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PATH INTEGRALS FOR SOLVING SOME ELECTROMAGNETIC EDGE DIFFRACTION PROBLEMS*

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Path integrals were introduced independently by N. Wiener in 1924 and by R. P. Feynman in 1948. They have since been used extensively in quantum mechanics and studied rigorously as a new branch of mathematics. An introduction to path integrals can be found in [1] and in the first three chapters of [2].

In the present paper, the path integral technique is applied to the calculation of the time-harmonic electromagnetic field on the incident shadow boundary in several edge-diffraction problems involving parallel half planes. The solutions thus obtained are asymptotically valid for a large wave number k (2π/wavelength) and include only the dominant terms of order k^0 relative to the incident field. Those solutions do agree, when available, with the exact asymptotic ones derived by analytical techniques [3]-[7] and/or by a uniform asymptotic theory of ray-optic techniques [8]. The latter techniques involve complicated mathematical manipulations. In contrast, the path integrals yield the asymptotic solutions in a few elementary steps.

All of the edge-diffraction problems treated in this paper depend only on two spatial variables (x,z). Consequently, the complete electromagnetic fields are derivable from a scalar u(x,z), which satisfies a scalar wave equation. Following Buslaev [9], we convert the problem of solving u to that of solving K(x,z,t), which satisfies a heat equation. The solution of u is related to that of K as follows

$$u(x,z) = \int_0^\infty K(x,z,t)\, e^{k^2 t}\, dt \quad , \quad \frac{\pi}{4} < \text{Arg } k < \frac{3\pi}{4} \quad . \tag{1}$$

Next, the heat equation for K together with appropriate boundary and initial conditions is approximately solved by a path integral over the Wiener measure.

As an example, consider the problem of two unstaggered half planes illuminated by an incident field u^i from a line source at Q (Figure 1). Using the path integral, we have determined asymptotically the total field u at point C, which is located on the incident shadow boundary, namely,

$$u(C) \sim u^i(C)\left\{\frac{1}{4} + \frac{1}{2\pi}\tan^{-1}\sqrt{\frac{ac}{b(a+c)}} + \frac{1}{2\pi}\sqrt{\frac{b(a+c)}{ac}}\sum_{p=1}^{\infty}\left(\frac{1}{\sqrt{2p}} - \frac{1}{\sqrt{2p+1}}\right) \cdot e^{i2pkb}\right\}, \quad k \to \infty,\ \frac{b}{a} \to 0,\ \frac{b}{c} \to 0 \quad . \tag{2}$$

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For the special case that (a/c) >> 1, (2) recovers the asymptotic expansion of the exact solution obtained by the Wiener-Hopf technique (see Eq. (11.68) or (A.6) in [8]). As another example, let the two half planes in Figure 1 be replaced by n staggered half planes as shown in Figure 2. For n = 3 (triple half planes), the asymptotic solution of the total field at C is found by the path integral, and the final result reads

$$u(C) \sim u^{i}(C)\left\{\frac{1}{8}+\frac{1}{4\pi}\tan^{-1}\left[\frac{ac}{2b(a+2b+c)}\right]^{1/2}+\frac{1}{4\pi}\tan^{-1}\left[\frac{(a+b)c}{b(a+2b+c)}\right]^{1/2}+\frac{1}{4\pi}\tan^{-1}\left[\frac{a(b+c)}{b(a+2b+c)}\right]^{1/2}\right\}, \quad k\to\infty . \qquad (3)$$

To our knowledge, the solution in (3) is new.

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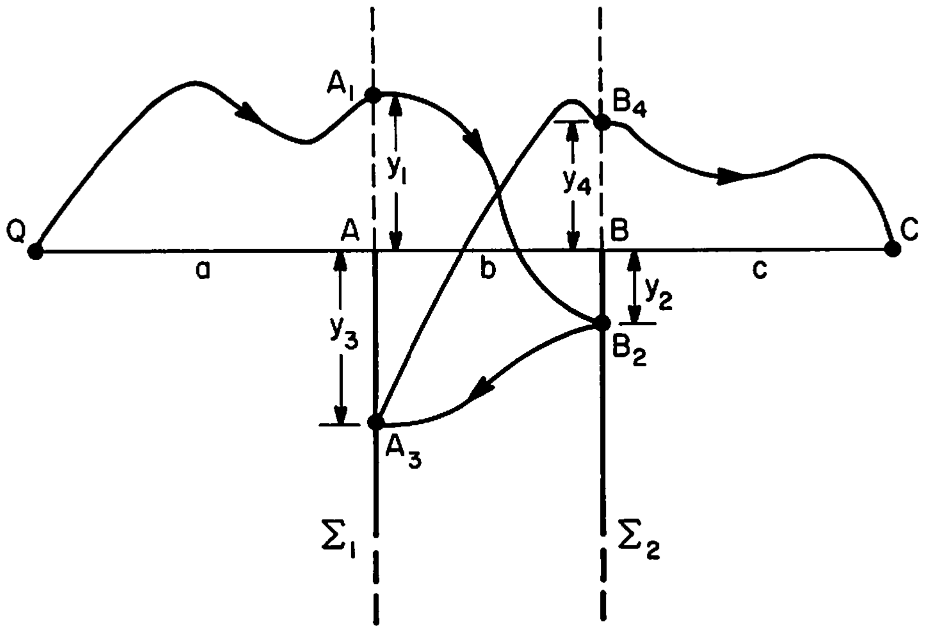

Figure 1. Two unstaggered half planes Σ_1 and Σ_2 illuminated by the incident field from a line source at Q.

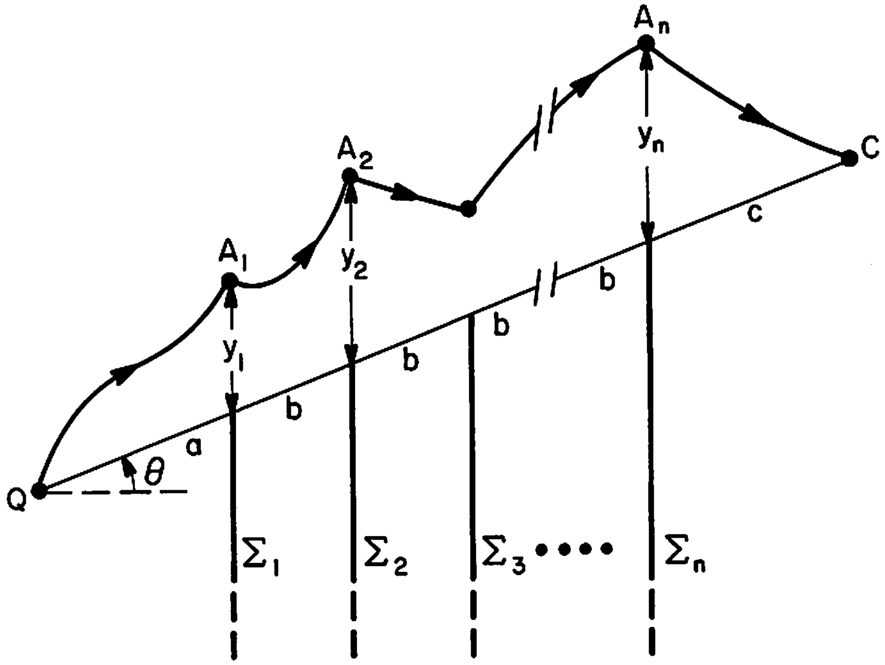

Figure 2. n equally spaced staggered half planes illuminated by the incident field from a line source at Q.