SCATTERING OF HERMITE-GAUSSIAN BEAMS FROM A SINUSOIDAL SURFACE

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Various analyses of the scattering of electromagnetic or acoustic waves from periodic surfaces have been achieved since the first work by Rayleigh. In most cases, as an incident wave, a plane wave has been treated. As far as we know, there seems to be only a few papers [2], [3] which have considered the case where the finite area of the scattering surface is illuminated by an incident wave such as a beam wave.

In our previous paper [3], we have briefly reported the scattering of a fundamental Gaussian beam from sinusoidal conducting surfaces. The purpose of this paper is to consider the more general case, i.e., the scattering of Hermite-Gaussian beams of an arbitrary order from a sinusoidal surface. The scattering problem like this seems to be not only an interesting subject in the field of electromagnetic wave theory but also an important one relating to the surface measurement or diagnostics by microwave, millimeter-wave, laser or ultrasonic beams.

The coordinate system of the problem is shown in Fig. 1. The scattering surface is assumed to be a perfect conductor and its height profile h(x) to be a sinusoid with period L and roughness amplitude d,

$$h(x) = -\frac{d}{2}\left[\,1 + \cos\frac{2\pi}{L}x\,\right]. \tag{1}$$

As an incident beam, we consider a two-dimensional Hermite-Gaussian beam of order m whose electric field is horizontally polarized. In a rectangular coordinate system (x', y', z') whose origin is located at $x=-x_0$ and $z=z_0$ and whose x' axis makes an angle θ_0 with the z axis, the field distribution of the beam waist located at x'=0 is expressed by

$$E^{i}_{ym}(x'=0,\ z') = H_m(\sqrt{2}z'/W_0)\exp(-z'^2/W_0^2) \qquad m=0,\ 1,\ 2\cdots \tag{2}$$

where H_m denotes the Hermite polynomial of order m and W_0 the beam radius or spot size. The time dependence $\exp(-i\omega t)$ is omitted here. In a rectangular coordinate system (x, y, z), the incident beam can be expressed by the plane wave spectrum representation as

$$E^i_{ym}(x, z) = \int_{-\infty}^{\infty} B_m(\alpha)\exp(ik\alpha x - ik\beta z)d\alpha \tag{3}$$

where the spectrum function $B_m(\alpha)$ is given by

$$B_m(\alpha) = \frac{kW_0}{2\sqrt{\pi}}(i)^m H_m[-\frac{kW_0}{2}(\alpha\cos\theta_0 - \beta\sin\theta_0)]$$
$$\exp[-\frac{(kW_0)^2}{4}(\alpha\cos\theta_0 - \beta\sin\theta_0)^2](\cos\theta_0 + \frac{\alpha}{\beta}\sin\theta_0) \tag{4}$$

and $\alpha^2+\beta^2=1$. It should be noted that the term $B_m(\alpha)\exp(ik\alpha x - ik\beta z)$ in (3) represents an elementary plane wave with the angle of incidence $\theta_i=\arcsin\alpha$. For such an elementary plane-wave incidence, the scattered waves are represented by the space harmonics

$$e^s_{ym}(x, z) = \sum_{n=-\infty}^{\infty} B_m(\alpha)R_n(\alpha)\exp(ik\alpha_n x + ik\beta_n z) \tag{5}$$

where $R_n(\alpha)$ is the scattering coefficient of the nth-order space harmonic for the case where a unit amplitude plane wave is incident on the sinusoidal surface. Considering the height profile in (1) together with the periodicity of the boundary value problem, the following grating equation is obtained:

$$\alpha_n = \alpha_0 + n\Lambda, \quad \beta_n = \sqrt{1-\alpha_n^2} \qquad n=\pm1, \pm2\cdots \tag{6}$$

where $\Lambda=\lambda/L$, $I_m(\beta_n) \geq 0$ and λ is the wave length. The scattering coefficient R_n^m can be calculated by applying Green's theorem with the Dirichlet boundary condition on $h(x)$ [1], [2].

The scattered waves for the beam-wave incidence can be obtained by integrating (5) with respect to α, i.e.,

$$E^s_{ym}(x, z) = \sum_{n=-\infty}^{\infty} E_{ymn}(x, z) \tag{7}$$

where

$$E_{ymn}(x, z) = \int_{-\infty}^{\infty} B_m(\alpha)R_n(\alpha)\exp(ik\alpha_n x + ik\beta_n z)d\alpha. \tag{8}$$

If the observing point of the scattered fields is sufficiently far from the origin, i.e., kr>>1, the integral of (8) can be evaluated by the saddle point method. Then the far-zone scattered fields for all space harmonics are represented by

$$E^s_{ym}(r, \theta) = C(kr)\sum_{n=-\infty}^{\infty} \Psi_{mn}(\sin\theta - n\Lambda) \tag{9}$$

where

$$C(kr)=\sqrt{\frac{2}{kr}}\exp(ikr-i\pi/4) \tag{10}$$

$$\Psi_{mn}(\sin\theta-n\Lambda)=B_m(\sin\theta-n\Lambda)R_n(\sin\theta-n\Lambda)\cos\theta \tag{11}$$

and

$$R_n(\sin\theta-n\Lambda)=-\frac{\beta_0^{(n)}}{\beta_n^{(n)}}(i)^n\exp(i\tau_n^{+(n)})\left[J_n(\tau_n^{+(n)})+\sum_{\ell\neq 0}a_\ell J_{n-\ell}(\tau_n^{+(n)})\right] \tag{12}$$

$$\beta_0^{(n)}=\sqrt{1-(\alpha_0^{(n)})^2}, \qquad \alpha_0^{(n)}=\sin\theta-n\Lambda \tag{13a, b}$$

$$\tau_n^{+(n)}=\Delta(\beta_0^{(n)}+\cos\theta), \qquad \Delta=\pi d/\lambda. \tag{13c, d}$$

a_ℓ are the solutions of the linear equations

$$J_\nu(\tau_\nu^{-(n)})+\sum_{\ell\neq 0}a_\ell J_{\nu-\ell}(\tau_\nu^{-(n)})=0 \qquad \nu=\pm1,\ \pm2\cdots\pm\ell \tag{14}$$

where

$$\tau_\nu^{-(n)}=\Delta(\beta_0^{(n)}-\beta_\nu^{(n)}) \tag{15}$$

$$\beta_\nu^{(n)}=\sqrt{1-(\alpha_\nu^{(n)})^2}, \qquad \alpha_\nu^{(n)}=\alpha_0^{(n)}+\nu\Lambda. \tag{16a, b}$$

and J_ν is the Bessel function of order ν.

The angular distribution of the scattered power is defined by the following pattern function $G_m(\theta)$

$$G_m(\theta)=\left|\sum_{n=-\infty}^{\infty}\Psi_{mn}(\sin\theta-n\Lambda)\right|^2. \tag{17}$$

In order to verify the accuracy of the computed values of $R_n(\alpha)$, we have used the same criteria as those in [2]. The number of space harmonics taken into account is large, so the errors for the energy conservation criterion become sufficiently small. An example of the calculated scattering pattern is shown in Fig. 2, which corresponds to the case where the first-order (m=1) Hermite-Gaussian beam is incident on the sinusoidal surface. In this figure we can observe a deep dip at a major peak located at the angle θ_n for each propagating space harmonic which will be produced if a plane-wave component propagating parallel to the beam axis is incident on the sinusoidal surface. It is obvious that this dip is caused by the zero on the beam axis in the amplitude distribution of the incident-beam field.

Especially such a tendency manifests itself strongly in the major peak at the specular angle θ=45°. On the contrary, there is no dip in the major peak at $\theta \simeq 80°$. This peak dose not correspond to any propagating space harmonic for the axial plane-wave component. However, we can confirm that this peak corresponds to the 1st-order propagating space harmonic for the elementary plane-wave component incident on the surface at the angle $\theta_i \simeq 40.7°$. As a result, it can be concluded that for the 1st-order Hermite-Gaussian beam incidence there appears a deep dip at a major peak corresponding to each propagating space harmonic which will be produced if the axial plane-wave component is incident on the surface.

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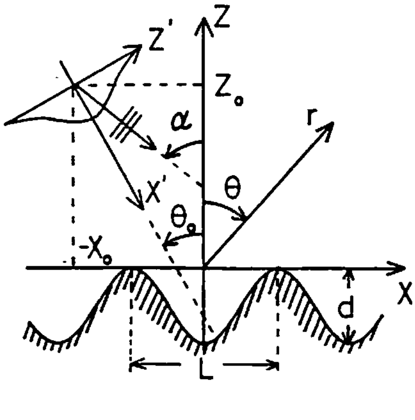

Fig. 1. Geometry of the problem

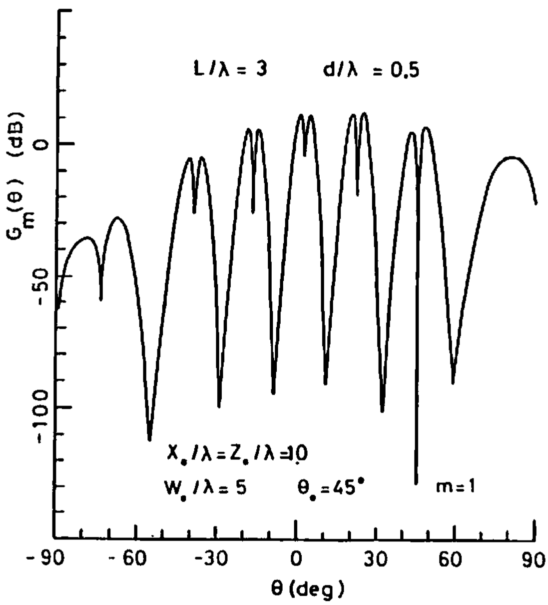

Fig. 2. An example of scattering pattern