Matching Limits for Single-Band Mobile Phone Antennas

[#]Yiqun Li ^{1,2,3}, Benoît Derat ¹, Daniel Pasquet ² and Jean-Charles Bolomey ³
¹SAGEM Communication, Mobile Phones R&D Department
2 rue du Petit Albi, 95801, Cergy-Pontoise, France, yiqun.li@sagem.com, benoit.derat@sagem.com
²ENSEA, Circuits, Instrumentation and Electronic Modeling Group
6 avenue du Ponceau, 95014, Cergy, France, pasquet@ensea.fr
³SUPELEC, Electromagnetics Research Department
3 rue Joliot-Curie, 91192, Gif-sur-Yvette, France, bolomey@supelec.fr

1. Introduction

Many researches on the limit of broadband impedance matching have been conducted, since the early work from Bode [1], Fano [2] and Youla [3], more recently extended by Carlin et al [4]. This paper applies the theory of Youla and Carlin et al. to a circuit model of a single-band PIFA (Planar Inverted-F Antenna) mounted on a chassis. Then, based on this theory, analytical limits of the minimum amplitude of the reflection coefficient (denoted S_{11} instead of $|S_{11}|$, for the sake of simplicity) in a given band and of the maximum relative bandwidth (*BW*) for a maximum authorized S_{11} , are computed in this circuit case. The obtained results are applied numerically to two different single-band mobile phone models at GSM 900 and DCS 1800. A good agreement is obtained between these limits and the lowest S_{11} or largest *BW* for the antenna system with a highorder matching network, optimized thanks to a least-square algorithm.

2. Principle of Youla-Carlin matching theory for an arbitrary load

Bode first showed a broadband impedance matching limit when the load consists of a resistance and a capacitor in parallel [1]. Then Fano [2], Youla [3] and Carlin et al. [4] extended the work on the feasibility of matching an arbitrary load, so as to obtain a target reflection coefficient as a function of frequency. The theory developed by Youla and Carlin can be briefly summarized as follows. Once the load is given, the zero(s) of transmission in the right half-plane of complex frequency can be calculated (the zero transmissions are defined as solutions of $S_{12} \times S_{21}=0$ [4]) and divided into several classes. After that, Taylor expansions of two expressions, one depending only on the load impedance and the other on both the S_{11} and BW, are computed around the obtained zeros. Then, depending on the different classes of these zeros, one or more constraints on the calculated Taylor coefficients have to be satisfied. If all these constraints can be satisfied simultaneously, the target reflection coefficient over the considered band can be realized by adding a matching circuit. Otherwise, the target $S_{11} - BW$ is not achievable. In general, when fixing either maximum S_{11} not to overcome or a maximum BW to cover, the largest achievable BW or respectively lowest S_{11} are unknown quantities. In this case, the previous constraints can be used to find the best trade-off (limits) between these two parameters.

3. Simple circuit model for a single-band PIFA-chassis combination

In order to apply the theory of Youla-Carlin to a single-band PIFA mounted on a chassis, the antenna impedance has to be either modelled by a lumped element circuit or more generally written in an analytical form (as a function of frequency). To this aim, Boyle's simplified circuit model is used here [5], as shown in Fig 1 (a). This model consists of a series RLC circuit in parallel with an inductor (note that the circuit shown in Fig 1 (b) could be used as well). Fig 1 (d) shows that the impedance at the input port of the antenna system represented in Fig. 1 (c), obtained with CST Microwave Studio (MWS), is effectively well-described by the chosen circuit model. A least-square

based optimization is used to find the values of the elements of the circuit in Fig 1 (a), allowing the best fitting to the antenna impedance in the GSM band. The quality factors are calculated for the PIFA – chassis combination and the circuit model. The quality factor of the numerical model in Fig 1 (c) is much lower than the quality factor of the circuit model outside the GSM band. An underestimate of the obtained limit will hence be expected. The impact of this out-of-band discrepancy on the derived limits will be discussed later in Section 5.

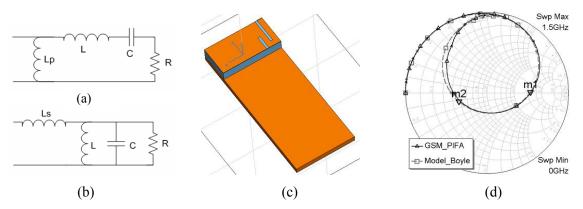


Figure 1 : (a) Boyle's simplified model for modelling a chassis-mounted single-band PIFA [5]. (b) Another circuit model. (c) Numerical model of a GSM chassis-mounted PIFA. (d) Impedance locii for the model of Fig. 1 (c) and the equivalent circuit of Fig. 1 (a), with least-square optimized component values. m1 at 880 MHz and m2 at 960 MHz indicate the GSM band.

4. Matching limits for the simple single-band PIFA-chassis circuit model

The target reflection coefficient, to be obtained by adding a matching circuit, is set to be less than S_{II} from ω to $n\omega$ for $n \ge 1$, as in [6]. Applying the theory of Youla-Carlin to the antenna model of Fig. 1 (a), the following system of equation-inequation is derived (see Appendix for more details):

$$\begin{cases} \frac{\ln(S_{11})}{\pi} \left(1 - \frac{1}{n}\right) = -RC\omega \\ \frac{\ln(S_{11})}{\pi} \left(1 - \frac{1}{n^3}\right) \ge -\left[\left(3RC^2L + 3RC^2L_p - R^3C^3\right)\omega^3\right] \end{cases}$$
(1)

Two different limits can be mathematically derived, by transforming the expressions above: first, eq. (2) shows the minimum S_{11} which can be achieved when BW is fixed; secondly, eq. (3) shows the maximum BW, which can be achieved when S_{11} is fixed:

$$\begin{cases} \ln(S_{11}) = -\frac{nRC\omega\pi}{n-1} \\ \ln(S_{11}) \ge -\frac{n^3(3RC^2L + 3RC^2L_p - R^3C^3)\omega^3\pi}{n^3 - 1} \\ n = \frac{\ln(S_{11})}{\ln(S_{11}) + RC\omega\pi} \\ \frac{1}{n^3} \ge 1 + \frac{(3RC^2L + 3RC^2L_p - R^3C^3)\omega^3\pi}{\ln(S_{11})} \end{cases}$$
(2)

5. Numerical application

To verify and illustrate the results obtained in Section 4, the limit is calculated numerically by substituting the values of the elements in the circuit model, computed by least-square fitting with the actual impedance of the antenna system. For the model of Fig. 1 (c), the obtained limits are the following: if *n* is fixed to 1.091 (=0.96/0.88 in the GSM band), the minimum achievable S_{11} over the whole band is -23.12dB; if S_{11} is fixed to be less than -6dB over the whole band, then the maximal achievable *n* is 1.47, which means a -6dB bandwidth from 880 MHz to 1295 MHz. Another classical single-band square PIFA-chassis combination at DCS (1710 – 1880 MHz) has also been studied, showing similar trends. Indeed, for a fixed *n* of 1.099 (=1.88/1.71 in the DCS band), the minimal S_{11} has been found to be -22.28dB; if S_{11} is chosen to be less than -6dB over the whole band, then the maximal achievable *n* is 1.503, meaning a -6dB bandwidth from 1710 MHz to 2570 MHz. At this step, it is worth noting that it can be expected to obtain a tri-band operation (DCS / PCS / UMTS) from a single-band system, by only adding an appropriate matching circuit.

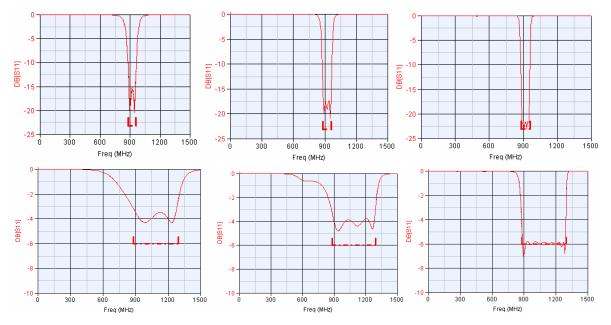


Figure 2 : Evolution of S_{II} (dB) for the circuit model of Fig. 1 (a), with a matching circuit consisting of 2, 3 or 10 low-pass filters (respectively columns 1, 2 or 3), optimized in the least-square sense to approach the limits of lowest S_{II} in the GSM band (line 1) or largest -6 dB bandwidth (line 2), represented in dashed lines.

Since the target S_{II} as a function of frequency is assumed to be rectangular, it can be achieved only by adding a lossless matching circuit with an infinite number of elements between the generator and the load. This is clearly observed in Fig. 2, which represents S_{II} , with a matching circuit consisting of 2, 3 or 10 low-pass filters, optimized in the least-square sense to reach either the best matching in the GSM band (optimization goal: $S_{II} < -23.12$ dB from 880 MHz to 960 MHz) or the largest – 6 dB bandwidth (optimization goal: $S_{II} < -6$ dB, from 880 MHz to 1295 MHz), as respectively defined by eq. 2 and 3. Since the circuit model impedance fits very well with the antenna impedance in the GSM band (see Fig. 1 (d)), there is a little difference between the limit obtained from eq. (2), and the amplitude of the reflection coefficient with the optimized 10 low-pass filters matching circuit. However, a discrepancy is observed for the largest achievable bandwidth with respect to eq. (3). Indeed, the -6 dB bandwidth *BW* of the chassis-PIFA of Fig. 1 (c) can be extended up to 1325 MHz, which is greater than what was expected due to the fact that the quality factor of the antenna is smaller than the one of the circuit model. On the contrary, if the quality factor of the antenna is higher than the one of the circuit model, the limit of eq. (3) would be an over-estimation with respect to what can actually be obtained.

6. Conclusion and perspectives

In this paper, the theory of Youla and Carlin has been applied to a simple circuit, modelling a single-band GSM or DCS PIFA-chassis combination. From these results, two limits have been deduced analytically: the minimal achievable amplitude of S_{11} over a given band, and the maximal bandwidth for a given S_{11} of the considered antenna system with matching circuit. The importance of finely modelling the antenna has been shown (especially quality factor), and the circuit model has been observed to allow a good evaluation of the limit for minimum S_{11} , but a less accurate estimation for the maximal *BW*. This discrepancy caused by a more approximate fitting of the impedance (or quality factor) out of the considered band could disappear if a more complex circuit were used to model the antenna. Note also that, despite of this small inaccuracy, the obtained results show interesting perspectives in making a single-band radiating system, becoming multi / wide – band.

Appendix

The key expressions are as follows. Notations used are the same as in [4].

$$z(s) = \frac{L_p L C s^3 + L_p R C s^2 + L_p s}{L C s^2 + L_p C s^2 + R C s + 1}, b = \pm \frac{d(-s)}{d(s)} = \frac{L C s^2 + L_p C s^2 - R C s + 1}{L C s^2 + L_p C s^2 + R C s + 1}$$
$$r = \frac{z(s) + z(-s)}{2} = \frac{L_p^2 R C^2 s^4}{(L C s^2 + L_p C s^2 + R C s + 1)(L C s^2 + L_p C s^2 - R C s + 1)}, F = \frac{4rb}{(z+1)^2}$$

There is a double zero of transmission at the origin, while $z(s=0) \neq \infty$. Therefore, these two zeros of transmission belong to "Class B" [4]. The coefficients of the Taylor expansion of $\ln(b)$, $\ln(S)$, r (being respectively the β_i , \sum_i , and ρ_i) at zero are shown below:

$$\beta_0 = \sum_0 = 0, \ \beta_1 = -2RC, \ \sum_1 = \frac{2\ln(S_{11})}{\pi\omega} \left(1 - \frac{1}{n}\right), \ \beta_2 = \sum_2 = 0$$

$$\beta_3 = 2RC^2L + 2RC^2L_p - \frac{2}{3}R^3C^3, \ \sum_3 = -\frac{2}{3}\frac{\ln(S_{11})}{\pi\omega^3} \left(1 - \frac{1}{n^3}\right), \ \rho_4 = L_p^{-2}RC^2$$

The constraints to be satisfied are: $\beta_0 = \sum_0$, $\beta_1 = \sum_1$, $\beta_2 = \sum_2$, $(\beta_3 - \sum_3)/\rho_4 \ge 0$. From these constraints, we end up at eq. (1). If the two expressions of eq. (1) cannot be satisfied simultaneously, an allpass factor $\eta = \prod \frac{s - \mu_i}{s + \mu_i}$ can be introduced, where μ_i is a zero of transmission in the right half-plane.

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