathbb{H}^{s2}. \tag{6}$$

At the two screens the boundary condition is

$$\mathbb{E} - (\mathbf{n}\cdot\mathbb{E})\mathbf{n} = \eta Z_0 \mathbf{n}\times\mathbb{H}, \tag{7}$$

where $\mathbf{n}$ is the unit vector outwared normal. Furthermore, the tangential components of scattered fields should be continuous across the regions y=0, x<0 and y=a, x<0.

After changing the variable from α to $\lambda = k_0\cos\alpha$, we obtain the integral equations on the angular spectra from the boundary conditions. For example, for x>0,

$$\int_{-\infty}^{\infty}\left[\frac{P^{1r}(\lambda)}{\sqrt{k_0^2-\lambda^2}} + \frac{P^{2t}(\lambda)}{\sqrt{k_0^2-\lambda^2}} + \left\{\frac{P^{1t}(\lambda)}{\sqrt{k_0^2-\lambda^2}} + \frac{P^{2r}(\lambda)}{\sqrt{k_0^2-\lambda^2}}\right\} e^{ia\sqrt{k_0^2-\lambda^2}}\right] \times \left(1 + \frac{\eta Z_0}{\omega\mu_0}\sqrt{k_0^2-\lambda^2}\right)e^{ix\lambda}\,d\lambda = -F_{(-)}(\lambda_0)e^{ix\lambda_0}, \tag{8}$$

$$\int_{-\infty}^{\infty}\left[\frac{P^{1r}(\lambda)}{\sqrt{k_0^2-\lambda^2}} - \frac{P^{2t}(\lambda)}{\sqrt{k_0^2-\lambda^2}} - \left\{\frac{P^{1t}(\lambda)}{\sqrt{k_0^2-\lambda^2}} - \frac{P^{2r}(\lambda)}{\sqrt{k_0^2-\lambda^2}}\right\} e^{ia\sqrt{k_0^2-\lambda^2}}\right] \times \left(1 + \frac{\eta Z_0}{\omega\mu_0}\sqrt{k_0^2-\lambda^2}\right)e^{ix\lambda}\,d\lambda = -G_{(-)}(\lambda_0)e^{ix\lambda_0}, \tag{9}$$

$$\int_{-\infty}^{\infty}\left\{\frac{P^{1t}(\lambda)}{\sqrt{k_0^2-\lambda^2}} + \frac{P^{2r}(\lambda)}{\sqrt{k_0^2-\lambda^2}}\right\} F_{(+)}(\lambda)e^{ix\lambda}\,d\lambda = -F_{(+)}(\lambda_0)e^{ix\lambda_0}, \tag{10}$$

$$\int_{-\infty}^{\infty}\left\{\frac{P^{1t}(\lambda)}{\sqrt{k_0^2-\lambda^2}} - \frac{P^{2r}(\lambda)}{\sqrt{k_0^2-\lambda^2}}\right\} G_{(+)}(\lambda)e^{ix\lambda}\,d\lambda = -G_{(+)}(\lambda_0)e^{ix\lambda_0}, \tag{11}$$

where

$$F_{(\pm)}(\lambda) = 1 + e^{ia\sqrt{k_0^2-\lambda^2}} \pm \frac{\eta Z_0}{\omega\mu_0}\sqrt{k_0^2-\lambda^2}\left(1 - e^{ia\sqrt{k_0^2-\lambda^2}}\right), \tag{12}$$

$$G_{(\pm)}(\lambda) = 1 - e^{ia\sqrt{k_0^2-\lambda^2}} \pm \frac{\eta Z_0}{\omega\mu_0}\sqrt{k_0^2-\lambda^2}\left(1 + e^{ia\sqrt{k_0^2-\lambda^2}}\right), \tag{13}$$

$P(\lambda)$'s are determined by solving these equations exactly. For example, the solutions are

$$P^{1t}(\lambda) = -\frac{\sqrt{k_0^2-\lambda^2}}{4\pi i(\lambda-\lambda_0)}V^{1t}(\lambda), \quad P^{2t}(\lambda) = \frac{\sqrt{k_0^2-\lambda^2}}{4\pi i(\lambda-\lambda_0)}V^{2t}(\lambda), \tag{14}$$

where

$$V^{1t}(\lambda) = \frac{F_{(+)}(\lambda_0)}{F_{(+)}(\lambda)}e^{f_{1+}(\lambda_0)-f_{1+}(\lambda)} + \frac{G_{(+)}(\lambda_0)}{G_{(+)}(\lambda)}e^{f_{2+}(\lambda_0)-f_{2+}(\lambda)}, \tag{15}$$

$$V^{2t}(\lambda) = V^{1t}(\lambda) e^{ia\sqrt{k_0^2-\lambda^2}} - \frac{F_{(-)}(\lambda_0) e^{f_{1+}(\lambda_0) - f_{1+}(\lambda)} - G_{(-)}(\lambda_0) e^{f_{2+}(\lambda_0) - f_{2+}(\lambda)}}{(\omega\mu_0 + \eta Z_0\sqrt{k_0^2 - \lambda^2})/\omega\mu_0}, \quad (16)$$

and $\exp\{-f_{1+}(\lambda)\}$ and $\exp\{-f_{2+}(\lambda)\}$ are the functions which are regular and non-zero in the upper half-plane of λ .

3 Numerical example

We can obtain the total field E_z^t $(y>a)$ from Eqs. (1), (3), (5), (6) and (14). If we apply the steepest descent method to E_z^t , the asymptotic expressions for E_z^t are derived. For example, E_z^t for θ , $\theta' > \alpha_0$ is written by

$$E_z^t \sim e^{ik_0 r\cos(\theta-\alpha_0)} - \frac{\sin\theta}{4\pi i(\cos\theta - \cos\alpha_0)} V^{1t}(k_0\cos\theta)\sqrt{\frac{2\pi}{k_0 r}}\, e^{ik_0 r - i\frac{\pi}{4}} + \frac{\sin\theta'}{4\pi i(\cos\theta' - \cos\alpha_0)} V^{2t}(k_0\cos\theta')\sqrt{\frac{2\pi}{k_0 r'}}\, e^{ik_0 r' - i\frac{\pi}{4}}, \quad (17)$$

and for θ , $\theta' \sim \alpha_0$,

$$E_z^t \sim \frac{1}{2}\left[1 - C(w) - S(w) - i\{S(w) - C(w)\}\right] \times e^{ik_0 r\cos(\theta-\alpha_0)}, \quad (18)$$

where $C(w)$ and $S(w)$ are Fresnel integrals, and

$$w = \sqrt{\frac{k_0 r}{\pi}\cos(\theta-\alpha_0)}\,\tan(\alpha_0 - \theta) \quad (19)$$

The numerical results of $|E_z^t|$ are shown in Fig.2, where the observation point is $P(r', \theta')$.

4 Conclusion

It has been shown that the scattered fields when E-polarized plane wave is incident can be solved exactly, and the numerical results for $|E_z^t|$ have been compared with the case of one screen.

References

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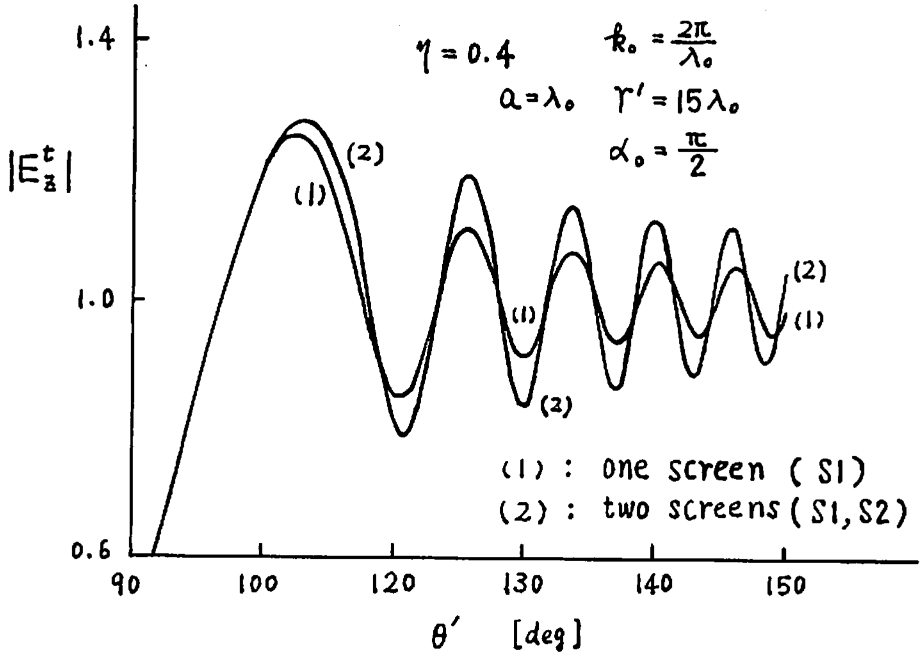

Fig.2 Numerical results for $|E_z^t|$.