B—9—2 METHOD OF STATIONARY PHASE FOR CALCULATION OF RADAR CROSS SECTIONS OF PERIODIC DEFORMED CYLINDER

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1. Introduction

Various numerical methods have been presented to solve scattering problems for somewhat complicated bodies [1],[2]. The fundamental limitation is the size of scatterer that may be handled using numerical methods. The physical optics approximation is useful for analyzing high frequency scattering problems [3]. Here we solve scattering problems for non-convex bodies within the physical optics approximation. The reduced phase integral can be evaluated by the method of stationary phase [3], [4]. In order to calculate the integral more precisely, we introduce the complex stationary points as well as the real ones: The solutions by the method of stationary phase well coincide with those by the numerical integration method. Moreover, the physical optics solutions for the radar cross sections are in good agreement with the numerically rigorous ones by the mode-matching method [5],[6].

2. Formulation of problem and complex stationary points

Consider a perfectly conducting, periodic deformed cylinder whose surface is described by

$$r(\phi) = a\,(\,1 - \delta \cos(\tau\phi)),\ a > 0,\ 1 > \delta > 0 \tag{1}$$

where τ is the integer. An incident plane wave with unit amplitude is represented by

$$E_i(\rho,\theta) = \exp[-jk\rho\cos(\theta-\alpha)] \tag{2}$$

where k is the wavenumber, and α the incident angle (see Fig.1). Then the physical optics solutions for the far scattered field can be represented in the form, for the E-polarized field,

$$E_s(\rho,\theta) = (k/2\pi\rho)^{1/2}\exp[j(k\rho-\pi/4)]\int_{L_0} g(\phi)\exp(-jkf(\phi,\theta))d\phi \tag{3}$$

with

$$g(\phi) = r(\phi)\cos(\alpha-\eta(\phi))/\cos(\phi-\eta(\phi)),$$

$$f(\phi,\theta) = 2r(\phi)\cos(\phi-(\theta+\alpha)/2)\cos((\theta-\alpha)/2) \tag{4}$$

where L_0 denotes the illuminated portion of the scatterer and $\eta(\phi)$ designates the angle between the outward normal direction **n** on L_0 and the positive x axis (see Fig.1).

The diffraction integral of the form (3) can be evaluated by the method of stationary phase [4]. The main contribution for (3) arises from near the points $\bar{\phi}$'s which satisfy

$$[df(\phi,\theta)/d\phi]_{\phi=\bar{\phi}} = 0\ . \tag{5}$$

The solutions of (5) are called the stationary points. By appealing to

Fig.1, we have a relation such that

$$\eta(\phi) = \phi - \tan^{-1}(dr/rd\phi) \ . \qquad (6)$$

When we redefine $\eta(\phi)$ through (6), the relation

$$\eta(\bar{\phi}) = (\theta+\alpha)/2 \qquad (7)$$

holds for the complex solutions of (5) as well as the real ones. For the complex solutions, we impose the constraint on $f(\bar{\phi},\theta)$ such that

$$\mathrm{Im}[f(\bar{\phi},\theta)] \leq 0 \qquad (8)$$

where $\mathrm{Im}[\cdot]$ denotes the imaginary part of the quantity in parentheses. When the complex solutions of (5) fulfill (8), they are called the complex stationary points. In this paper, we evaluate the diffraction integral (3) by the method of stationary phase in which the complex stationary points are considered as well as the real ones.

3. Radar cross section for non-convex body

We calculate the radar cross section of the body of the form (1). The radar cross section is defined as follows:

$$\sigma(\alpha,\alpha) = \lim_{\rho \to \infty} 2\pi\rho |E_s(\rho,\alpha)|^2 .$$

The body (1) is non-convex for the parameters τ, δ which satisfy the condition as $1 > \delta > 1/(\tau^2+1)$. When the plane wave incidents on the locally concave part of the scatterer, in particular $\alpha=0$, the radar cross section of the body with $\tau=10$ and $\delta=0.04$ is given by, (see the Appendix),

$$\begin{aligned}\sigma(0,0)/2a = {} & 2.21 + 1.82 \sin(0.080ka) + \exp(-0.80ka) \\ & [3.33 \cos(0.552ka-0.574) + 1.75 \cos(0.472ka+0.996)] \\ & + \exp(-0.319ka)[4.44 \cos(1.50ka+0.122) \\ & + 2.33 \cos(1.44ka-1.69)] + 4.28 \exp(-0.399ka) \\ & \cdot\cos(0.968ka+0.697) + 1.61 \exp(-0.161ka) \\ & + 2.85 \exp(-0.638ka) \ . \end{aligned} \qquad (9)$$

The radar cross section without consideration of the complex stationary points is as follows:

$$\sigma(0,0)/2a = 2.21 + 1.82 \sin(0.080ka) \ . \qquad (10)$$

These results are illustrated in Fig.2 together with those by the direct numerical evaluation of (3) and the numerically rigorous solutions by the mode-matching method [6]. By considering the complex stationary points, the method of stationary phase can well estimate the diffraction integral for the non-convex body. The physical optics solutions are in good agreement with those by the mode-matching method in moderately high frequency regions. The second term of (9) and (10) can be explained by the interference phenomena among the scattered waves from the real stationary points on L_0.

4. Conclusion

Consideration of the complex stationary points permits us to evaluate the diffraction integral for the non-convex body by the method of stationary phase. The physical optics approximation is useful for handling scattering problems for non-convex bodies. The radar cross section of the non-convex body shows the essentially different feature from the convex body: The radar cross section varies with frequency in a complicated way. This, however, is interpreted by the interference of the scattered waves from each stationary points.

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Appendix

Let us calculate the complex stationary points as well as the real ones. In this example, we have an algebraic equation of the form

$$[5632\ \delta\cos^{10}\bar{\phi} - 11520\ \delta\cos^{8}\bar{\phi} + 7840\ \delta\cos^{6}\bar{\phi} - 2800\ \delta\cos^{4}\bar{\phi} + 150\ \delta\cos^{2}\bar{\phi} - \delta - 1]\sin\bar{\phi} = 0 \qquad (A.1)$$

The stationary points can be obtained by solving (A.1) numerically. For such case $\delta=0.04$, the physically reasonable solutions of (A.1), which satisfy (8), are given by

$$\sin\bar{\phi} = 0,\ \cos\bar{\phi} = 0.970;\ 0.750\exp(-j1.33);\ 0.353\exp(-j0.746) \qquad (A.2)$$

with the aid of the Bairstow method [7]. Considering the symmetry of the problem under consideration, from (A.2) we have seven stationary points as follows:

$$\bar{\phi} = 0;\ \pm0.244;\ \pm(0.745+j0.146);\ \pm(1.316+j0.245)\ . \qquad (A.3)$$

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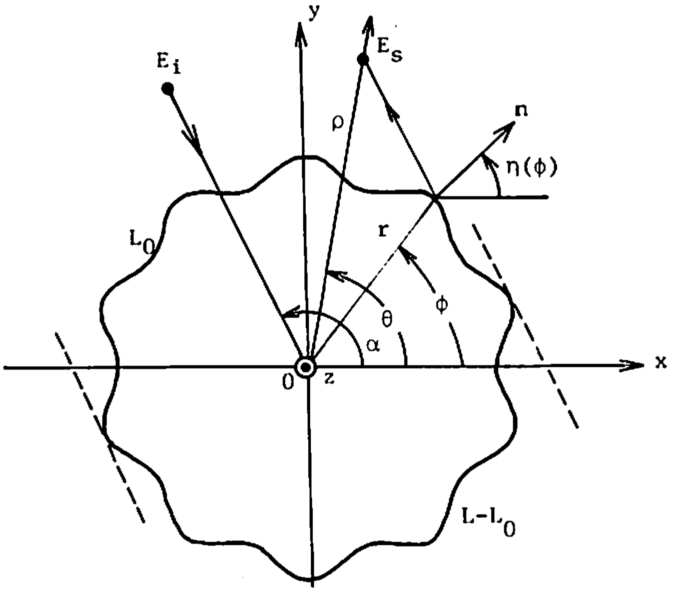

Fig.1-Coordinate system and a periodic deformed cylinder.

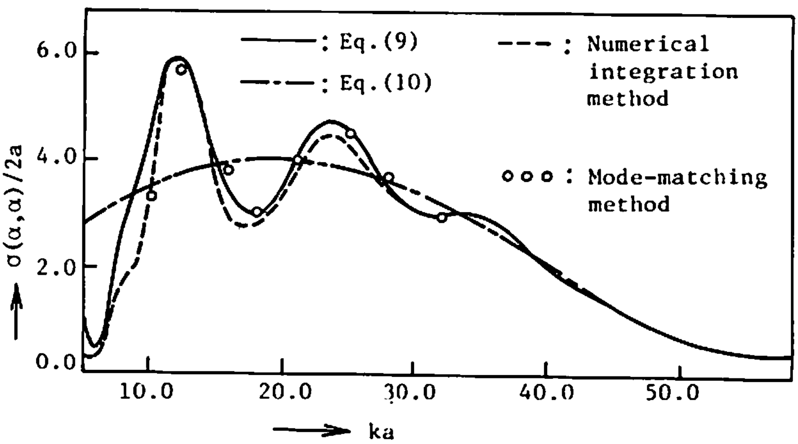

Fig.2-Frequency dependence of radar cross section (τ=10, ρ=0.04, α=0).