COUPLING ANALYSIS OF NONUNIFORM TRANSMISSION LINES

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1 Introduction

Knowledge of the currents and voltages induced at terminal loads of transmission lines excited by external electromagnetic waves is of fundamental importance in electromagnetic compatibility (EMC). The excitation of uniform transmission lines(UTLs), has been widely covered in the literature (e.g. [1]-[3] and ch. 7 in [4]). Relatively fewer papers consider coupling of external waves to nonuniform transmission lines(NTLs)[5]. However, there are many occasions when the transmission line is not uniform/parallel to the ground plane, e.g. connection of indoor wires with power lines, wiring of electronic equipments, etc. Even a line which consists of straight conductors above a ground plane can be nonuniform if the line axis itself or the ground plane has an inhomogenous structure. In any case if the position of the conductor in the cross-sectional plane varies along the longitudinal axis(X-axis in Fig. 1(a)), the per-unit-length(p.u.l.) line parameters are no longer constants along the X-axis. This causes the transmission and coupling parameters of the line to be functions of x.

The objective of this study is to gain further insight into the coupling mechanism to a NTL above a conducting ground plane. The NTL excited by an external field is approximated by a cascaded series of UTLs. To confirm the validity of this approach, both laboratory experiments and a completely different approach that employs the method of moment(MoM) to solve the electromagnetic problem of a wire structure immersed in an external field, are considered.

2 Theory

The problem under consideration is shown in Fig. 1(a), where a NTL of length L(along the X-axis) and radius a is suspended over a conducting ground (X-Z plane). The line is assumed to be externally excited by a uniform plane wave, having angles of incidence θ and ϕ , of time-dependence exp($j\omega t$), and thus induced current flows in the line. For convenience, this field can be decomposed into parallel (θ -polar.) and perpendicular (ϕ -polar.) polarizations[3]. We are interested in calculating the load currents flowing in the impedances Z_1 and Z_2 . Under the quasi-TEM approximation, the telegrapher's equations for the line current, $I(x,\omega)$, and line voltage, $V(x,\omega)$ are expressed as [6]

$$-\frac{d}{dx}V(x,\omega) = Z(x,\omega)I(x,\omega) + V_f(x,\omega)$$
 (1)

$$-\frac{d}{dx}V(x,\omega) = Z(x,\omega)I(x,\omega) + V_f(x,\omega)$$

$$-\frac{d}{dx}I(x,\omega) = Y(x,\omega)V(x,\omega) + I_f(x,\omega)$$
(2)

subject to terminal (or boundary) conditions $V(0,\omega) = -Z_1I(0,\omega)$ and $V(L,\omega) = -Z_1I(0,\omega)$ $Z_2I(L,\omega)$ at the loads x=0 and x=L. The parameters $Z(x,\omega)$ and $Y(x,\omega)$ are the p.u.l. inductive reactance and capacitive admittance of the line shown in Fig. 1(b), which depend on the longitudinal variable x. The forcing terms, $V_f(x,\omega)$ and $I_f(x,\omega)$ represent, respectively, the equivalent distributed voltage and current sources along the line (Fig. 1(b)), which are derived from Maxwell's field equations [1]. These

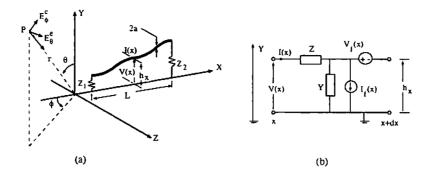


Figure 1: A nonuniform transmission line excited by an external electromagnetic wave: (a) geometry, and (b) equivalent circuit of an infinitesimal section

terms, expressible in terms of E_{θ}^{e} and E_{ϕ}^{e} in Fig. 1(a), are computed by summing the directly incident waves and the reflected fields from the ground plane[3].

3 Equivalent Representation of NTL with Distributed Sources

For NTLs, Eqs. (1) and (2) almost cannot be analytically solved. In this study, we model the NTL of Fig. 1(a) by a cascade connection of N sections(see Fig. 2(a)) of lengths $\Delta x_k (k = 1, 2, ..., N)$, each characterized by a different characteristic impedance and chain parameter matrix $F_k(\Delta x_k)$. If N is large enough, each section can be approximated to be uniform and the p.u.l. parameters are constant within every section and the chain parameter matrix, $F_k(\Delta x_k)$, can be derived analytically:

$$F_k(\Delta x_k) = \begin{bmatrix} \cosh \xi_k & -Z_0^k \sinh \xi_k \\ -\sinh \xi_k / Z_0^k & \cosh \xi_k \end{bmatrix} , \text{ with } \xi_k = \gamma_k \Delta x_k$$
 (3)

where $\gamma_k (= \sqrt{Z_k Y_k})$ and $Z_0^k (= \sqrt{Z_k / Y_k})$ are the complex propagation constant and characteristic impedance of the kth section, respectively.

A solution to Eqs. (1) and (2) can now be obtained by the use of state variables through the following expression[3]:

$$\begin{bmatrix} V(\Delta x_k) \\ I(\Delta x_k) \end{bmatrix} = F_k(\Delta x_k) \begin{bmatrix} V(\Delta x_{k-1}) \\ I(\Delta x_{k-1}) \end{bmatrix} + \begin{bmatrix} v_k(\Delta x_k) \\ i_k(\Delta x_k) \end{bmatrix}$$
(4)

where $V(\Delta x_k)$ and $I(\Delta x_k)$ denote the voltage and current at the kth segmenting point, respectively. Similarly, $v_k(\Delta x_k)$ and $i_k(\Delta x_k)$ represent, respectively, the contribution of the external wave to NTL for the kth segment in terms of ideal voltage and current sources ([3] eq. 17).

From Eq. (4), since the output variables of the kth section are readily related to the input variables defined for the next section, the expression for the terminal voltages and currents of the entire line, interpreted as a cascade of N segments, can be easily obtained. The induced currents at x=0 and x=L can be put into a compact form by transferring the sources to one end of the line(see Fig. 2(b)). The resultant expression for induced current in Z_1 becomes[3]:

$$I(0) = \frac{-\tilde{\mathbf{v}}_L + Z_2\tilde{\mathbf{i}}_L}{\Delta(L)} \tag{5}$$

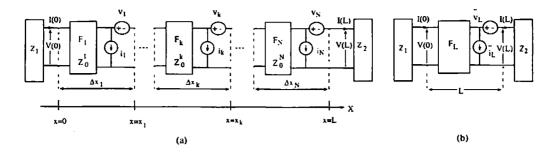


Figure 2: Piecewise uniform representation of nonuniform transmission: (a) cascaded-form, and (b) equivalent circuit for the cascade-model

where $\Delta(L) = (Z_1D^t + Z_2A^t) + (B^t + Z_1Z_2C^t)$ with A^t, B^t, C^t and D^t being the elements of the overall chain parameter matrix, F_L , of the entire line, and

$$\begin{bmatrix} \tilde{\mathbf{v}}_L \\ \tilde{\mathbf{i}}_L \end{bmatrix} = \begin{bmatrix} v_N \\ i_N \end{bmatrix} + F_N \begin{bmatrix} v_{N-1} \\ i_{N-1} \end{bmatrix} + \dots + F_N F_{N-1} \dots F_2 \begin{bmatrix} v_1 \\ i_1 \end{bmatrix}$$
 (6)

where $\hat{\mathbf{v}}_L, \hat{\mathbf{i}}_L$ are overall equivalent representations of sources due to all sections. The schematic form of the equivalent circuit of field-to-NTL coupling is shown in Fig. 2(b). If the lengths of the sections are taken to be identical, then the matrix products appearing in Eq. (6) can be computed quite efficiently (F_L becomes just the Nth power of the chain parameter matrix of each section) using, for example, the Cayley-Hamilton theorem for powers of a matrix([4] ch. 4).

4 Comparison between numerical and experimental results

To verify the predicted induced currents on a NTL, the numerical results from the evaluation of Eq. (5) are compared to MoM and laboratory experiments for the configuration described in Fig 3(a). The experiment was carried out by using the same setup as in [3, 5]. Parameters used here are as follows: L = 20 cm, $f = 1.5 \text{GHz} (L = \lambda)$. $Z_1 = Z_2 = 50\Omega$, and a = 0.25 mm. The line is situated above a perfect ground plane and a 1V/m plane wave is impinging onto it at a fixed angle of incidence of 45°. Figures 3(b) and (c) show the comparison between the computed values and the measured values for θ - and ϕ -polar., respectively. The corresponding results for the case of a uniform transmission line of length L = 20 cm suspended at a height h = 3 mm over a perfect ground are given in Fig. (d) and (e). (Other results not shown for reasons of space). As can be concluded from these figures, our results are in very good agreement with those of MoM and measurement.

5 Conclusions

Analysis of field-to-nonuniform transmision line coupling, under quasi-TEM assumption, is carried out by a cascaded series of uniform transmision lines. This allows one to derive analytical expressions for the chain parameter matrix of the line from the solutions of modified telegrapher's equations in the frequency-domain.

Theoretical expectations are in good agreement with the laboratory experiments. A further validation of this methodology was done against an alternative and completely independent formulation that employs MoM. This implies that the computational method presented here is applicable for a variety of NTLs.

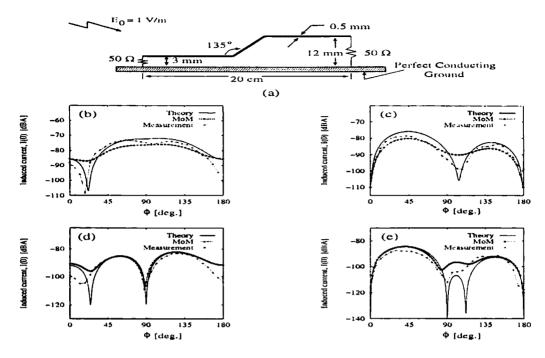


Figure 3: A nonuniform transmission line excited by a plane electromagnetic wave: (a) geometry, (b) and (c) induced currents at the beginning of the line for parallel, and perpendicular polarizations, respectively. (d) and (e) show the corresponding results for the case of uniform transmission line.

Finally, by making segment's length electrically small, i.e. $\Delta x \ll \lambda$, a very simple but approximate model can be further developed. In this model the line is approximated with the lumped-circuit iterative structures such as lumped-pi or lumped-T structures containing lumped sources representing the effect of external wave. It is very easy to solve the resulting structure in both frequency- and time-domain using a CAD circuit simulator, such as SPICE, which seems to be a powerful tool to analyze transients produced by EMP, lightning, and ESD. We are currently working towards the extension of the model to the general case of lossy multiconductor NTLs excited by an external wave.

References

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