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ON THE INTEGRATION IN THE METHOD OF EQUIVALENT EDGE CURRENTS

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Introduction

The Method of Equivalent Edge Currents (EEC) is now finding increasing applications to the diffraction and antenna engineering.[1] This paper deals with the integration of the equivalent currents along the edge of the scatterer which possesses axially symmetric geometry.

The radiation integral in EEC is characteristic in that the integrand has the form of the product of the function which oscillates very rapidly, and the function which contains the two-dimensional diffraction coefficients obtained easily for the vicinity of the stationary phase points. These condition suggest that this integration should be evaluated by the Method of Stationary Phase (SP), and this produces the same results as the Geometrical Theory of Diffraction (GTD).[2] This process is self-consistent, since EEC in itself comes from the optical understanding that the diffracted ray emanates from the edge of the scatterer. But there are two main limitations to the straightforward application of the SP. One is the case where not only the vicinity of the stationary phase points but also every part of the interval of the integration contribute to the fields, as in the case of the axial caustics. The other is the case where the value of the function in the integrand is small at the stationary phase points. For these cases, it is necessary to evaluate the integral more carefully.

But as mentioned before, it should be noted that the equivalent currents to be integrated can not generally be obtained for all the interval and various approximations have been proposed. To overcome the first limitation in the region near the axis, it is proposed to put the diffraction coefficients out of the integration assuming that they are constant along the edge and equal to the value on the axis. This method which is called the Axial Solution (AS) is physically reasonable and has been examined experimentally with a fine agreement. For the region which is not so near the axis, the Extended Axial Solution (EAS) has given a good approximation. In this solution the interval of the integral is separated appropriately into subsections and the diffraction coefficient at the nearest stationary phase point is used for integrating each subsection. For the second case, however, where the value of the function is small at the stationary phase points, the situation is more complicated and detailed study has not been done. One attempt was made by applying the EAS directly[3], but it has not examined analytically nor experimentally.

It is the purpose of this paper to point out that as the frequency becomes higher, the EAS produces unreasonable results from the optical point of view, in the case where the value of the function is small at the stationary phase points. Then a method of new type is proposed in place of the EAS and it is shown that the solution by this method approaches to that by the SP in the lateral region, while it approaches to the AS in the region near the axis. This means that fields in almost all the region can be obtained at once by this method, while the conventional methods are effective only in the relatively narrow regions.

Analysis

In the method of EEC, magnetic fields for example can be represented by the radiation integrals of the equivalent electric and magnetic currents as follows

$$\tilde{H}_\phi=\int_0^{2\pi} K(\phi')\cos(\phi-\phi')\, e^{j\nu\cos(\phi-\phi')}\, d\phi' - \eta\cos\theta\int_0^{2\pi} I(\phi')\sin(\phi-\phi')\, e^{j\nu\cos(\phi-\phi')}\, d\phi' \quad (1)$$

$$(\nu = ka\ \sin\theta)$$

$$\frac{I}{K}(\phi') = \frac{i}{k}(\phi',\theta)R^e_h(\rho_0,\theta_0,\phi') + \frac{i^s}{k^s}(\phi',\theta)\left[-\frac{\partial}{\partial\theta} R^e_h(\rho_0,\theta,\phi')\right]_{\theta=\theta_0} \quad (2)$$

where a is the radius of the circle on which the equivalent currents $I(\phi')$ and $K(\phi')$ are assumed, and other parameters are shown in Fig.1. $k(\phi',\theta)$, $i(\phi',\theta)$ are the corresponding two-dimensional diffraction coefficients while $k^s(\phi',\theta)$, $i^s(\phi',\theta)$ are the slope wave correction[3]. R^e and R^h are the radiation patterns of the source for E_ϕ and H_ϕ respectively. The integral to be focused on is simply expressed in the form of Eq.(3)

$$F=\int_0^{2\pi} i(x)\cdot f(x)\, e^{j\nu\cos x}\, dx \qquad \text{where } i(x) \text{ is known only for } x=0,\pi \quad (3)$$

To begin with, three types of the methods stated above are expressed together with the concrete results for $f_e(x)=\sin(\phi-x)\sin(x)$, as the typical example where the function vanishes at the stationary phase points, that is, $f(0)=f(\pi)=0$.

<u>The Method of Stationary Phase</u> (for $\nu \gg 1$)

$$F \simeq i(0)\left(\frac{2\pi}{\nu}\right)^{\frac{1}{2}} f(0)e^{j\nu-j\frac{\pi}{4}} + i(\pi)\left(\frac{2\pi}{\nu}\right)^{\frac{1}{2}} f(\pi)e^{-j\nu+j\frac{\pi}{4}} \quad (4)$$

<u>The Axial Solution</u> (for $\theta\simeq 0,\pi$)

$$F \simeq i(\phi)\Big|_{\theta=\binom{0}{\pi}}\int_0^{2\pi} f(x)\, e^{j\nu\cos x}\, dx \quad \text{where } i(\phi) \text{ is independent of } \phi \text{ for } \theta=0,\pi \quad (5)$$

<u>The Extended Axial Solution</u>

$$F \simeq i(0)\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} f(x)\, e^{j\nu\cos x}\, dx + i(\pi)\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} f(x)\, e^{j\nu\cos x}\, dx \quad (6)$$

Example

$$F \equiv \int_0^{2\pi} i(x)\cdot f_e(x)\, e^{j\nu\cos x}\, dx = \begin{cases} 0 & \text{by SP} \quad (7)\\ -\frac{\pi}{2}\cos\phi[J_0(\nu) + J_2(\nu)]\, i(\phi)\big|_{\theta=0,\pi} & \text{by AS} \quad (8)\\ -\frac{\pi}{2}\cos\phi[G_1(\nu)i(0) + G_2(\nu)i(\pi)] & \text{by EAS} \quad (9)\end{cases}$$

$$\text{where}\quad \frac{G_1}{G_2} = J_0(\nu) + J_2(\nu) \pm \frac{8}{\pi}\, j\sum_{\substack{1\\ n:\text{odd}}}^{\infty}\frac{n^2-2}{n(n^2-4)}\, J_n(\nu) \quad (10)$$

Here we pay attention to the solution (9)(10). It should be noted that the summation has appeared as a result of separating the integral at $x=\pm\pi/2$, in other words, making the artificial discontinuities for $i(x)$. Furthermore it is this term that plays an important role for large ν. This fact is proved by calculating the phase of G_1. As is shown in Fig.2, it is almost constant except for extremely small θ. This means physically that the main contribution comes from the vicinity of $x=\pm\pi/2$, the artificial end points. These results can also be expected mathematically by the asymptotic theory of the SP[4], which points out that the contribution from the end points are $O(\nu^{-1})$, while that from the vicinity of the end points are $O(\nu^{-1.5})$ in case of $f(0)=f(\pi)=0$. From the facts described above, we may conclude that the main contribution comes from the end points,so long as the artificial end points are made in the integration. But this conclusion contradicts the optical point of view.

All of these considerations indicate the necessity of the approximation which does not make any artificial end points. The method we propose sets the starting point on the AS. In the first place, the AS which consists of Bessel functions is separated into two components which consist of the first and second Hankel functions. The resulting components are understood to be the contributions from the corresponding stationary phase points. To finish the process, these are multiplied by the value of $i(x)$ at each stationary phase point. This method is expressed as follows. Suppose $f(x)$ is expanded in the finite Fourier series.

Axial Solution

$$\int_0^{2\pi} f(x)\, e^{j\nu\cos x}\, dx = 2\pi\sum_{n=0}^{m} j^n a_n J_n(\nu) \tag{11}$$

$$a_n = \frac{1}{2\pi}\int_0^{2\pi} \varepsilon_n f(x)\cos nx\, dx \tag{12}$$

$\varepsilon_n = 1$ for n=0, 2 for n≠0

$$\pi\sum_{n=0}^{m} j^n a_n H_n^1(\nu) + \pi\sum_{n=0}^{m} j^n a_n H_n^2(\nu) \tag{13}$$

from $x=0$ from $x=\pi$

$$F \equiv i(0)\pi\sum_{n=0}^{m} j^n a_n H_n^1(\nu) + i(\pi)\pi\sum_{n=0}^{m} j^n a_n H_n^2(\nu) \tag{14}$$

Separated Axial Solution (14)

It is interesting to examine the behavior of this solution for the limiting values of ν ($=ka\sin\theta$).

$\nu\gg m, \theta \neq 0,\pi$ The asymptotic form for $H_n(\nu)$ can be substituted, to obtain

$$F \to i(0)\left[f(0)+\frac{1}{2j\nu}f''(0)\right]\left(\frac{2\pi}{\nu}\right)^{\frac{1}{2}} e^{j\nu-j\frac{\pi}{4}} + i(\pi)\left[f(\pi)-\frac{1}{2j\nu}f''(\pi)\right]\left(\frac{2\pi}{\nu}\right)^{\frac{1}{2}} e^{-j\nu+j\frac{\pi}{4}} \tag{15}$$

which coincides exactly with the result by the SP of the second order. It is clear that the wave comes from the vicinity of the stationary phase points, even if $f(0)$ and $f(\pi)$ vanish.

$\nu \not\gg m, \theta \simeq 0,\pi$ It approaches to the AS by taking the limit for $i(x)$ first, provided $i(0)\simeq i(\pi)$. But if $i(0) \neq i(\pi)$ for $\theta\to 0,\pi$, it diverges, as is also the case with the AS when the axis happens to be in the direction of the specular reflection.

Finally in order to compare those methods, the radiation pattern of the paraboloidal reflector antenna is calculated by the new method [SAS] and three conventional methods [EAS],[SP] and [AS] as shown in Fig.3. In this calculation, an X-directed electric dipole is located at the origin. This means that R^h becomes zero and that the second term in Eq.(1) which has the integrand of the type discussed above, that is $f(x)=\sin(\phi-x)\cdot\sin(x)$, plays an important role for H_ϕ. Fig.3 shows that in the backlobe region, SAS and EAS coincide with AS, while in the lateral region they are quite different one another.

For $\theta \simeq 90°$, EAS seems to approach to the SAS and SP, but this is the consequence of the factor $\cos\theta$ in Eq.(1). On the other hand, SAS would approach to the SP in the lateral region even if it was not for the factor $\cos\theta$.

Conclusion

It has been indicated that the conventional Extended Axial Solution produces unreasonable results in the case where the function in the radiation integral vanishes at the stationary phase points. A solution of the new type named the Separated Axial Solution is proposed and its behavior is studied to get to know that it approaches to the Axial Solution in the axial region and to the solution by the Method of Stationary Phase of the second order in the lateral region. Though the superiority of this new method has not been examined experimentally yet, we have proved that the exact solution for the diffraction by a sphere, which is modified by the Watson transformation, has just the same form as Eq.(15)

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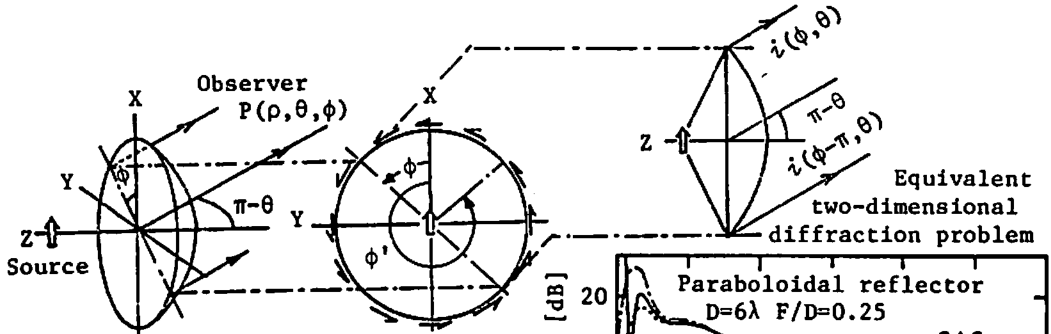

Fig.1 Coordinates used for the scatterer with axial symmetric geometry.

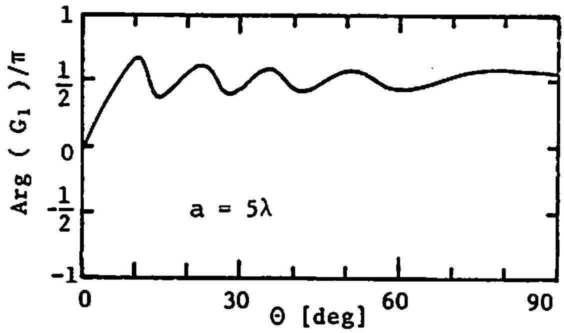

Fig.2 Variation of the phase of G_1 .

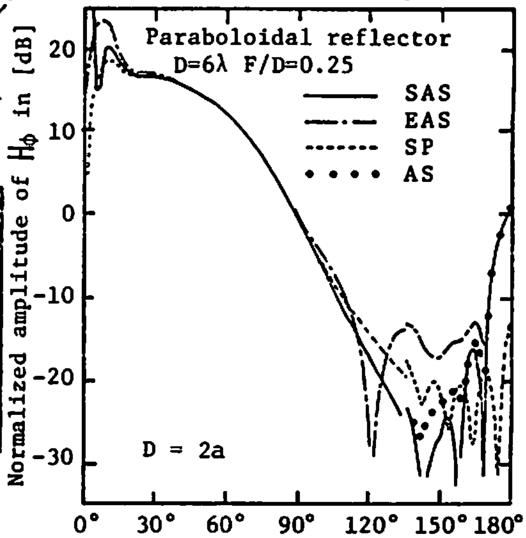

Fig.3 Comparison of the radiation pattern calculated by four methods.